श्रीवटेश्वराचार्यविरचितः गोलोपेतः

वटेश्वरसिद्धान्तः

VAṬEŚVARA-SIDDHĀNTA AND GOLA

OF VAṬEŚVARA

Critically Edited with

English Translation and Commentary

by

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PART II

ENGLISH TRANSLATION AND COMMENTARY

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FOREWORD

One of the main objectives of the National Commission for the Compilation of History of Sciences in India is to publish original texts in Sanskrit, Arabic, Persian etc. and their translation into English. Texts like *Āryabhaṭīya* with commentaries (3 vols.), *Śiṣyadhīvṛddhida* with commentary (2 vols.), *Rasārṇava-kalpa* and the *Śulbasūtras* of Baudhāyana, Āpastamba, Kātyāyana and Mānava are some of the prestigious publications of the Commission in fulfilment of the objectives. The present text, *Vaṭeśvara-siddhānta* by Vaṭeśvara, a very important work in astronomy written towards the beginning of the tenth century, is a new addition to this series of texts.

The *Vaṭeśvara-siddhānta* is the largest and most comprehensive work on Indian astronomy and throws full light on the various methods and processes employed by Indian astronomers up to the tenth century. Besides, it is sufficiently original and incorporates new methods and techniques devised by the author himself. It was studied as a standard text in astronomy during the tenth, eleventh and twelfth centuries in India. Some of the rules and examples of this work were adopted by the celebrated astronomers like Śrīpati and Bhāskara II. The works of Vaṭeśvara were available to the great Persian scholar Al-Bīrūnī who had referred to Vaṭeśvara and cited several of the rules in his own writings.

The work of editing the Sanskrit text of *Vaṭeśvara-siddhānta* and translating it into English was taken up by Dr. K. S. Shukla, retired Professor of Mathematics, Lucknow University, who had earlier edited and translated the *Āryabhaṭīya* for the Commission. Only two manuscripts of *Vaṭeśvara-siddhānta*, both full of errors and omissions, were available. These were utilized for the present edition. Dr. Shukla has rectified the entire text

filling up the gaps wherever they occurred, and has translated it adding explanatory and critical notes and comments where necessary. The text of the first five chapters of Vaṭeśvara's *Gola* ("Spherics") occurred in one of the manuscripts used. This has been appended to the text of the *Vaṭeśvara-siddhānta* and its translation is also given.

It is hoped that this publication will prove useful towards a better understanding of the development of astronomy in medieval times.

S. K. Mukherjee
Vice-Chairman
National Commission for the
Compilation of History of
Sciences in India

CONTENTS

TRANSLITERATION

VOWELS

Short:	अ	इ	उ	ऋ	लृ		
	a	i	u	ṛ	l'		
Long:	आ	ई	ऊ	ए	ओ	ऐ	औ
	ā	ī	ū	e	o	ai	au

Anusvāra : ं = ṁ

Visarga : : = ḥ

Non-aspirant : ऽ = '

CONSONANTS

Classified :	क्	ख्	ग्	घ्	ङ्			
	k	kh	g	gh	ṅ			
	च्	छ्	ज्	झ्	ञ्			
	c	ch	j	jh	ñ			
	ट्	ठ्	ड्	ढ्	ण्			
	ṭ	ṭh	ḍ	ḍh	ṇ			
	त्	थ्	द्	ध्	न्			
	t	th	d	dh	n			
	प्	फ्	ब्	भ्	म्			
	p	ph	b	bh	m			
Unclassified :	य्	र्	ल्	व्	श्	ष्	स्	ह्
	y	r	l	v	ś	ṣ	s	h
Compound :	क्ष्	त्र्	ज्ञ्					
	kṣ	tr	jñ					

ABBREVIATIONS

Ā	*Āryabhaṭīya* of Āryabhaṭa I (499 A. D.)
BṛSaṁ	*Bṛhat-saṁhitā* of Varāhamihira (d. 587 A. D.)
BrSpSi	*Brāhma-sphuṭa-siddhānta* of Brahmagupta (628 A. D.)
GK	*Gaṇita-kaumudī* of Nārāyaṇa (1356 A. D.)
GSS	*Gaṇita-sāra-saṅgraha* of Mahāvīra (c. 850 A. D.)
GLā	*Graha-lāghava* of Gaṇeśadaivajña (1520 A. D.)
IJHS	*Indian Journal of History of Science*
JC	*Jyotiś-candrārka* of Rudradeva Śarmā (1735 A. D.)
KKau	*Karaṇa-kaustubha* of Kṛṣṇa-daivajña (1653 A. D.)
KKu	*Karaṇa-kutūhala* of Bhāskara II (1150 A. D.)
KK	*Khaṇḍa-khādyaka* of Brahmagupta
KK (BC)	*Khaṇḍa-khādyaka*, ed. Bina Chatterjee
KPr	*Karaṇa-prakāśa* of Brahmadeva (1092 A. D.)
KR	*Karaṇa-ratna* of Deva (689 A. D.)
KT	*Karaṇa-tilaka* of Vijayanandī (966 A. D.)
L	*Līlāvatī* of Bhāskara II
LBh	*Laghu-Bhāskarīya* of Bhāskara I (629 A. D.)
LG	Lalla's *Gola*
LMā	*Laghu-mānasa* of Mañjula (Muñjāla)(932 A. D.)
MBh	*Mahā-Bhāskarīya* of Bhāskara I
MSi	*Mahā-siddhānta* of Āryabhaṭa II (c. 950 A. D.)
MuCi	*Muhūrta-cintāmaṇi* of Rāmadaivajña (1600 A. D.)
PSi	*Pañca-siddhāntikā* of Varāhamihira
RajT	*Raja-taraṅgiṇī* of Kalhaṇa (1148 A. D.)
SiDa	*Siddhānta-darpaṇa* of Chandra Shekhara Singh (1869 A. D.)
ŚiDVṛ	*Śiṣya-dhī-vṛddhida* of Lalla
SiSā	*Siddhānta-sārvabhauma* of Munīśvara (1646 A. D.)
SiŚe	*Siddhānta-śekhara* of Śrīpati (1039 A. D.)

SiŚi	*Siddhānta-śiromaṇi* of Bhāskara II
SiTi	*Siddhānta-tilaka* of Lalla
SiTV	*Siddhānta-tattva-viveka* of Kamalākara (1658 A. D.)
SK	*Sarvānanda-karaṇa* of Govinda Sadashiva Apte
SMT	*Sumati-mahā-tantra* of Sumati
SuSi	*Sundara-siddhānta* of Jñānarāja (1503 A. D.)
SūSi	*Sūrya-siddhānta*
TS	*Tantra-saṅgraha* of Nīlakaṇṭha (c. 1500 A. D.)
VāK	*Vākya-karaṇa*
VSi	*Vaṭeśvara-siddhānta* of Vaṭeśvara (904 A. D.)
VVSi	*Vṛddha-vasiṣṭha-siddhānta.*

INTRODUCTION

This volume, issued as Part II of "*Vaṭeśvara-siddhānta* and *Gola*", gives the English translation of the *Vaṭeśvara-siddhānta* and of the first five chapters of Vaṭeśvara's *Gola* along with explanatory and critical notes and comments etc.

VAṬEŚVARA

Astronomer Vaṭeśvara has been famous as a critic of Brahmagupta. Although his works were not available to earlier scholars, references to him and his works were found to occur in the writings of later writers. The earliest references are found in *Rasā'ilul'Bīrūnī*[1] and *Kitāb fi Taḥqīq mā li'l-Hind*[2] of the Persian scholar Al-Bīrūnī (b. 973 A.D.) and in the *Siddhānta-śekhara*[3] of the Hindu astronomer Śrīpati (A.D. 1039). Al-Bīrūnī has quoted some passages from the *Karaṇa-sāra*, another work of Vaṭeśvara which has not been discovered so far.[4] According to Al-Bīrūnī, Vitteśvara (=Vaṭeśvara) was the son of Mihidatta (=Mahadatta) and a resident of the city of Nāgarapura.[5] Śrīpati has mentioned the name of Vaṭeśvara amongst the first-rate astronomers of India—Āryabhaṭa I, Brahmagupta, Lalla, Sūrya and Dāmodara.[6] He has also utilized the *Vaṭeśvara-siddhānta* in writing his own *Siddhānta*, the *Siddhānta-śekhara*.

HIS DATE AND PLACE

In the *Vaṭeśvara-siddhānta*[7], Vaṭeśvara expressly states the year of his birth and his age at the time of composition of the *Vaṭeśvara-siddhānta*. He writes :

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1. See Mohammad Saffouri and Adnan Ifram, "Al-Bīrūnī on Transits", pp. 32, 142. "Al-Bīrūnī on Transits" is an English translation of the third treatise included in *Rasā'ilul'Bīrūnī* published by the Osmania Oriental Publications Bureau, Hyderabad-Deccan, in 1948.
2. See Al-Bīrūnī's *India*, (English translation of *Kitāb fi Taḥqiq mā li'l-Hind* by E. C. Sachau), Vol. I, pp. 156, 392.
3. xviii. 18.
4. See Al-Bīrūnī's *India*, Vol. I, pp. 317, 392; Vol. II, pp. 54, 60, 79; and "Al-Bīrūnī on Transits", p. 32. Also see Al-Bīrūnī's "Exhaustive Treatise on Shadows", ch. xxiii.
5. See Al-Bīrūnī's *India*, Vol. I, p. 156.
6. See *Siddhānta-śekhara*, xviii. 18.
7. Ch. I, sec. 1, vs. 21.

"When 802 years had elapsed since the commencement of the Śaka era, my birth took place; and when 24 years had passed since my birth, this (*Vaṭeśvara-*) *siddhānta* was written by me by the grace of the heavenly bodies."

Obviously, Vaṭeśvara was born in Śaka 802 or A.D. 880 and the *Vaṭeśvara-siddhānta* was written 24 years later in A. D. 904.

From a passage quoted by Al-Bīrūnī from Vaṭeśvara's *Karaṇa-sāra,*[1] we find that this work adopted the beginning of Śaka 821 as the starting point of calculation. This shows that the *Karaṇa-sāra* was written in Śaka 821 or A.D. 899, i.e., five years before the composition of the *Vaṭeśvara-siddhānta.*

In the opening verse of the *Vaṭeśvara-siddhānta,* Vaṭeśvara has called himself "son of Mahadatta." The colophons at the ends of the various chapters of the *Vaṭeśvara-siddhānta* go a step further and declare him as being "the son of Bhaṭṭa Mahadaṭṭa belonging to Ānandapura." This shows that Vaṭeśvara was the son of Bhaṭṭa Mahadatta and belonged to the place called Ānandapura.

Ānandapura has been identified by Sir Alexander Cunningham and Nundo Lal Dey with the town of Vaḍnagar in northern Gujarat situated to the south-east of Sidhpur (lat. 23°·45N, long. 72°·39E).[2] "Ānandapura or Vaḍnagara," writes Dey, "is also called Nāgara which is the original home of the Nāgara Brāhmaṇas of Gujarat. Kumārapāla surrounded it with a rampart. Bhadrabāhu Svāmī, the author of the *Kalpa-sūtra,* composed in A.D. 411, flourished at the court of Dhruvasena II, King of Gujarat, whose capital was at this place."[3] That Vaṭeśvara's Ānandapura was the

1. See Al-Bīrūnī's *India,* Vol. I, p. 392; vol. II, p. 54.

2. See Sir Alexander Cunnnigham, "The Ancient Geography of India," p. 416; Nundo Lal Dey, "The Geographical Dictionary of Ancient and Mediaeval India," p. 6.

On Ahmedabad-Delhi line of the Western Railway, at a distance of 43 miles from Ahmedabad, there is Mehsana railway station. On Mehsana-Taranga Hill line, at a distance of 21 miles from Mehsana, lies Vadnagar railway station. The station next to it is Sidhpur. This Vadnagar, which has been identified with our Ānandapura, is the famous seat of God Śiva, called Hāṭakeśvara, the tutelary deity of the Nāgara Brāhmaṇas who originally belonged to this place. (For further history and religious importance of this place, see *Kalyāṇa, Tīrthāṅka,* Year 31, No. 1, pp. 403-4)

There is another place called Barnagar on Ratlam-Indore line, but it is a totally different place and should not be confused with our Vadnagar.

3. See Nundo Lal Dey, *ibid.*

same place as Vaḍnagar or Nāgara is confirmed by the testimony of Al-Bīrūnī who has written that Vaṭeśvara belonged to the city of Nāgarapura. Nāgara and Nāgarapura are obviously one and the same.

Ānandapura seems to have been a great seat of Sanskrit learning. It was visited by the Chinese traveller Hiuen Tsiang. Āmarāja (c. A D.1200), who wrote a commentary on the *Khaṇḍa-khādyaka* of Brahmagupta, and his nephew Mahādeva (1264 A.D.), who wrote a commentary on the *Jyotiṣa-ratna-mālā* of Śrīpati, belonged to this place. According to both these writers, the equinoctial midday shadow at this place was 5 *aṅgulas* and 20 *vyaṅgulas* and the hypotenuse of the equinoctial midday shadow, 13 *aṅgulas* and 8 *vyaṅgulas*,[1] which shows that the latitude of Ānandapura was 24° north, approximately. The latitude of Vaḍnagar is also approximately the same.

The above identification of Vaṭeśvara's place viz. Ānandapura with Vaḍnagar in northern Gujarat is further confirmed by Vaṭeśvara's own reference to his place in the closing stanza of Section 9 of Chapter III of the *Vaṭeśvara-siddhānta.* In that stanza Vaṭeśvara refers to his local place and to Daśapura and says that at both these places the distance of the midday Sun from the Sun's rising-setting line (viz. the *dhṛti*), at summer solstice, amounts to :

$$\frac{R \times \text{hypotenuse of equinoctial midday shadow}}{12},$$

where R denotes the radius of the celestial sphere. This is possible only when the midday Sun at summer solstice is at the zenith, and this happens if the latitude of the place is equal to the Sun's greatest declination. This clearly shows that the latitude of Vaṭeśvara's place of residence (viz. Ānandapura) and also that of Daśapura must have been 24 degrees north, for, according to Vaṭeśvara, the Sun's greatest declination equals 24°. We have already shown that the latitude of Ānandapura or Vaḍnagar is 24° north, approximately. The latitude of Daśapura also is 24° north, approximately, for Daśapura is the same place as modern Mandasore in Madhya Pradesh.[2] Its latitude is 24° 3′ north, and longitude 75° 8′ east.

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1. See Āmarāja's commentary on *Khaṇḍa-khādyaka*, iii. 1, Babuājī's edition, p. 87; and *Bhāratīya Jyotiṣaśāstra* (in Marathi) by S. B. Dikshit, Second Edition, p. 471.
2. See Mm. Dr. V. V. Mirashi, "Further light on Yaśodharman-Viṣṇuvardhana," *Bhāratī-Bhānam* ("Light of Indology"), V. V. B. I. S. & I. S., Hoshiarpur (1980), p. 405. Daśapura has been mentioned by astronomer Lalla also. See Lalla's *Gola*, ix. 10.

Views of other scholars. Ram Swarup Sharma[1] conjectured that Vaṭeśvara's Ānandapura was probably the Ānandapura of the Panjab. But this has been rightly refuted by R.N. Rai[2] on the ground that Ānandapura situated in the Panjab was known as Mākhoval before A.D. 1664 when Guru Tegh Bahadura bought it from the hill states and built a Gurudvārā there. And it is he who named it Ānandapura.

R.N. Rai himself,[3] on the other hand, expressed the view that Vaṭeśvara belonged to Kashmir and lived at Nāgarapatharī, a village situated in latitude 33°55′ between Srinagar and Punch. He argues : "The evidence of *Karaṇasāra* points to the fact that he (Vaṭeśvara) belonged to Kashmir as he gives the latitude of Kashmir as 34° 9′ which is very nearly the latitude of Srinagar. Also the name Vaṭeśvara is not very common in the rest of India and we have on the evidence of *Rājataraṅgiṇī* that there was a *Śivaliṅga* of the name of Vaṭeśvara near Srinagar which one of the kings of Kashmir used to worship daily. Also Al-Bīrūnī says that he belonged to the city of Nāgarapura. Now names are liable to change a little during the course of one thousand years. But there is a village between Srinagar and Punch of the name of Nāgarapatharī, of which the latitude is 33°55′. This latitude is so very close to 34° 9′ that I am tempted to believe that this was the native place of Vaṭeśvara."

This view is unacceptable on the ground that the village of Nāgarapatharī in Kashmir was never called Ānandapura whereas Vaṭeśvara belonged to Ānandapura.

R.N. Rai's assertion that the name Vaṭeśvara is not very common in the rest of India is not correct. For, we know of two persons called Vaṭeśvara, one a saint who lived in Maharastra in the thirteenth century and the other a painter who lived at Lucknow in Uttar Pradesh in the first half of the nineteenth century. Similarly, we know of two places bearing the name Vaṭeśvara, one in Uttar Pradesh in the district of Agra and the other in Bihar between Multanganj and Bhagalpur.

Mention of the latitude of Kashmir in the *Karaṇasāra*, according to Al-Bīrūnī, led S.B. Dikshit also to believe that our Vaṭeśvara belonged to

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1. See the opening lines of the introduction to his edition of the *Vaṭeśvara-siddhānta*, published by Indian Institute of Astronomical and Sanskrit Research, Gurudvara Road, New Delhi—5, 1962.
2. See R. N. Rai, "*Kāraṇasāra* of Vaṭeśvara," *IJHS*, vol. 6, no. 1, 1971, p. 34.
3. *Ibid.*

Kashmir.[1] But it is not known in what connection the latitude of Kashmir was mentioned in the *Karaṇasāra*. Until the *Karaṇasāra* is discovered nothing definite can be said in this regard.

There is, however, no doubt that Vaṭeśvara's father Mahadatta belonged to Ānandapura or Vaḍnagar in northern Gujarat and that Vaṭeśvara wrote his *Vaṭeśvara-siddhānta* there.

HIS WORKS

Vaṭeśvara wrote at least three works on astronomy, viz.

(1) *Karaṇa-sāra*

(2) *Vaṭeśvara-siddhānta*

(3) *Gola*.

The *Karaṇa-sāra* has not survived but it has been mentioned and quoted at several places in the writings of Al-Bīrūnī. The epoch used in this work shows that it was written in the year 899 A.D. when Vaṭeśvara was only 19 years of age. From the name of this work, it is obvious that it was a *karaṇa* work meant for *Pañcāṅga*-makers.

The *Vaṭeśvara-siddhānta* is evidently a *siddhānta*. It is the largest of the *Siddhāntas* available to us. Whereas the *Āryabhaṭīya* contains in all 121 verses, the *Brāhma-sphuṭa-siddhānta* of Brahmagupta (A.D. 628) 1008 verses, the *Śiṣya-dhī-vṛddhida* of Lalla 322 verses, the *Sūrya-siddhānta* 500 verses, the *Siddhānta-śekhara* of Śrīpati 890 verses and the *Grahagaṇita* section of the *Siddhānta-śiromaṇi* of Bhāskara II (1150 A.D.) 460 verses, the *Vaṭeśvara-siddhānta* (excluding *Gola*) contains as many as 1326 verses. As has been already mentioned it was written in A.D. 904 when Vaṭeśvara was 24 years of age.

Vaṭeśvara's *Gola* has not survived completely. Fragments of the first five chapters are found towards the end of MS A in highly disturbed arrangement. These fragments have been collected and arranged, as systematically as possible, and appended to Part I of this work. The available five chapters of this work bear the titles : (1) *Gola-praśaṁsā*, (2) *Chedyaka*,

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1. See *Bhāratīya Jyotiṣaśāstra* by S. B. Dikshit, Second Edition (Marathi), p. 313.

(3) *Gola-bandha*, (4) *Gola-vāsanā* and (5) *Bhūgola*. The contents of these chapters are strikingly similar to those of the chapters of the same titles in Lalla's *Gola*.

From the colophons occurring at the ends of the various chapters of the *Vaṭeśvara-siddhānta* and those occurring at the ends of the available chapters of Vaṭeśvara's *Gola*, it appears that these were two independent compositions and did not form parts of the same work. The same is seen to be true in the case of Lalla's *Gola* also. For we see that : (1) the colophons occurring at the ends of the various chapters of Lalla's *Gola* do not treat it as forming part of Lalla's *Śiṣya-dhī-vṛddhida*, (2) the manuscripts of the *Śiṣya-dhī-vṛddhida* and Lalla's *Gola* are found independent of each other, and (3) Bhāskara II (A.D. 1150) and Mallikārjuna Sūri (A.D. 1178) who wrote commentaries on the *Śiṣya-dhī-vṛddhida*, have not commented on Lalla's *Gola*.

VAṬEŚVARA-SIDDHĀNTA

The *Vaṭeśvara-siddhānta* reckons the day from sunrise at Laṅkā and belongs to the Brahma school of Hindu astronomy. The author commences the work with obeisance to Brahmā. At several places in the work he mentions the name of Brahmā and declares some of the teachings to have come directly from the mouth of Brahmā. He is thus a staunch exponent of the Brahma school. However, he has not confined himself to the teachings of Brahmā alone. In writing this work he has utilized all the important works on the subject that existed in his time and has produced an encyclopaedic work by compiling most of the relevant material contained in them. Explaining the scope of this work, he himself says :

"This (science of astronomy) was (first) taught by the divine sages whose excellent intellect was purified by the vast and deep knowledge of *kālakriyā* ("reckoning with time"), *gaṇita* ("mathematics") and *gola* ("the celestial sphere, or spherics"), the subjects of that great science. When we, ignorant people, consulting their teachings, write on the subject, the credit is theirs. But to those who by virtue of their own intellect have difference of views, the *yuga* prescribed by Brahmā does not always lead to equally correct results. So the essence of the teachings of all the *śāstras* ("texts on astronomy") is being set out, excepting all based on erroneous views."

The most important works on Hindu astronomy that existed in the time of Vaṭeśvara were the *Āryabhaṭīya* of Āryabhaṭa I, the *Mahā-Bhāskar-*

īya and the *Laghu-Bhāskarīya* of Bhāskara I, the *Brāhma-sphuṭa-siddhānta* and the *Khaṇḍa-khādyaka* of Brahmagupta, the *Śiṣya-dhī-vṛddhida* of Lalla and the *Sūrya-siddhānta*. Vaṭeśvara consulted all these works and utilized some of their teachings which he considered correct and adaptable. But he gave preference to the works of Āryabhaṭa I and his followers Bhāskara I and Lalla who were also the exponents of the Brahma school. Lalla seems to have been his favourite astronomer whom he has followed to a greater extent. Vaṭeśvara has not only borrowed a number of rules from Lalla's *Śiṣya-dhī-vṛddhida*, but has also copied certain interesting ideas and poetic fancies from that work. Even the incorrect rules for computing the *valana* and *dṛkkarma* have been taken from Lalla. It is surprising that Vaṭeśvara has given preference to these incorrect rules over the corresponding correct rules given by Brahmagupta. These incorrect rules were later criticised by Bhāskara II.

Vaṭeśvara was not happy with the way Brahmagupta had criticised Āryabhaṭa I in his *Brāhma-sphuṭa-siddhānta*. He took it rather seriously, and so in a section[1] of the *Vaṭeśvara-siddhānta*, which he has specially reserved for this purpose, he has defended Āryabhaṭa I from the criticism of Brahmagupta and has condemned Brahmagupta and levelled counter-allegations against him. But he has borrowed some rules and ideas from Brahmagupta too.

Although Vaṭeśvara has consulted the works of earlier writers and utilized their contents, it should not be inferred that everything that Vaṭeśvara gives in the *Vaṭeśvara-siddhānta* is derived from the anterior works. There is plenty of material in the *Vaṭeśvara-siddhānta* which is original and the production of Vaṭeśvara's own mind. The general layout of the work, the arrangement of the contents in the different chapters under different sections, and the treatment of the topics in a systematic sequence, fully and exhaustively, that we find in the *Vaṭeśvara-siddhānta* is Vaṭeśvara's own. A major portion of the *Vaṭeśvara-siddhānta* is the result of Vaṭeśvara's own imagination. Quite a large number of rules stated by Vaṭeśvara have no counterpart in any other work on Hindu astronomy. There is also sufficient matter which seems to have inspired his successors in the field like Śrīpati and Bhāskara II. For example, Bhāskara II's rule for obtaining the *lambana* directly, without taking recourse to the method of iteration, is really Vaṭeśvara's method which Bhāskara II has borrowed from him without mentioning his name. The chapter on seasons that occurs in the *Siddhānta-śiromaṇi* of Bhāskara II was also written probably under the influence of

1. Sec. 10 of Chap. I.

Vaṭeśvara. Quite a few rules and examples occurring in the *Siddhānta-śekhara* of Śrīpati are either exactly the same or almost the same as those occurring in the *Vaṭeśvara-siddhānta.*

CONTENTS OF THE *VAṬEŚVARA-SIDDHĀNTA*

Vaṭeśvara divides the contents of the *Vaṭeśvara-siddhānta* into eight chapters, which are further subdivided topicwise into a number of sections, as follows :

Chapter I. Mean Motion

Sec. 1. Revolutions of the planets.

Sec. 2. Time-measures.

Sec. 3. Calculation of the *Ahargaṇa.*

Sec. 4. Computation of mean planets.

Sec. 5. *Śuddhi* or intercalary fraction, for solar year etc.

Sec. 6. Methods of a *karaṇa* work.

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Sec. 9. Examples on Chapter I.

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Chapter II. True Motion

Sec. 1. Correction of Sun and Moon.

Sec. 2. Correction of planets under the epicyclic theory.

Sec. 3. Correction of planets under the eccentric theory.

Sec. 4. Correction of planets without using the Rsine table.

Sec. 5. Correction of planets by the use of *mandaphala* and *śīghraphala* tables.

Sec. 6. Elements of the *Pañcāṅga.*

Sec. 7. Examples on Chapter II.

Chapter III. Three Problems

Sec. 1. Cardinal directions and equinoctial midday shadow.

Sec. 2. Latitude and colatitude.

The division of chapters into small sections earmarked for different topics is a unique feature of this work. No other work is known to have divided its chapters into sections as done in this work. Bhāskara II has indeed divided the first chapter of his *Siddhānta-śiromaṇi* into seven sections but he has not done so in the case of the other chapters.

The topics treated under each section are dealt with systematically, fully and exhaustively. Rules for all possible hypotheses are formulated and each rule is followed by numerous alternatives. In this respect too this work stands unique and has no parallel except the *Sumati-mahātantra* of Sumati which also gives numerous alternative rules. Inclusion of too many alternative rules has enlarged the bulk of this work so much that this work is about eleven times as large as the *Āryabhaṭīya* of Āryabhaṭa I.

A set of unsolved examples at the end of every chapter is another unique feature of this work. The *Mahā-Bhāskarīya* of Bhāskara I, the *Śiṣya-dhī-vṛddhida* of Lalla, the *Siddhānta-śekhara* of Śrīpati, the *Mahā-siddhānta* of Āryabhaṭa II, the *Siddhānta-śiromaṇi* of Bhāskara II, and the *Siddhānta-tattva-viveka* of Kamalākara do give a set of examples towards the end of their works, but no work on Hindu astronomy other than the *Siddhānta* of Vaṭeśvara gives a set of examples at the end of every chapter.

LANGUAGE, METRES AND TECHNICAL TERMS ETC.

The language used in the *Vaṭeśvara-siddhānta* is simple, straightforward and easily understandable. Obscurities that seem to occur at places are due to the difficulties of the subject matter.

The text of the *Vaṭeśvara-siddhānta* has been garbed in a variety of metres, the metres used being *Anuṣṭubh, Āryā, Indravajrā, Utpalamālikā, Udgīti, Upagīti, Upajāti, Upendravajrā, Gīti, Tāmarasa, Toṭaka, Dodhaka, Drutavilambita, Pañcakāvalī, Pañcacāmara, Puṣpitāgrā, Pramāṇikā, Bhujaṅgaprayāta, Mandākrāntā, Mālabhāriṇī* (or *Vasantamālikā*), *Mālinī, Rathoddhatā, Rucirā* (or *Prabhāvatī*), *Vaṁśastha, Vasantatilakā, Vāhinī, Viyoginī, Vaitālīya, Śārdūlavikrīḍita, Śālinī, Śikhariṇī, Sragdharā, Svāgatā, Hariṇī,* and four anonymous metres.[1] In his *Gola* (Spherics), Vaṭeśvara uses the *Daṇḍaka* metre also.

Of the terms used in the *Vaṭeśvara-siddhānta*, the following seem to be new and deserve notice :

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1. See V, 1. 4; VI, 12. 15; and VII, 1. 3.

Technical terms

1. *Alpa-bhūkhaṇḍa*, meaning "smaller segment of *bhū*" or "*smaller dṛggati*". For smaller *dṛggati* and larger *dṛggati*, see p. 459.
2. *Iṣṭadhṛti*, used in the sense of the usual term "*iṣṭahṛti*".
3. *Khaṇḍa*, used in the sense of "one-half".
4. *Kṣiti*, used in the sense of "*bhū*". See *Bhū*.
5. *Dyumūḍha*, used in the sense of "*avamaśeṣa*".
6. *Dṛglagna*, used in the sense of "the planet corrected for *ayanadṛkkarma*.". The usual term is "*ayanagraha*."
7. *Dṛgvilagna*. Same as *dṛglagna*.
8. *Dhṛti*, meaning "*iṣṭadhṛti* for the meridian.". Also used for *iṣṭadhṛti*.
9. *Bheda*, used in the sense of "one-half".
10. *Bhū*, used in the sense of "larger *dṛggati* + smaller *dṛggati*."
11. *Bhūya-bhū-khaṇḍa*, meaning "larger segment of *bhū*" or "larger *dṛggati*.".
12. *Bhūyasī-dṛggati*. Same as *mahatī dṛggati* or "larger *dṛggati*."
13. *Mahatī-dṛggati*, meaning "larger *dṛggati*."
14. *Mahat-lambana*, meaning "larger *lambana*" and used in the sense of "*lambana* calculated from the larger *dṛggati*."
15. *Laghīyasī dṛggati*, meaning "smaller *dṛggati*."
16. *Laghu-dṛggati*, meaning "smaller *dṛggati*."
17. *Laghu-lambana*, meaning "smaller *lambana*" and used in the sense of "*lambana* calculated from the "smaller *dṛggati*."
18. *Svadhṛti* meaning "own *dhṛti*."
19. *Svalpa-dṛggati*, meaning "smaller *dṛggati*."
20. *Yuti* ("Sum"), used to denote "the sum of the longitudes of the Moon and the Moon's ascending node", the latter measured westwards."

Word-numerals

21. *Kha* ("Brahma"), used to denote "1".
22. *Khaga* ("arrow"), used to denote "5".

23. *Puṁ*, used to denote "1".
24. *Praṇimnageśa* ("ocean"), used to denote "4".
25. *Bhuvanyu* ("Moon"), used to denote "1".
26. *Raviputra* ("Yama"), used for "2"
27. *Vāk* ("Speech"), used to denote "1".
28. *Vidhṛti* ("*Dhṛti* minus 1"), used to denote "17".
29. *Sukha* ("the name of the fourth house of the horoscope"), used to denote "4".

In a scientific work like the *Vaṭeśvara-siddhānta*, there is hardly any scope to indulge into poetic fancies. But Vaṭeśvara has found occasion to do so. The Sun and the Moon on the full moon day, one lying on the eastern horizon and the other on the western, appear to him like "two huge gold bells (hanging from the two sides) of Indra's elephant."[1] The first digit of the Moon appears to his eye like "the creeper of the Cupid's bow," and gives to him "the false impression of the beauty of the eyebrows of a fair-coloured lady with excellent eye-brows."[2] The half-phased Moon looks to him like "the forehead of a lady belonging to the Lāṭa-deśa (northern Gujarat)."[3] And the higher horn of the Moon while rising or setting appears to him as "bearing the beauty (seen) at the tip of the Ketaka flower on account of its association with the black bees."[4]

SPECIAL FEATURES OF THE *VAṬEŚVARA-SIDDHĀNTA*

Amongst the special features of the *Vaṭeśvara-siddhānta* which deserve special notice of the historian, mention may be made of the following :

1. Linear measures. (I, 7. 1-3)

Vaṭeśvara defines an *aṇu* as a particle seen floating in the beam of sunlight coming into a room through an aperture, and gives the following table of linear measures:

8 *aṇus*	=	1 *kacāgra*
8 *kacāgras*	=	1 *likṣā*
8 *likṣās*	=	1 *yūkā*

1. Chap. VII, sec. 1, vs. 8.
2. Chap. VII, sec. 1, vs. 51.
3. Chap. VII, sec. 1, vs. 49.
4. Chap. VII, sec. 1, vs. 50.

8 *yūkās*	=	1 *yava*
8 *yavas*	=	1 *aṅgula* (digit)
12 *aṅgulas*	=	1 *vitasti*
2 *vitastis*	=	1 *kara* (cubit)
4 *karas*	=	1 *nṛ*
1000 *nṛs*	=	1 *krośa*
8 *krośas*	=	1 *yojana.*

The measures from *aṅgula* to *yojana* are the same as those given by Āryabhaṭa I. The smaller measures were not mentioned by him.

A similar table of linear measures is given by Śrīpati also, but it differs from the above one in some cases. Śrīpati defines a *paramāṇu* in the same way as Vaṭeśvara defines an *aṇu*, but he gives

8 *paramāṇus*	=	1 *reṇu*
8 *reṇus*	=	1 *bālāgra* (*kacāgra*)
8 *bālāgras*	=	1 *likṣā*
. . .		. . .
4 *karas*	=	1 *cāpa* or *dhanu* (same as *nṛ*)
2000 *dhanus*	=	1 *krośa*
4 *krośas*	=	1 *yojana.*

2. Time-measures. (I, 1. 7-9)

Vaṭeśvara defines a *truti* as the time taken by a sharp needle to pierce a petal of a lotus flower, and gives the following table of time-measures :

100 *truṭis*	=	1 *lava*
100 *lavas*	=	1 *nimeṣa* ("twinkling of the eye")
4½ *nimeṣas*	=	1 long syllable
4 long syllables	=	1 *kāṣṭhā*
2½ *kāṣṭhās*	=	1 *asu* (= 4 seconds)
6 *asus*	=	1 sidereal *pala* (= 24 seconds)
60 *palas*	=	1 *ghaṭikā* (= 24 minutes)
60 *ghaṭikās*	=	1 *day*

30 days = 1 month

12 months = 1 year.

The measures from *asu* to year are the same as given by Āryabhaṭa I. The smaller measures not were mentioned by him.

Similar tables have been given by Śrīpati and Bhāskara II, but according to them

100 *truṭis* = 1 *tatparā*

30 *tatparās* = 1 *nimeṣa* ("twinkling of the eye").

As regards the larger measures of time, Vaṭeśvara is a follower of Āryabhaṭa I. Like Āryabhaṭa I, he defines :

4320000 years = 1 *yuga*

72 *yugas* = 1 Manu

14 Manus = 1 *kalpa*.

Thus, like Āryabhaṭa I's *kalpa*, his *kalpa* too contains 1008 *yugas*, each of 4320000 years. But Vaṭeśvara goes beyond *kalpa* and defines :

2 *kalpas* = 1 day-and-night of Brahmā

30 day-and-nights of Brahmā = 1 month of Brahmā

12 months of Brahmā = 1 year of Brahmā

100 years of Brahmā = life-span of Brahmā.

The total age of Brahmā, according to Vaṭeśvara, thus comes out to be equal to $72576 \times 432 \times 10^7$ years.

Although Vaṭeśvara adopts the same lengths of a *kalpa* and a *yuga* as stated by Āryabhaṭa I, the beginnings of the current *kalpa* and the current *yuga* according to them are not the same. The current *kalpa* according to Vaṭeśvara began on Saturday whereas that according to Āryabhaṭa I, on Thursday. This is so because Vaṭeśvara's *yuga* is 60 days longer than that of Āryabhaṭa I. The current *kalpa* of Vaṭeśvara started 27585 days earlier than that of Āryabhaṭa I, and 27585 ≡ 5 (mod 7). Similarly, the current *yuga* of Vaṭeśvara began on Sunday whereas the same according to Āryabhaṭa I, 45 days later on Wednesday.

But this difference is immaterial, because according to all Hindu astronomers the beginning of the current Kaliyuga occurred on Friday, February 18, B. C. 3102, at sunrise at Laṅkā in the beginning of the month Caitra, Laṅkā being the hypothetical place where the Hindu prime meridian ("the meridian of Ujjain") intersects the equator. And what has been said above is in conformity with this.

3. Age of Brahmā. (I, 1, 10)

According to Vaṭeśvara, the age of Brahmā in the beginning of the current *kalpa* was equal to

8 years of Brahmā + $6\frac{1}{2}$ months of Brahmā

= 6150×1008 *yugas*, or 26780544×10^6 years.

It is noteworthy that although Vaṭeśvara, like Puliśa and Lalla, is an exponent of the Brahma school, he differs from both Puliśa and Lalla in regard to the age of Brahmā. For, according to Puliśa, the age of Brahmā in the beginning of the current *kalpa*

= 8 years of Brahmā + 5 months of Brahmā + 4 days of Brahmā

= 6068×1008 *yugas*, or 2642347008×10^4 years;

and, according to Lalla, it is

= 8 years of Brahmā + $6\frac{1}{2}$ months of Brahmā

= 6150×1000 *yugas*, or 26568×10^9 years.

The difference between the views of Vaṭeśvara and Lalla is due to the fact that while Vaṭeśvara, following Āryabhaṭa I, takes a *kalpa* as consisting of 1008 *yugas*, Lalla, following the orthodox Hindu tradition, takes a *kalpa* as made up of 1000 *yugas*. But what makes the difference between the views of Vaṭeśvara and Puliśa is not known.

The view of the *Sūrya-siddhānta* in this respect is totally different. According to it, half of Brahmā's life (i.e., 50 years of Brahmā) had passed in the beginning of the current *kalpa:*

Āryabhaṭa I is silent on this point, whereas Śrīpati and Bhāskara II have expressed their inability to say anything.

4. Astronomical parameters.

(1) Revolutions of the planets, their apogees and nodes. (I, 1. 11-14, 16-17)

The following table gives the revolutions of the Sun, Moon, and the planets and stars (in a period of 4320000 years) as stated by Āryabhaṭa I and Vaṭeśvara :

Revolutions of	according to Āryabhaṭa I	according to Vaṭeśvara
Sun	43,20,000	43,20,000
Moon	5,77,53,336	5,77,53,336
Moon's apogee	4,88,219	4,88,211
Moon's asc. node	– 2,32,226	– 2,32,234
Mars	22,96,824	22,96,828
Śīghrocca of Mercury	1,79,37,020	1,79,37,056
Jupiter	3,64,224	3,64,220
Śīghrocca of Venus	70,22,388	70,22,376
Saturn	1,46,564	1,46,568
Stars	1,58,22,37,500	1,58,22,37,560

The revolutions stated by Vaṭeśvara differ from those given by Āryabhaṭa I, but they have been derived from those of Āryabhaṭa I by applying to them the *Bīja* correction prescribed by Lalla, a notable follower of Āryabhaṭa I. In addition to the *Bīja* correction, Vaṭeśvara has made some adjustment to preserve the characteristic features of the revolutions of Āryabhaṭa I. Thus, the *Bīja*-corrected revolutions of the Moon, the *Śīghrocca* of Mercury and Saturn have been increased by 2 so that, like the revolutions of Āryabhaṭa I, they may become divisible by 4. Similarly, the revolutions of the Moon's apogee have been increased by 1 so that they may become odd and prime to the number of civil days in a *yuga*, as in the case of Āryabhaṭa I. See the table below.

	Āryabhaṭa I's revolutions	*Bīja* correction	Corrected revolutions	Adjustment	Vaṭeśvara's revolutions
Sun	43,20,000	Nil	43,20,000		43,20,000
Moon	5,77,53,336	– 2	5,77,53,334	+ 2	5,77,53,336
Moon's apogee	4,88,219	– 9·12	4,88,210	+ 1	4,88,211
Moon's asc. node	–2,32,226	– 7·68	– 2,32,234		– 2,32,234
Mars	22,96,824	+ 3·84	22,96,828		22,96,828
Śīghrocca of Mercury	1,79,37,020	+33·60	1,79,37,054	+ 2	1,79,37,056
Jupiter	3,64,224	– 3·76	3,64,220		3,64,220
Śīghrocca of Venus	70,22,388	–12·24	70,22,376		70,22,376
Saturn	1,46,564	+ 1·6	1,46,566	+ 2	1,46,568

It is noteworthy that the revised *Sūrya-siddhānta*, which was utilized by Vijayanandī (A. D. 966) in writing his *Karaṇa-tilaka* and used by Parameśvara (A. D. 1432) in writing his commentary thereon, gives the same revolutions as stated by Vaṭeśvara excepting those of Mars and the *Śīghrocca* of Mercury, in which cases the *Sūrya-siddhānta* gives 4 revolutions more than those given by Vaṭeśvara. See the next table.

	Revolutions according to		
	Āryabhaṭa I	Vaṭeśvara	*Sūrya-siddhānta*
Sun	43,20,000	43,20,000	43,20,000
Moon	5,77,53,336	5,77,53,336	5,77,53,336
Moon's apogee	4,88,219	4,88,211	4,88,211
Moon asc. node	– 2,32,226	– 2,32,234	– 2,32,234
Mars	22,96,824	22,96,828	22,96,832
Śīghrocca of Mercury	1,79,37,020	1,79,37,056	1,79,37,060

Jupiter	3,64,224	3,64,220	3,64,220
Śīghrocca of Venus	70,22,388	70,22,376	70,22,376
Saturn	1,46,564	1,46,568	1,46,568
Stars	1,58,22,37,500	1,58,22,37,560	1,58,22,37,828

It seems that the redactor of the *Sūrya-siddhānta* has borrowed the revolutions of the planets from the *Vaṭeśvara-siddhānta*, adopting those of the Moon's apogee, the Moon's node, Jupiter, the *Śīghrocca* of Venus and Saturn without any alteration and those of Mars and the *Śīghrocca* of Mercury after suitable modification.

In the case of the apogees and nodes of the planets, Vaṭeśvara's revolutions are quite different from those given by the other astronomers as the following table will show :

	Revolutions according to		
	Brahmagupta (for 432 × 10⁷ years)	*Sūrya-siddhānta* (for 432 × 10⁷ years)	Vaṭeśvara (for 72576 × 432 × 10⁷ years)
Apogee of :			
Sun	480	387	1,65,801
Mars	292	204	81,165
Mercury	332	368	4,77,291
Jupiter	855	900	13,948
Venus	653	535	1,52,842
Saturn	41	39	6,774
Asc. Node of :			
Mars	−267	−214	−20,684
Mercury	−521	−488	−1,62,719
Jupiter	−63	−174	−3,802
Venus	−893	−903	−60,895
Saturn	−584	−662	−1,542

Vaṭeśvara has evidently derived the revolutions of the apogees and ascending nodes of the planets on the basis of his own observations. Using these revolutions, Vaṭeśvara has calculated the positions of the apogees and ascending nodes of the planets for the beginning of Kaliyuga. These positions differ from those given by Āryabhaṭa I and other astronomers.

(2) Diameters and distances of the Earth, Sun, Moon and the planets, (IV, 5. 7 (c-d); VII, 1. 4)..

The diameters of the Earth, Sun and Moon stated by Vaṭeśvara differ from those given by Āryabhaṭa I, the difference being large in the case of the Moon's diameter.

(3) *Manda* and *śīghra* epicycles. (II, 1. 52-53)

The *manda* and *śīghra* epicycles of the planets, stated by Vaṭeśvara, are invariable like those given in the old *Sūrya-siddhānta* and *Khaṇḍa-khādyaka,* but their dimensions differ from them. They also do not agree with those given by any other astronomer. Agreement, wherever it occurs, is only accidental.

(4) Distances for heliacal visibility and inclinations of orbits. (VI. 3-4)

The distances of the planets from the Sun for their heliacal visibility, given by Vaṭeśvara, are exactly the same as those stated by Āryabhaṭa I, Brahmagupta and Lalla, but the orbital inclinations (to the ecliptic) of the planets given by Vaṭeśvara do not agree with those stated by any other astronomer.

5. The zero point of calculation. (I, 3. 1-2; I, 4. 56-62).

The *Siddhāntas* generally take the beginning of creation or the beginning of the current *kalpa* as the zero point of calculation. Vaṭeśvara has deviated from this practice. He has left it at the discretion of his reader to choose either the time of birth of Brahmā, or the beginning of the current *kalpa,* or the beginning of the current *yuga,* or the beginning of Kaliyuga, for the epoch of calculation.

6. The Jovian year. (I, 5. 76-95)

The Jovian year seems to have been more popular in the locality where Vaṭeśvara lived, for he has given undue prominence to it. He has devised

methods to find *śuddhi* for the beginning of a Jovian year in terms of civil days and also in terms of lunar days. He has given rules to find the lord of the Jovian year and the shorter *Ahargaṇa* reckoned from the beginning of the Jovian year. He has also stated rules to compute the longitudes of the planets for the end of a Jovian year.

7. Lords of the 30 degrees of a zodiacal sign. (I, 5. 117-120)

The names of the lords presiding over the thirty degrees of a sign were first noticed in the *Pañca-siddhāntikā*[1] of Varāhamihira. But the text of the *Pañca-siddhāntikā* being faulty, the names given there could not be deciphered correctly. The same names with minor difference appear in the *Vaṭeśvara-siddhānta* also. These names have now been correctly deciphered and it is found that they are the Hinduised names of the gods and angels after whom the thirty days of the Parsi months are known. The names of the thirty days of the Parsi months and the Hindu names by which they have been called by Varāhamihira and Vaṭeśvara are given on p. 115.

8. Value of π. (I, 8. 3)

Vaṭeśvara, following Āryabhaṭa I, gives $\pi = \frac{3927}{1250}$ and makes use of this value in his calculations. He remarks that this value of π is better than the value $\pi = \sqrt{10}$, which was considered accurate by Brahmagupta.

9. Trigonometrical relations. (Chap. III)

The earlier Hindu astronomers knew the following relations between the sine, cosine and versed sine functions:

1. $R\sin\theta = R\cos(90^\circ - \theta)$, or $\sin\theta = \cos(90^\circ - \theta)$

   $R\cos\theta = R\sin(90^\circ - \theta)$, or $\cos\theta = \sin(90^\circ - \theta)$

2. $(R\sin\theta)^2 + (R\cos\theta)^2 = R^2$, or $\sin^2\theta + \cos^2\theta = 1$

3. $R\sin\theta + R\text{vers}(90^\circ - \theta) = R$, or $\sin\theta + \text{vers}(90^\circ - \theta) = 1$

   $R\cos\theta + R\text{vers}\,\theta = R$, or $\cos\theta + \text{vers}\,\theta = 1.$

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1. i. 24-25.

Therefore, they could express Rsin θ and Rcos θ in the following ways :

1. $R\sin\theta = R\cos(90°-\theta)$

   $R\cos\theta = R\sin(90°-\theta)$.

2. $R\sin\theta = \sqrt{R^2-(R\cos\theta)^2}$, or $\sqrt{(R-R\cos\theta)(R+R\cos\theta)}$.

   $R\cos\theta = \sqrt{R^2-(R\sin\theta)^2}$, or $\sqrt{(R-R\sin\theta)(R+R\sin\theta)}$.

3. $R\sin\theta = R - R\text{vers}(90°-\theta)$

   $R\cos\theta = R - R\text{vers}\,\theta$.

Vaṭeśvara knew the following relations also :

1. $(R\sin\theta)^2 + (R\text{vers}\,\theta)^2 = 2R.\,R\text{vers}\,\theta$

   or $\sin^2\theta + \text{vers}^2\theta = 2\,\text{vers}\,\theta$.

2. $2\,R\sin\theta.\,R\cos\theta + [R\text{vers}\,\theta - R\text{vers}(90°-\theta)]^2 = R^2$

   or $2\sin\theta\cos\theta + [\text{vers}\,\theta - \text{vers}(90°-\theta)]^2 = 1$.

3. $(R+R\sin\theta).\,R\text{vers}(90°-\theta) = (R\cos\theta)^2$

   or $(1+\sin\theta).\text{vers}(90°-\theta) = \cos^2\theta$.

4. $2\,R\sin\theta - [R\text{vers}\,\theta - R\text{vers}(90°-\theta)]$

   $$= \sqrt{2R^2 - [R\text{vers}\,\theta - R\text{vers}(90°-\theta)]^2}$$

   or $2\sin\theta - [\text{vers}\,\theta - \text{vers}(90°-\theta)]$

   $$= \sqrt{2 - [\text{vers}\,\theta - \text{vers}(90°-\theta)]^2}$$

5. $(R\cos\theta + R\sin\theta)^2 + [R\text{vers}\,\theta \sim R\text{vers}(90°-\theta)]^2 = 2R^2$

   or $(\cos\theta + \sin\theta)^2 + [\text{vers}\,\theta \sim \text{vers}(90°-\theta)]^2 = 2$.

Therefore, he has expressed Rsin θ and Rcos θ in the following new forms also:

1. $$\text{Rsin}\,\theta = \sqrt{2\text{R. Rvers}\,\theta - (\text{Rvers}\,\theta)^2}$$

$$\text{Rcos}\,\theta = \sqrt{2\text{R. Rvers}\,(90^\circ - \theta) - [\text{Rvers}\,(90^\circ - \theta)]^2}$$

and $$\text{Rsin}\,\theta = \sqrt{\text{Rvers}\,\theta\,(2\text{R} - \text{Rvers}\,\theta)}$$

$$\text{Rcos}\,\theta = \sqrt{\text{Rvers}\,(90^\circ - \theta)\,[2\text{R} - \text{Rvers}\,(90^\circ - \theta)]} \qquad \text{[III, 2. 18]}$$

2. $$\text{Rsin}\,\theta = \frac{\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2}{2\,\text{Rcos}\,\theta}$$

$$\text{Rcos}\,\theta = \frac{\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2}{2\,\text{Rsin}\,\theta} \qquad \text{[III, 2. 19]}$$

3. $$\text{Rsin}\,\theta = \frac{(\text{Rcos}\,\theta)^2}{\text{Rvers}\,(90 - \theta)} - \text{R}$$

$$\text{Rcos}\,\theta = \frac{(\text{Rsin}\,\theta)^2}{\text{Rvers}\,\theta} - \text{R}. \qquad \text{[III, 2. 17]}$$

4. $$\text{Rsin}\,\theta = \tfrac{1}{2}[\sqrt{2\,\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2} + [\text{Rvers}\,\theta - \text{Rvers}\,(90^\circ - \theta)]$$

$$\text{Rcos}\,\theta = \tfrac{1}{2}[\sqrt{2\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2} - [\text{Rvers}\,\theta - \text{Rvers}\,(90^\circ - \theta)]$$

[III. 2, 20]

5. $$\text{Rsin}\,\theta = \sqrt{2\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2} - \text{Rcos}\,\theta$$

$$\text{Rcos}\,\theta = \sqrt{2\text{R}^2 - [\text{Rvers}\,\theta \sim \text{Rvers}\,(90^\circ - \theta)]^2} - \text{Rsin}\,\theta.$$

[III, 2, 21]

These expressions do not occur in any earlier work, while Vaṭeśvara has used them in more than one context.

In the geometry of the celestial sphere, Vaṭeśvara has made new experiments and has employed mathematical artifices to express a formula in a variety of forms. See, for example, Sec. 9 of Chap. III. He has also been

able to frame some ingenious rules which were unknown to his predecessors. For example, he states the following formula for the midday shadow of the gnomon:

$$\text{midday shadow of the gnomon} = \frac{(D \sim R)(D + R)\,H}{A \times D} \sim P,$$

where

R = radius of the celestial sphere,

P = midday shadow of the gnomon at an equinox,

H = hypotenuse of the equinoctial midday shadow,

A = distance of the rising Sun from the east-west line,

and D = distance of the midday Sun from its rising-setting line.

[III, 9. 39]

10. The sine table. (II, 1. 2-50)

The sine tables of the earlier astronomers were generally constructed under the assumption that the 24th part of the quadrant of a circle was straight like a rod and they gave the values of the Rsines and Rversed-sines for the 24 multiples of 225′ (i. e., 225′, 450′, 675′,……,5400′) in terms of the nearest minutes of arc. So these sine tables were very approximate. Vaṭeśvara has criticised Brahmagupta for taking the Rsine of the 24th part of the quadrant as equal to the 24th part of the quadrant itself, but the Rversed-sine of the same arc as equal to 7′.

To ensure greater accuracy, Vaṭeśvara has constructed his sine table under the assumption that the 96th part of the quadrant of a circle was straight like a rod. He has divided the quadrant into 96 parts, each equal to 56′15″, and has stated the values of the Rsines and the Rversed-sines for the 96 multiples of 56′15″ (i. e., 56′15″, 112′30″,…,5400′) correct up to seconds of arc. His table gives the radius R = 3437′44″, Rsin 56′15″ = 56′ 15″, and Rversin 56′15″ = 0′ 27″, as they should be.

Vaṭeśvara has also given a number of short and simple methods to compute the desired Rsine from his table.

11. Second order interpolation. (II, 1. 65-92)

When the Rsine-differences

$$\Delta f(n) = f(n+1) - f(n) = \text{Rsin}\,(n+1)h - \text{Rsin}\,nh, \quad n = 1, 2, 3, \ldots\ldots$$

are known, the values of the intermediate Rsines

$$\text{Rsin}\,(nh+\lambda),\ \lambda < h$$

are obtained by taking recourse to interpolation. The usual formula of interpolation, which occurs in almost every work on Hindu astronomy, is

$$\text{Rsin}\,(nh+\lambda) = \text{Rsin}\,nh + \frac{\lambda}{h}\,\Delta f(n). \qquad (1)$$

This is based on the rule of three and is known as the first order interpolation formula.

Brahmagupta (A. D. 628) was the first Hindu astronomer who gave the second order interpolation formula :

$$\text{Rsin}\,(nh+\lambda) = \text{Rsin}\,nh + \frac{\lambda}{h}\left[\frac{\Delta f(n-1) + \Delta f(n)}{2} \pm \frac{\lambda}{h}\,\frac{\Delta f(n-1) \sim \Delta f(n)}{2}\right] \qquad (2)$$

where + or − sign is taken according as

$$\frac{\Delta f(n-1) + \Delta f(n)}{2} \lessgtr \Delta f(n)$$

i. e., according as

$$\Delta f(n-1) \lessgtr \Delta f(n).$$

This formula of Brahmagupta is now known as Stirling's formula of interpolation up to the second order terms. It may be expressed in the following two forms :

$$\text{Rsin}\,(nh+\lambda) = \text{Rsin}\,nh + \frac{\lambda}{h}\,\Delta f(n) - \frac{\lambda}{h}\left(\frac{\lambda}{h} - 1\right)\frac{\Delta f(n-1) - \Delta f(n)}{2} \qquad (3)$$

and

$$\text{Rsin}\,(nh+\lambda) = \text{Rsin}\,nh + \frac{\lambda}{h}\,\Delta f(n-1) - \frac{\lambda}{h}\left(\frac{\lambda}{h} + 1\right)\frac{\Delta f(n-1) - \Delta f(n)}{2}. \qquad (4)$$

Formula (3) is a particular case (up to second order terms) of Newton-Gauss forward interpolation formula and formula (4) is a particular case (up to second order terms) of Newton-Gauss backward interpolation formula.

Formula (3) occurs in Govinda Svāmi's commentary on the *Mahā-Bhāskarīya* (iv. 2) of Bhāskara I, where it has been prescribed for interpolating the value of Rsin $(nh+\lambda)$ when $30° < nh+\lambda < 60°$. It occurs again in Parameśvara's commentary (A. D. 1408) on the *Laghu-Bhāskarīya* [ii. 2(c-d)-3 (a-b)] of Bhāskara I, where it has been prescribed for interpolating the value of Rsin $(nh+\lambda)$, irrespective of the value of $nh+\lambda$.

Formula (4) has not been found to occur in any work on Hindu astronomy, so far. It has now been discovered for the first time in the *Vaṭeśvara-siddhānta*, where it has been displayed in a variety of ways. For details, see chap. II, sec. 1, vss. 65-82.

Conversely, when Rsin $(nh+\lambda)$ is given, λ can be obtained by solving (4) as a quadratic equation in λ. It is this technique that has been employed by Vaṭeśvara. Here also, Vaṭeśvara has expressed λ in a number of ways. See chap. II, sec. 1, vss. 85-92.

12. *Bhujāntara* correction. (II, 1. 93-94; II, 2. 27-28)

The *bhujāntara* correction is the correction for the equation of time due to the eccentricity of the ecliptic. It was applied to the planets by all the earlier astronomers and the formula used by them was :

$$\textit{bhujāntara}\text{ correction} = \frac{\text{Sun's } \textit{bhujāphala} \times \text{ planet's daily motion}}{21600} \text{ mins.}$$

Vaṭeśvara improved this formula and stated it in the form :

$$\textit{bhujāntara}\text{ correction} = \frac{\text{Sun's } \textit{bhujāphala} \times \text{ planet's daily motion}}{21600 + \text{Sun's daily motion}} \text{ mins.}$$

Vaṭeśvara's formula is better than the earlier one, because in this formula the Sun's diurnal motion per day has been taken to be equal to

(21600 + Sun's daily motion) minutes of arc

in place of 21600 minutes of arc taken by the earlier astronomers.

13. The stationary points of a planet's orbit. (II, 5. 9-22).

"Where the thread stretched from the initial point of Capricorn or Cancer, on the *śīghra* epicycle, to the centre of the Earth meets the *śīghra* epicycle, there," says Vaṭeśvara, "lies the centre of the planet when it takes up direct or retrograde motion."

Starting with this hypothesis, Vaṭeśvara treats the topic of the stationary points of a planet's orbit systematically and in all its details.

Such a treatment of the stationary points does not occur in any other work on Hindu astronomy and forms a unique feature of the *Vaṭeśvara-siddhānta.*

14. *Śīghrakendras* for the time of heliacal rising. (II. 5. 28-29)

The problem of finding the *śīghra-kendras* of the planets for the time of their heliacal rising does not occur in the works of Āryabhaṭa I, Brahmagupta and Bhāskara II. Vaṭeśvara is the only ancient Hindu astronomer known to us who deals with this problem.

15. Motion of the solstices. [III, 2. 24(d)-27]

The earlier Hindu astronomers were under the wrong impression that the solstices were fixed and had no motion. Bhāskara I criticized the followers of the *Romaka-siddhānta* who believed in the motion of the solstices and put forward the following argument in favour of their view :

"The sages of ancient times remarked that the winter solstice and the summer solstice occurred at the beginning of Dhaniṣṭhā and the middle of Āśleṣā (respectively). But now they are seen to occur at the beginnings of Capricorn and Cancer (respectively). How can it be so unless they have motion ?"

Brahmagupta, too, criticized Viṣṇucandra for giving the period of the solstitial motion.

Vaṭeśvara not only believes in the motion of the solstices but also tells us how to find that motion and apply it to the longitudes of the planets.

16. Description of the seasons. (III, 13. 17-25)

Vaṭeśvara is perhaps the earliest Hindu astronomer who has described the six seasons, giving the characteristic features of each of them, so that one could infer the quadrant in which the Sun stood at that time. It is probably the influence of Vaṭeśvara that Bhāskara II has devoted a chapter of his *Siddhānta-śiromaṇi* (*Golādhyāya*) to the description of the seasons. Following Bhāskara II, Jñānarāja has also described the seasons in one of the chapters of his *Siddhānta-sundara.*

17. Computation of *lambana* directly, without the iteration process. (V, 1. 27-28 and 32-33)

There is an ingenious method given by Bhāskara II in his *Siddhānta-śiromaṇi* (I, vi. 8-9) which tells us how to find the *lambana* directly, without taking recourse to the process of iteration. This is equivalent to the methods devised for the purpose by Vaṭeśvara and has indeed been borrowed by Bhāskara II from Vaṭeśvara.

The other peculiarities of the *Vaṭeśvara-siddhānta* are of more technical nature and need not be mentioned here. They have been pointed out in the English translation and the interested reader is referred to it.

IMPORTANCE

The greatest importance of the *Vaṭeśvara-siddhānta* is that it highlights the achievements made by the Hindu astronomers from the sixth century A.D. right up to the end of the ninth century A. D., and provides a good document of the astronomical knowledge of the Hindus in the beginning of the tenth century A.D. This work marks the end of one era and heralds the beginning of a new era in the history of Hindu astronomy. For soon afterwards the Calculus began to be employed and certain new refinements in the form of new corrections and techniques came to be introduced in Hindu astronomy. This was done by the Hindu astronomers Mañjula, Śrīpati and Āryabhaṭa II who succeeded Vaṭeśvara.

The *Vaṭeśvara-siddhānta,* coming between the *Śiṣya-dhī-vṛddhida* of Lalla on the one hand, and the *Siddhānta-śekhara* of Śrīpati on the other, also enables us to have a better and more precise assessment of the

gradual achievements of the Hindu astronomers. We can now make the following inferences perhaps with definiteness :

(i) Vaṭeśvara was the earliest astronomer who gave the method for finding the *lambana* directly, without taking recourse to the process of iteration. Bhāskara II borrowed this method from Vaṭeśvara.

(ii) Vaṭśevara was the earliest Hindu astronomer to give a methematically correct method for finding the motion of the solstices or equinoxes and applying it to the longitudes of the planets.

(iii) Vaṭeśvara was also the first to give a precise method, depending on the decrease and increase of the midday shadow, for the purpose of finding the quadrant of the Sun at any given time. So for the credit of this was given to Śrīpati.

(iv) Śrīpati was the first to introduce the *udayāntara* correction (i.e., correction for the equation of time due to the obliquity of the ecliptic).

POPULARITY

The *Vaṭeśvara-siddhānta* due to its bulky size did not prove to be a suitable text-book for the beginners in astronomy and nobody was tempted to write a commentary on it. It was studied by more advanced students. There are reasons to believe that Govinda, son of Vahnika, who lived in Daurаṇḍa, was a research student working on "the determination of the Sun's altitude" (*Śaṅkvānayana*). He had made a deep study of the relevant chapters of the *Vaṭeśvara-siddhānta* and had made them the background of his research work. The five chapters written by him, which are found appended to the *Vaṭeśvara-siddhānta* in MS A, in my opinion, formed his doctoral dissertation. There can be no other justification for writing those chapters.

There is also sufficient ground to suppose that Śrīpati and Bhāskara II had studied the *Vaṭeśvara-siddhānta* and were influenced by some of its teachings. Śrīpati has actually referred to Vaṭeśvara as one of the foremost astronomers. There are certain rules and examples in Śrīpati's *Siddhānta-śekhara* which are exactly the same or similar to those found in the *Vaṭeśvara-siddhānta*. They were probably taken from the *Vaṭeśvara-siddhānta*. Bhāskara II, as already mentioned, has borrowed the method of finding

the *lambana* directly, without applying the process of iteration, from Vaṭeśvara. The chapter on the seasons occurring in his *Siddhānta-śiromaṇi* was also probably written under the influence of Vaṭeśvara. The idea of *ayanasandhi*.[1] occurs for the first time in the *Vaṭeśvara-siddhānta*. It is probable that this idea too was borrowed by Bhāskara II from Vaṭeśvara.

There is evidence to show that the *Vaṭeśvara-siddhānta* was studied at places which were far distant from the place where Vaṭeśvara lived. Sundararāja (c. A.D. 1500), who belonged to the Tamil country in South India, in his commentary on the *Vākya-karaṇa* mentions Vaṭeśa (=Vaṭeśvara) along with Āryabhaṭa I, Lalla, and other Hindu astronomers. Quotations from the *Vaṭeśvara-siddhānta* have been discovered in Māhārāṣṭra and Kashmir. For example, verse 14 of sec. 4, Chap. II, of the *Vaṭeśvara-siddhānta* is found to occur in MS No. 6670 of the *Khaṇḍa-khādyaka* of Brahmagupta, belonging to Ānandāśrama, Poona.[2] Verses 10, 11, 14 and 15 of Sec. 4, Ch. II, of the *Vaṭeśvara-siddhānta* occur in MS No. 1664 (written in the *Śāradā* script of Kashmir) of the Akhila Bharatiya Sanskrit Parisad Library, Lucknow. It may also be added that MS A was purchased in a village fair at Takiyā, near Tirawā in Uttar Pradesh, and MS. B was acquired from Lahore.

OTHER REFERENCES TO VAṬEŚVARA AND HIS *SIDDHĀNTA*

References to Vaṭeśvara and *Vaṭeśvara-siddhānta* are found to occur also in the writings of the scholars hailing from the Āndhra State of South India, viz. Mallikārjuna Sūri and Yallaya. Mallikārjuna Suri, in his commentary on the *Sūrya-siddhānta* (xi. 1-6), and Yallaya, in his commentary on the *Laghu-mānasa* (vi. 1) of Mañjula, ascribes the following four verses to *Vaṭeśvara-siddhānta* :

विषमपदगस्य शशिनः चरमधिकं चेद्रवेश्चराद्[1] भूतः ।
पातो भाव्यूनं तद्यदि समपदगेऽन्यथा शशिनि ॥१॥
ऋणधनसमविषमतया युतिविवरं वैधृतेऽन्यथा पाते ।
योऽसौ प्रथमो राशिः स्वेष्टघटीभिस्तथाऽन्योऽपि ॥२॥
द्वावपि भूतैष्यौ चेत् तदन्तरेणान्यथा हरेद्युत्या ।
आद्येष्टघातम'द्यह्राप्तं नाड्योऽसकृत्ताभिः ॥३॥
निश्चलघटिकागुणिता विषुवच्छाया विभाजिताऽऽद्येन ।
पातस्य मध्यकालात् शोध्या योज्या तदाद्यन्तौ ॥४॥

1. सूर्यचरगुणाद् (मल्लिकार्जुनसूरिपाठः) ।

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1. See Chap. II, sec. 6, vss. 16(c-d)-17.
2. See *Khaṇḍa-khādyaka* (edited by Bina Chatterjee), vol. I, p. 76, footnote.

These verses do not occur in the *Vaṭeśvara-siddhānta* available to us. But the counterparts of all these verses do occur there, the corresponding verses of the *Vaṭeśvara-siddhānta* being :

विषमपदगे यदीन्दोः क्रान्तिर्महती सहस्रगुक्रान्तेः ।
भूतोऽन्यथा तु भावी समपदगे व्यत्ययात्पातः ॥
विवरयुती व्यतिपाते युतिविवरे वैधृते समान्यदिशोः ।
क्रान्त्योः प्रथमो राशिः स्वेष्टघटीभिस्तथाऽन्योऽपि ॥ [II, 6. 19-20]
यदि भूतो भावी वा द्वयोर्विशेषोऽन्यथा युतिर्हारः ।
आद्येष्टहृतेर्नाड्यः प्रथमवशान्मध्यमेताभिः ॥
तात्कालिकैर्ग्रहैस्तैरसकृत्त्ववशिष्टमध्यनाडीघ्नम् ।
मानैक्यार्धं भक्तं प्रथमेनाप्तघटिकाभिराद्यन्तौ ॥ [II, 6. 29-30]

Yallaya, in his commentary on the *Sūrya-siddhānta* (xi. 1-6) and also in his commentary on the *Laghu-mānasa* (vi. 1), attributes the following verse to Vaṭeśvara :

समाशयोः शीतकरार्कयोः स्याद्
भार्धं युतिश्चेदयने विभिन्ने ।
योगो व्यतीपात इहान्यदिक्कयोः
तुल्यायने चक्रमितौ तु वैधृतिः ॥

This verse too does not occur in the *Vaṭeśvara-siddhānta*, but its counterpart does exist there in the form :

एकदिशोर्व्यतिपातः क्रान्त्योर्विदिशोस्तु वैधृतं भवति ।
दिग्भेदेऽपक्रमणं महदप्यूनं विधोर्ज्ञेयम् ॥ [II, 6. 18]

In case the verses ascribed to Vaṭeśvara or *Vaṭeśvara-siddhānta* by Mallikārjuna Sūri and Yallaya are really from the pen of our Vaṭeśvara they must be from his *Karaṇasāra* or some other work on astronomy written by him. It is probable that Vaṭeśvara, like Āryabhaṭa I and Lalla, wrote two works on astronomy, besides his *Karaṇasāra* and *Gola*.

Yallaya has also made an important statement which shows that the special visibility correction (*dṛkkarma-viśeṣa*) for the Moon which consists of the evection and the deficit of the Moon's equation of the centre, the same as stated by Mañjula in the *Laghu-mānasa*[1] too occurred in the *Vaṭeśvara-siddhānta*. Yallaya has also stated this correction in five verses composed by himself. These verses were earlier supposed to be composed by Vaṭeśvara.[2]

This special visibility correction too does not occur in the *Vaṭeśvara-siddhānta* available to us. This too must have occurred in the *Karaṇasāra* or some other work on astronomy written by Vaṭeśvara.

VAṬEŚVARA'S *GOLA*

The Sanskrit text of the first five chapters of Vaṭeśvara's *Gola* that have survived in fragments has been given in Part I of this work. In these chapters there is no specific mention of Vaṭeśvara, nor the colophons at their ends mention his name. But the occurrence of these chapters towards the end of MS A and the mention, in vs. 24 of ch. III, of the terms *laghu-bhū-khaṇḍa*, *mahad-bhū-khaṇḍa*, *bṛhad-bhū-khaṇḍa* and *mahat-ku-śakala*, which have been used by Vaṭeśvara only, have led us to believe that the author of these chapters was Vaṭeśvara.

These chapters do not reveal any significant originality of the author. The author seems to have formally fulfilled his duty of writing a *Gola* besides a *Siddhānta*, because the author of a *Siddhānta* must write on *Gola* too. It is found that these chapters of Vaṭeśvara's *Gola* are undoubtedly based on Lalla's *Gola*. Most of the verses of these chapters have their counterparts in Lalla's *Gola* and occur in almost the same sequence. Sometimes the language and words are also the same. The borrowing is evident. The influence of Bhāskara I and Brahmagupta is also visible in one or two places. Parallel passages of Lalla's *Gola* and other works have been noted in the footnotes.

The errors comitted by Lalla in his *Gola* have also been copied. Thus, following Lalla, Vaṭeśvara says :[3]

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1. See *LMā* (Ānandāśrama edition), iii. 1-2.
2. See K. S. Shukla "The evection and the deficit of the equation of the centre of the Moon in Hindu astronomy", *Proc. Benares Math. Soc.*, N. S., Vol. VII, No. 2, Dec. 1945.
3. See *VG*, iv. 16-17.

"The sign whose right ascension is equal to its ascensional difference at a place is always visible at that place, and that sign remains (permanently) invisible at that place which is at the same declination (southwards) as the sign (of north declination) which is always visible there.

"Where the latitude amounts to 66 degrees, there the signs Capricorn and Sagittarius are not visible; and where the latitude amounts to 75 degrees, there the signs Aquarius, Scorpio, Sagittarius and Capricorn are always invisible."

But this is mathematically incorrect and was criticized by Bhāskara II. The same erroneous statement was made by Śrīpati too. It seems that the *Pañca-siddhāntikā,* which deals with this topic correctly, was not a popular work; at least Lalla, Vaṭeśvara and Śrīpati had not seen it. Otherwise, they would have saved themselves from this serious error.

Again, following Lalla, Vaṭeśvara writes :[1]

"When a planet is at the intersection of the *kakṣyāvṛtta* and *mandaprativṛtta*, its mean motion itself is its true motion."

Śrīpati has also said the same, but Bhāskara II has rightly criticised this statement.

It is noteworthy that Lalla, though a follower of Āryabhaṭa I believes that the Earth is stationary, but he does not say so specifically. Vaṭeśvara expressly states that the Earth is stationary.

NOTABLE FEATURES

The following features of the five chapters of Vaṭeśvara's *Gola* deserve notice :

1. *Two great circles added to Khagola :*

Lalla's *Khagola* consists of six great cireles only, viz. (1) the prime vertical, (2) the meridian (3,4) the two vertical circles through the intermediate cardinal points, (5) the horizon, and (6) the six o'clock circle. Vaṭeśvara adds two more great circles, viz. (7) the vertical circle through the planet observed and (8) the vertical circle through that point of the ecliptic which lies three signs behind the horizon-ecliptic point. Bhāskara II has followed Vaṭeśvara in this matter.

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1. See *VG*, ii. 7.

2. *A list of right-angled triangles added* :

Unlike Lalla, Vaṭeśvara gives a list of eight right-angled triangles associated with the armillary sphere, including the declination triangle, the latitude triangles, and the *lambana* triangles. Bhāskara II has also given a list of declination and latitude triangles.

3. *Absence of hypotenuse-proportion in mandakarma and iteration of mandakarṇa explained* :

Vaṭeśvara explains why the hypotenuse is not used in finding the equation of the centre, why the *mandakarṇa* is obtained by the process of iteration, and also why the process of iteration is employed in deriving the mean longitude of the Sun or Moon from its true longitude. This was not done by Lalla.

ENGLISH TRANSLATION

The question of translating technical material written in Sanskrit into English presents considerable difficulty. It requires thorough knowledge of both the languages, which few can claim. Effort has been directed towards giving, as far as possible, a literal version of the text in English. At the same time care has been taken to ensure that it is clear and easily understandable. The portions of the English translation enclosed within brackets do not occur in the text and have been given in the translation to make it understandable and are, at places, explanatory. Without these portions, translation, at these places, might appear meaningless to a reader who cannot consult the original for lack of knowledge of Sanskrit. Attempt has been made to keep the spirit of the original and as for as possible the sequence of the text has been unaltered. Sanskrit technical terms having no equivalents in English have been given as such in the translation. They have been explained in the subjoined notes.

Verses dealing with the same rule, have been translated together and are prefixed by an introductory heading briefly summarising their contents.

The translation is followed by short notes and comments comprising : (1) elucidation of the text where necessary, (2) rationale of the rule given in the text, (3) illustrative solved examples, where necessary, (4) critical notes, and (5) other relevant matter, depending on the passage translated. In doing so vast literature has been consulted and

parallel passages occurring elsewhere have been noted in the footnotes. This has been of considerable help in translating the text; without it quite a number of passages would have remained obscure.

For the convenience of the reader, the chapter-heading has been mentioned at the top on the left hand page and the section-heading at the top on the right hand page. The chapter-number and the section-number are also mentioned at the top.

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K. S. Shukla

Vaṭeśvara-Siddhānta

English Translation and Commentary

Chapter I

MEAN MOTION

Section 1 : Revolutions of the Planets

HOMAGE AND INTRODUCTION

1. Having paid obeisance to Brahmā, the Earth, the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn, the asterisms, the teacher, and to parents, I, Vaṭeśvara, son of Mahadatta, very clearly set out the entire science of astronomy (lit. mathematics pertaining to the planets and the asterisms) that was promulgated by Brahmā.

Homage to Brahmā and reference to Brahmā as the promulgator of the science of astronomy show that Vaṭeśvara was a follower of the Brahma school of Hindu astronomy.

ACKNOWLEDGEMENT TO DIVINE SAGES AND AIM OF THE WORK

2. This (science of astronomy) was (first) taught by the divine sages whose excellent intellect was purified by the vast and deep knowledge of *kālakriyā* ("reckoning with time"), *gaṇita* ("mathematics") and *gola* ("the celestial sphere, or spherics"), the subjects of that great science. When we, ignorant people, consulting their teachings, write on the subject, the credit is theirs.

3. But to those who by virtue of their intellect have a difference of views, the *yuga* prescribed by Brahmā does not always lead to equally correct results. So the essence of the teachings of all the *śāstras* ("texts on astronomy") is being set out, excepting all based on erroneous views.

ASTRONOMY—THE EYE OF THE VEDA AND HIGHLY HONOURED SCIENCE

4. It is this science (of astronomy) that has been regarded as the eye of the Veda, for the reason that the Vedic sacrifices are performed at the specified times defined by *ayana* ("northward or southward course of the Sun"), season, *tithi*, *parva* ("full moon or new moon") and day etc., and the sacrificial altars, the cardinal points, the fire-pits (meant for

offering oblation to fire) and the distances involved therein, etc., are to be correctly known (by the Vedic priests);[1] and so this science stands highly honoured amongst the Vedic scholars.

DEFINITION OF *SIDDHĀNTA*

5. An astronomical work which describes all measures of time as well as the determination of longitudes of the planets, which treats all mathematics including the theory of the pulveriser, etc., and which correctly states the configurations and positions of the planets, the asterisms, and the Earth, is verily called a true *Rāddhānta* (or *Siddhānta*) by the distinguished sages.[2]

CREATION OF ASTERISMS AND PLANETS

6. In the beginning (of creation), Brahmā created the ever-revolving circle of asterisms—a net of twinkling stars, fastened to the Pole star lying in front of the orbits of the planets ranging from Saturn to the Moon, together with the planets lying at the junction of the signs Pisces and Aries, lorded over by the Sun and the Moon (the planets by the Sun and the stars by the Moon).[3]

TIME - MEASURES AND CIRCULAR MEASURES.
DURATION OF BRAHMĀ'S LIFE

7. The time taken (by a sharp needle) to pierce (a petal of) a lotus flower is called a *truṭi*;[4] one hundred times that is called a *lava*; one hunded times that is a *nimeṣa* ; four and a half times that is a "long syllable" (i. e., time required for pronouncing a long syllable by a healthy person with a moderate flow of voice); four times that is a *kāṣṭhā*; and one half of five times that is an *asu*.

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1. *Cf. SiŚi*, I, i (a). 9. According to the ***Pañca-siddhāntikā***, the reason why the science of astronomy should be studied by a Brāhmaṇa is as follows :
   कालापेक्षाविधयः श्रौताः स्मार्तश्च तदपचारेण ।
   प्रायश्चित्ती भवति द्विजो यतोऽतोऽधिगम्येदम् ॥ (***PSi***, iii, 36)
2. *Cf. SiŚe*, i. 3; *SiŚi*, I, i(a). 6-8.
3. *Cf. BrSpSi*, i. 3; *SiŚe*, i. 9; *MSi*, i. 3; *SiŚi*, I, i (a). 13-14.
   In ancient times, people thought that the stars and planets were attached to the Pole by means of strings of air, of varying lengths, and as the Pole rotated, the stars and planets moved and made revolutions around the Earth. See *Matsya-purāṇa*, ch, 124, vss. 2, 3, 5, 9 (a-b).
   चन्द्रमा नक्षत्राणामधिपतिः
   See *Atharva-veda, kāṇḍa* 5, *Sūkta* 24, *Mantra* 10.
4. तीक्ष्णसूच्याऽब्जपुष्पस्य दलछेदस्त्रुटिर्भवेत् ।

8. Six *asus* make a sidereal *pala*; sixty *palas* make a *ghaṭikā*; sixty *ghaṭikās* make a day; thirty days make a month; and twelve times that is a year.[1] The divisions of the circle too have been defined in the same manner as those of time excepting those up to *asu*.

9. Solar years amounting to 432 multiplied by 10000 make a *yuga*; a period of 72 *yugas* is called a *manu*; a period of 14 *manus* is a *kalpa*; a couple of them is a day-and-night of Brahmā; and a century of Brahmā's own years is stated to be the duration of his life.

The above-mentioned divisions of time may be stated in the tabular form as follows:

Table 1. The time-divisions

lotus-pricking time	=	1 *truṭi*
100 *truṭis*	=	1 *lava*
100 *lavas*	=	1 *nimeṣa* (twinkling of eye)
4½ *nimeṣas*	=	1 long syllable
4 long syllables	=	1 *kāṣṭhā*
2½ *kāṣṭhās*	=	1 *asu* (respiration) = 4 seconds
6 *asus*	=	1 sidereal *pala* (*caṣaka*, *vināḍī* or *vighaṭikā*)
60 *palas*	=	1 *ghaṭikā* = 24 minutes
60 *ghaṭikās*	=	1 day
30 days	=	1 month
12 months	=	1 year
4320000 years	=	1 *yuga*
72 *yugas*	=	1 *manu*
14 *manus*	=	1 *kalpa*
2 *kalpas*	=	1 day of Brahmā (including night)
30 days of Brahmā	=	1 month of Brahmā
12 months of Brahmā	=	1 year of Brahmā

1. Similar time-divisions are found to be stated in *BrSpSi*, i. 5-6; *SiŚe*, i. 11-15; i. 3 (Makkibhaṭṭa's comm.) ; *SiŚi*, I, i (a). 16-18. Also see *Skanda-purāṇa*, *Nāgara-khaṇḍa*, ch. 184, vss. 11-26 ; *Mārkaṇḍeya-purāṇa* , ch. 43, vss. 23-44.

1 year of Brahmā	=	$72 \times 14 \times 2 \times 30 \times 12$ *yugas*
	=	725760 *yugas*
100 years of Brahmā	=	life of Brahmā, or *mahākalpa.*

The divisions of a circle may be stated in the tabular form as follows:

Table 2. The circular divisions

60 *vikalās* (seconds)	=	1 *kalā* or *liptā* (minute)
60 *kalās* (minutes)	=	1 *bhāga* or *aṁśa* (degree)
30 *bhāgas* (degrees)	=	1 *rāśi* (sign)
12 *rāśis* (signs)	=	1 *bhagaṇa* or *cakra* (revolution)

This division is evidently similar to that of time from *pala* to year.

AGE OF BRAHMĀ IN THE BEGINNING OF THE ŚAKA ERA

10. Since the birth of Brahmā up to the beginning of the *Śaka* era, $8\frac{1}{2}$ years (of Brahmā), $\frac{1}{2}$ month (of Brahmā), 6 *manus* of the (current) day (of Brahmā), $27\frac{3}{4}$ *yugas*, and 3179 years of the (current) Kali era had gone by.

There are two theories regarding the age of Brahmā. According to one, 50 years of Brahmā's life had elapsed at the beginning of the current *kalpa*; and according to the other about eight and a half years of Brahmā had then elapsed. The authors of the *Sūrya-siddhānta*[1], the *Siddhānta-śiromaṇi*[2], *the Mārkaṇḍeya-purāṇa*,[3] and the *Viṣṇu-purāṇa*[4] are the exponents of the former, and Puliśa,[5] Lalla,[6] Sumati,[7] Vaṭeśvara, and the author of the *Skanda-purāṇa*,[8] the exponents of the latter.

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1. i. 21.
2. I, i (a). 26.
3. ch. 43, vss. 43-44.
4. *Aṁśa* 1, ch. 3, vs. 27.
5. See *Albīrūnī's India*, Vol. 1, p, 374.
6. See Yallaya's comm. on *SūSi*, i. 21.
7. See *Sumati-mahātantra*, ch. 1.
8. *Nāgara-khaṇḍa*, ch. 184, vss. 22-23 (i) ; ch. 228, vs. 8.

According to Vaṭeśvara: Brahmā's age at the beginning of the current *kalpa*

= 8½ years of Brahmā + ½ month of Brahmā

= 6150 *kalpas* or 6150 × 1008 *yugas* or 6150 × 1008 × 4320000 years.

= 26780544000000 years;

Brahmā's age at the beginning of the current Kaliyuga

= 8½ years of Brahmā + ½ month of Brahmā + 6 *manus* + 27¾ *yugas*

= 6199659¾ *yugas*

= 26782530120000 years;

and Brahmā's age at the beginning of the Śaka era

= 8½ years of Brahmā + ½ month of Brahmā + 6 *manus* + 27¾ *yugas* + 3179 years

= 26782530123179 years.

According to Puliśa:[1]

Brahmā's age at the beginning of the current *kalpa*

= 8 years of Brahmā + 5 months of Brahmā + 4 days of Brahmā

= 6068 *kalpas* or 6068 × 1008 *yugas*

= 26423470080000 years.

According to Lalla:[2]

Brahmā's age at the beginning of the current *kalpa*

= 8½ years of Brahmā + ½ month of Brahmā

= 6150 *kalpas* or 6150 × 1000 *yugas*

= 26568000000000 years.

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1. See *Albīrūnī's India*, Vol. 1, p. 374.
2. See Yallaya's comm. on *SūSi*, i. 21. In his *Siddhānta-tilaka*, Lalla is said to have written :

स्वयं महाकल्प इति स्वसंख्यया स्वजन्मनोऽष्टौ सदलाः समाः ययुः ।
तथाऽर्धमासोऽथ गता नवत्सरा नवाम्बराण्यष्टषडर्थषड्यमाः(२६५६८०००००००००) ॥

According to Sumati :[1]

Brahmā's age at the beginning of the current *kalpa*
= 8 years of Brahmā

According to the *Skanda-purāṇa* :[2]

Brahmā's age
= 8 years 6 months and 1/4 of a day of Brahmā.

REVOLUTIONS OF PLANETS

11. 43,20,000 are stated to be the revolutions performed in a *yuga* by Venus, Mercury, and the Sun and also by the *Śīghroccas* of Saturn, Jupiter, and Mars.

12-14. The revolutions of the Moon, as stated by the learned, are 5,77,53,336; of Mars, 22,96,828; of Jupiter, 3,64,220; of Saturn, 1,46,568; of the *Śīghrocca* of Mercury, 1,79,37,020 plus 36 (i.e., 1,79,37,056); of the *Śīghrocca* of Venus, 70,22,376; of the Moon's apogee, 4,88,211; and of the Moon's node, 2,32,234.

These revolutions, as suggested by Roger Billard,[3] were probably obtained by the application of the *Bīja* correction, prescribed by Lalla, to the revolutions given by Āryabhaṭa I. See the table below.

	Āryabhaṭa-I's revolutions	*Bīja* correction	Corrected revolutions	Actual revolutions stated by Vaṭeśvara
Sun	4320000	Nil	4320000	4320000
Moon	57753336	—2	57753334	57753336
Moon's apogee	488219	—9.12	488210	488211
Moon's node	—232226	—7.68	—232234	—232234
Mars	2296824	+3.84	2296828	2296828
Mercury's *Śīghrocca*	17937020	+33.60	17937054	17937056
Jupiter	364224	—3.76	364220	364220
Venus' *Śīghrocca*	7022388	—12.24	7022376	7022376
Saturn	146564	+1.6	146566	146568

1. See *Sumati-mahā-tantra*, ch. 1.
2. See *Nāgara-khaṇḍa*, ch. 184, vss. 22-23 ; ch. 227, vs. 8.
3. See *L'Astronomie Indienne*, Paris (1971), p. 149.

Since the revolutions of the Sun, Moon and the planets stated by Ārya-bhaṭa-I were divisible by 4, so, Vaṭeśvara, in order to preserve this chracteristic feature of Āryabhaṭa-I's revolutions, increased the *Bīja*-corrected revolutions of the Moon, Mercury's *Śīghrocca* and Saturn by 2. Similarly, he added 1 to the revolutions of the Moon's apogee to make them odd and prime to the number of civil days is a *yuga* as in Āryabhaṭa. This explains the difference of the revolutions of Vaṭeśvara from the *Bīja*-corrected revolutions of Āryabhaṭa-I.

It is noteworthy that the revolutions stated by Vaṭeśvara are the same as those given in the revised *Sūrya-siddhānta* which was utilized by Vijaya-nandī in writing his *Karaṇa-tilaka* and found to be stated in Parameśvara's version of the *Sūrya-siddhānta*, except in the cases of Mars and the *Śīghrocca* of Mercury, where the *Sūrya-siddhānta* gives 4 revolutions more than Vaṭeśvara.

It seems that the redactor of the *Sūrya-siddhānta* has borrowed the revolutions of the planets from the *Vaṭeśvara-siddhhānta*, adopting those of Moon's apogee, Moon's node, Jupiter, the *Śīghrocca* of Venus and Saturn without any alteration and those of Mars and the *Śīghrocca* of Mercury after suitable modification.

Table 3. Revolutions of the planets in a *yuga*

Planet	Revolutions according to		
	Āryabhaṭa-I	*Sūrya-siddhānta*	Vaṭeśvara
Sun	4320000	4320000	4320000
Moon	57753336	57753336	57753336
Moon's apogee	488219	488211	488211
Moon's node	—232236	—232234	—232234
Mars	2296824	2296832	2296828
Mercury's *Śīghrocca*	17937020	17937060	17937056
Jupiter	364224	364220	364220
Venus's *Śīghrocca*	7022388	7022376	7022376
Saturn	146564	146568	146568

REVOLUTIONS OF THE SEVEN SAGES (OR STARS OF THE GREAT BEAR)

15. Endowed with the boon acquired from the planets, I now state the revolutions performed by the Sages in a *yuga*, which were pronounced in clear words by the lotus-like mouth of the lotus-seated Brahmā, as 1692.[1]

The stars of the constellation of the Great Bear do not have a motion relative to the *nakṣatras*. So the statement of their revolutions is not correct.

REVOLUTIONS OF PLANETS' APOGEES AND NODES

16-17. During the life of Brahmā, the revolutions performed by the apogee of the Sun are 1,65,801; of Mars, 81,165; of Jupiter, 13,948; of Saturn, 6,774; of Mercury, 4,77,291; and of Venus, 1,52,842.

18-19. 98,82,71,45,64,18,719; 19,61,27,64,06,36,895; 20,684; 3,802; and 1,542 are, in order, the revolutions performed by the nodes of Mercury, Venus, Mars, Jupiter, and Saturn, during the life of Brahmā.

NODES OF MERCURY AND VENUS : THEIR ACTUAL REVOLUTIONS

20. In the case of Mercury and Venus, it is the remainder obtained by dividing the revolutions of the planet's node (stated above) by the revolutions of the planet's *śīghra* epicycle (i.e., by the revolutions of the planet's *śīghra* anomaly) that really gives the actual revolutions of the

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1. Albīrūnī (*India* I, p. 392) quotes a rule from Vaṭeśvara's *Karaṇasāra* which gives the method of computing the position of the Great Bear (called Saptarṣi or the seven sages). This runs as follows :

" Multiply the basis (i.e., the years elapsed since the beginning of Śaka 821) by 47 and add 68000 to the product. Divide the sum by 10,000. The quotient represents the zodiacal signs and fractions of them, i.e., the position of the Great Bear which was sought."

According to this rule, the Great Bear has a motion of 47 signs per 10,000 years, which is equivalent to 1692 revolutions per 43,20,000 years, as given above.

The position of the Great Bear in Śaka 821 (i.e. Kali year 4000) was

$$\frac{1692\times12\times4000}{4320000}\text{ signs} = 1\text{ rev.} + \frac{68000}{10000}\text{ signs},$$

which accounts for the addition of 68000 in the rule quoted by Albīrūnī from the *Karaṇasāra*.

planet's node but the learned (astronomers) declare the sum of the actual revolutions of the planet and the revolutions of the planet's *śīghra* anomaly as the revolutions of the planet's node.[1]

This means that the revolutions of the nodes of Mercury and Venus which have been stated in vs. 18 above are not the actual revolutions of the nodes of Mercury and Venus, but the sum of the actual revolutions of the nodes of Mercury and Venus and the revolutions of the *śīghra* anomalies of those planets. The actual revolutions of the nodes of Mercury and Venus are 1,62,719 and 60,895 respectively which are the remainders obtained by dividing the revolutions of the nodes of Mercury and Venus, stated in vs. 18, by the revolution-numbers of their *sīghra* anomalies, or, what is the same thing, by subtracting the latter from the former.

Below are given the revolutions of the apogees and nodes of the planets according to Vaṭeśvara in the tabular form:

Table 4. Revolutions of the apogees and nodes of the planets during the life of Brahmā (i. e., in 72576000 *yugas*)

	Revolutions of apogee	Revolutions of node
Sun	165801	—
Mars	81165	—20684
Mercury	477291	—162719
Jupiter	13948	—3802
Venus	152842	—60895
Saturn	6774	—1542

It is to be noted that these revolutions of the apogees and nodes of the planets are not the actual revolutions performed by them in the period of 72576000 *yugas*. They have been obtained as the least positive integral solutions of the pulverisers formed from their approximate positions for certain known time. They are far less than their actual values and are of no utility in astronomical calculations.

1. Also see *SiŚi*, II, vi. 23.

The following table gives the annual motions in longitude of the apogees and nodes of the planets according to modern astronomy.

Table 5. Annual motions (*nirayaṇa*) of the apogees and nodes of the planets

	Practical Astronomy by Loomis	Positional Astronomy Centre Calcutta
Apogee of:		
Sun	+11.24″	+11.63″
Mars	+15.46″	+15.99″
Mercury	+ 5.81″	+ 5.73″
Jupiter	+ 6.65″	+ 7.72″
Venus	— 3.24″	+ 0.38″
Saturn	+19.31″	+ 22.25″
Node of:		
Mars	—25.32″	—22.524″
Mercury	—10.07″	— 7.606″
Jupiter	—15.90″	—13.888″
Venus	—20.50″	—17.870″
Saturn	—19.54″	—18.850″

TIME OF AUTHOR'S BIRTH AND AGE AT COMPOSITION OF THIS WORK

21. When 802 years had elapsed since the commencement of the Śaka era, my birth took place; and when 24 years had passed since my birth, this *Siddhānta* was written by me by the grace of the heavenly bodies.

This shows that the author, Vaṭeśvara, was born in 880 A.D. and that the present work, the *Vaṭeśvara-siddhānta*, was written in 904 A.D. when the author had attained the age of 24 years.

Section 2 : Time-measures

SIDEREAL AND CIVIL DAYS, LUNAR MONTH AND SOLAR YEAR, INTERCALARY MONTHS AND OMITTED DAYS

1. 1,58,22,37,560 is the number of risings of the asterisms in a *yuga*. This diminished by the revolutions of a planet gives the number of risings in the east of that planet (in a *yuga*).[1] The risings of the Sun are called the terrestrial civil days.

2. The difference between the revolutions of two planets gives the number of conjunctions of those two planets (in a *yuga*). The conjunctions of the Sun and Moon are the lunar (or synodic) months. The revolutions of the Sun are the solar years. The risings of the asterisms stated above are the sidereal days.

3-4(a-b). The difference between the revolutions of a planet and its (*manda* or *śīghra*) apogee gives the so called revolutions of the (*manda* or *śīghra*) epicycle of that planet.[2] The difference between the lunar and solar months gives the so called intercalary months. The difference between the lunar and civil days is called the omitted days.

Table 6. Years, months, and days in a *yuga*

Type of years, etc.	Number in a *yuga*	Remark
Solar years	43,20,000	Same as in *Ā* and *SūSi*
Solar months	5,18,40,000	,,
Solar days	1,55,52,00,000	,,
Lunar months	5,34,33,336	,,
Lunar days	1,60,30,00,080	,,
Intercalary months	15,93,336	,,
Civil days	1,57,79,17,560	60 more than in *Ā*
Omitted days	2,50,82,250	60 less than in *Ā*

1. *Cf. SiŚe*, i. 35.
2. *Cf. Ā*, iii. 4 (a-b) ; *SiŚe*, i. 42 (c-d).

DAYS OF MANES, GODS AND DEMONS

4 (b-d). The solar year is called the year of men, and the lunar month is called the day of the manes (who are supposed to reside on the opposite side of the Moon). The solar year is also called the day of the gods (residing at the North Pole) and the demons (residing at the South Pole).

JOVIAN YEARS AND *VYATIPĀTAS*

5. The product of the revolutions performed by Jupiter by 12 gives the (elapsed) Jovian years beginning with Vijaya or Āśvina.[1] Two times the sum of the revolutions performed by the Sun and the Moon gives (the number of elapsed) ***Vyatipātas.*** [2]

A Jovian year is the time taken by Jupiter in passing through one sign of the zodiac. There are two cycles of Jovian years, one consisting of 12 Jovian years and the other consisting of 60 Jovian yeats. The years of the 12-year cycle bear the same names as the 12 lunar months, but the 12-year cycle begins with Āśvina. The years of the 60-year cycle bear the names Vijaya etc. (or Prabhava etc.).[3] According to Vaṭeśvara, as also according to the author of the *Sūrya-siddhānta,* these cycles started together, the former with Āśvina and the latter with Vijaya, in the beginning of a *yuga.* Both the cycles took a new round and started with Āśvina and Vijaya respectively in the beginning of *Kaliyuga* (i.e.. at sunrise at Laṅkā on Friday, February 18, B.C. 3102).

The 60-year cycle of Jupiter is divided into 12 sub-cycles, each consisting of 5 Jovian years. These sub-cycles are called Nārāyaṇādi *yugas* and are given the following names after their presiding deities :

1. Nārāyaṇa or Viṣṇu	5. Tvaṣṭrā	9. Soma
2. Bṛhaspati	6. Ahirbudhnya	10. Indrāgni
3. Indra	7. Pitṛ	11. Aśvi
4. Agni	8. Viśva (or Viśvedeva)	12. Bhaga

See *Ratnamālā*, i. 14; or *BṛSaṁ*, ch. viii. 23.

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1. *Cf. SiŚe*, i. 43.
2. *Cf. BrSpSi*, xiii. 41.
3. For the names of the 60 Jovian years of the 60-year cycle, the reader is referred to my notes on *A*, lii. 4 (c-d).

As regards *Vyatipāta*, the reader is referred to my edition of the *Karaṇa-ratna* or to ch. xi of the *Sūrya-siddhānta*.

UTSARPIṆĪ, APASARPIṆĪ, SUṢAMĀ, AND *DUṢṢAMĀ*

6. The first half of a *yuga* is called *Utsarpiṇī* and the second half is called *Apasarpiṇikā* (or *Apasarpiṇī*). *Suṣamā* occurs in the middle of a *yuga* and *Duṣṣamā* at the beginning and end (i.e., the second and third quarters of a *yuga* are called *Susamā* and the first and fourth quarters of a *yuga* are called *Duṣṣamā*). (The time elapsed or to elapse is to be reckoned) from the position of the Moon's apogee.

This is exactly what Āryabhaṭa-I has written. For details, the reader is referred to my notes on *Ā*, iii. 9.

CONSTANTS FOR *KALPA* OF LIFE OF BRAHMĀ

7. The revolutions etc., which have been stated above for a *yuga*, when multiplied by 1008, correspond to a *kalpa*, and, when further multiplied by 72000, correspond to the life of Brahmā.

This is so, because

1 *kalpa* = 1008 *yugas*

and life of Brahmā = 72000 *kalpas*.

See *supra*, sec. 1, vs. 9.

THE ZERO POINT

8. The cycle of time, commencing with *truṭi* and ending with the life of Brahmā, was started by Him (i. e. by Brahmā) on Saturday, in the beginning of the light half of Caitra, when the planets, situated on the horizon of Laṅkā were at the junction of the signs Pisces and Aries.[1]

NINE MODES OF TIME-RECKONING

9. The imperishable time is measured by sidereal, lunar, solar, civil *Brāhma*, Jovian, Paternal, divine, and demoniacal reckonings. This is why (these) nine varieties of time-reckoning have been defined.

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1. Similar statements are made in *BrSpSi*, i. 4; *MSi*, i. 5 and *SiŚi*, I, i (a), 15.

Table 5. Units of nine varieties of time-reckoning

Reckoning	Unit used
sidereal	sidereal day : one star-rise to the next
lunar	lunar month : one new moon to the next
solar	solar year : period of one solar revolution
civil	civil day : one sunrise to the next
Brāhma	day of Brahmā : period of 2 *kalpas* or 2016 *yugas*
Jovian	Jovian year : period of Jupiter's motion through a sign
paternal	day of manes : one lunar month
divine and demoniacal	day of gods and demons : one solar year

USE OF TIME-RECKONINGS

10-11. The knowledge of *parva* (new moon and full moon days), *avama* (omitted days), *tithi*, *karaṇa*, and *adhimāsa* (intercalary month) is acquired on the basis of lunar reckoning. The sixty (Jovian) years, Prabhava etc., and the *yugas*, Nārāyaṇa etc.,[1] — the knowledge of these is acquired on the basis of the Jovian reckoning. The Pitṛ-yajña (i.e., sacrificial rites pertaining to the deceased ancestors) is performed on the basis of the paternal reckoning. On the basis of the Brāhma, divine and demoniacal reckonings are determined the life-spans (and other sub-measures) of the lives of Brahmā, gods and demons.

12-13. The study of the Vedas, duties specified under *niyama*,[2] (rites performed to get over) the impurities caused by birth or death, the sacrificial rites, penance, medical treatment, *horā* (=hour), *muhūrta* (=a period of 2 *ghaṭīs*), and *yāma* (=*prahara*, or a period equal to one-eighth of a day), atonement of sin, fast, duration of man's life, and

1. See supra, vs. 5, notes.

२. शौचमिज्या तपो दानं स्वाध्यायोपस्थनिग्रहः ।
व्रतमौनोपवासं च स्नानं च नियमा दश ॥

departure and return are based on the civil reckoning The seasons, northward and southward courses of the Sun, equinoxes, years, and *yuga* as well as the increase and decrease of day are ascertained on the basis of the solar reckoning.

14. Computation of sines and determinations pertaining to the revolutions of the Moon are performed on the basis of the sidereal reckoning. On the same are also based the nomenclatures of the months, years, and days, as well as the knowledge of good and bad consequences.[1]

For more details regarding the uses of the various time-reckonings, the reader is referred to Bhāskara-I's commentary on *Ā*, iii. 5.

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1. Also see *BrSpSi*, *mānādhyāya*, xxiii. vss. 1-6.

Section 3 : Calculation of the *Ahargaṇa*

METHOD 1. GENERAL METHOD

1-2. Multiply the years elapsed since the birth of Brahmā, or since the beginning of the current *Kalpa*, or since the beginning of the current *yuga*, by 12 ; (to the product) add the number of lunar months (elapsed since the beginning of the light half of Caitra); multiply (the sum) by 30; (to the product) add the number of lunar days (elapsed since the beginning of the current lunar month); and set down the result at two places. At one place, multiply that by the number of intercalary months (in a *yuga*) and divide the product by the number of solar days (in a *yuga*). Add the days corresponding to the resulting intercalary months to the result at the other place; and write down the sum at two places. At one place, multiply that by the number of omitted days (in a *yuga*) and divide the product by the number of lunar days (in a *yuga*). By the resulting omitted days, diminish the result at the other place: the result thus obtained is the *Ahargaṇa*. This being divided by seven and the remainder counted from Saturn, Saturn, or Sun (respectively), (depending on whether the epoch chosen is the birth of Brahmā, the beginning of the current *kalpa*, or the beginning of the current *yuga*), gives the lord of the current day.[1]

The *rationale* of this rule is similar to that given in my notes on *MBh*, i. 4-6. The interested reader is referred to it. It must, however, be noted that, according to Vaṭeśvara, the birth of Brahmā took place on Saturday, the present *kalpa* commenced on Saturday, the current *yuga* commenced on Sunday, and the current Kaliyuga commenced on Friday.

According to Āryabhaṭa-I, the current *kalpa* commenced on Thursday, and the current *yuga* commenced on Wednesday. This difference is due to the difference in the number of civil days in a *yuga* according to the two authors.

METHOD 2. SHORTER METHOD
(without using intercalary months and omitted days)

3. Multiply the number of solar months elapsed (since the epoch chosen)[2] by the number of lunar days (in a *yuga*) and divide the product

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1. Same method occurs in *ŚiDVr*. i. 12-14; *SiŚe*, ii. 1-2 (a-b) ; *SiŚi*, I, i (c). 1-3.
2. What is meant is : the number of solar years elapsed since the epoch multiplied by 12, plus the number of lunar months elapsed since the beginning of Caitra.

by the number of solar days (in a *yuga*) ; multiply the resulting quotient (denoting the lunar months elapsed since the epoch) by 30 and to the sum obtained add the number of lunar days elapsed (since the beginning of the current lunar month). Then multiply that by the number of civil days (in a *yuga*) and divide the product by the number of lunar days (in a *yuga*) : the quotient thus arrived at, or that increased by *one*, gives the *Ahargaṇa*.[1]

When the final quotient being divided by seven and the remainder counted as directed in the previous rule one obtains the lord of the current day, the final quotient itself is the *Ahargaṇa*. In case one obtains the lord of the preceding day, the final quotient increased by *one* is the *Ahargaṇa* ; and in case one obtains the lord of the succeeding day, the final quotient diminished by *one* is the *Ahargaṇa*.

It would have been simpler if the number of solar months elapsed (since the epoch chosen) was multiplied by the number of lunar months in a *yuga* and the resulting product divided by the number of solar months in a *yuga*, as done by Brahmagupta, Āryabhaṭa II, and Śrīpati.

METHOD 3. WHEN *ADHIMĀSA-ŚEṢA* IS KNOWN

4. Multiply the number of solar days elapsed (since the epoch chosen)[2] **by the number of civil days in a *yuga* ; (then) diminish (the product) by 30 times the residue of the intercalary months (*adhimāsa-śeṣa*) ; and (then) divide (the remainder) by the number of solar days in a *yuga*. The quotient thus obtained, or that increased by *one*, gives the *Ahargaṇa*.**

That is,

$$Ahargaṇa = \frac{\text{solar days elapsed} \times \text{civil days in a } yuga - 30 \times adhimāsaśeṣa}{\text{solar days in a } yuga}$$

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1. *Cf. BrSpSi*, xiii. 18; *ŚiDVr*, i. 15-16; (abraded numbers are used here) ; *MSi*, xvii. 19-20(a-b); *SiŚe*, ii. 3. A similar rule occurs in *SiŚe*, ii. 6-7. Also see the problem set in *SiŚe*, xx. 3 (a-b).

2. What is meant is : the number of solar years elapsed since the epoch multiplied by 12, then increased by the number of lunar months elapsed since the beginning of Caitra, then multiplied by 30, and then increased by the number of lunar days elapsed since the beginning of the current lunar month.

METHOD 4. ANOTHER SHORTER METHOD

5. Multiply the number of solar days elapsed (since the epoch chosen) by the difference between the intercalary and omitted days (in a *yuga*) and divide the product by the number of solar days (in a *yuga*). Add the resulting quotient to the number of solar days elapsed (since the epoch): this sum, or this sum increased by *one*, is the *Ahargaṇa*.

The *rationale* of this rule is as follows:

Since

civil days in a *yuga* = lunar days in a *yuga* − omitted days in a *yuga*

and

lunar days in a *yuga* = solar days in a *yuga* + intercalary days in a *yuga*,

therefore

civil days in a *yuga* = solar days in a *yuga* + (intercalary days in a *yuga* − omitted days in a *yuga*).

Multiplying both sides by the number of solar days elapsed (since the epoch) and dividing by the number of solar days in a *yuga*, we obtain

$$\textit{Ahargaṇa} = \text{solar days elapsed} + \frac{\text{solar days elapsed} \times (\text{intercalary days in a } yuga - \text{omitted days in a } yuga)}{\text{solar days in a } yuga}$$

METHOD 5. WHEN *ADHIMĀSAŚEṢA* AND *AVAMAŚEṢA* ARE KNOWN

6. (At one place) multiply the residue of the intercalary months (*adhikaśeṣa*) by the number of civil days (in a *yuga*) and (at another place) multiply the residue of the omitted days (*avamaśeṣa*) by the number of intercalary months (in a *yuga*). Take the sum of these and divide that sum by the number of lunar days (in a *yuga*): then is obtained the corrected residue of the intercalary months.

7(a-b) Then multiply the number of civil days (in a *yuga*) by the number of intercalary months elapsed; to the product add the corrected

residue of the intercalary months ; and divide the resulting sum by the number of intercalary months (in a *yuga*). The quotient thus obtained is the *Ahargaṇa*.[1]

Let A be the *Ahargaṇa*. Also let C, L, O, and I denote the numbers of civil days, lunar days, omitted days, and intercalary months in a *yuga*. Then

$$\frac{A\times O}{C} = \text{omitted days elapsed} + \frac{R_o}{C},$$

where R_o is the residue of the omitted days.

$$\therefore \frac{A\times O - R_o}{C} = \text{omitted days elapsed}$$

$$\therefore \frac{A\times O - R_o}{C} + A = A + \text{omitted days elapsed}$$
$$= \text{lunar days elapsed}$$

$$\text{or} \quad \frac{A(O+C) - R_o}{C} = \text{lunar days elapsed}$$

$$\text{or} \quad \frac{A\times L - R_o}{C} = \text{lunar days elapsed}$$

$$\therefore \frac{A\times L - R_o}{C} \times \frac{I}{L} = \frac{\text{lunar days elapsed} \times I}{L}$$

$$= \text{intercalary months elapsed} + \frac{R_i}{L},$$

where R_i is the residue of the intercalary months.

$$\therefore \frac{A\times L}{C} = \frac{\text{intercalary months elapsed} \times L}{I} + \frac{R_i}{I} + \frac{R_o}{C}.$$

$$\therefore A = \frac{\text{intercalary months elapsed} \times C + \dfrac{R_i\times C + R_o\times I}{L}}{I}$$

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1. Same rule occurs in *BrSpSi*, xiii. 15-16 and *MSi*, xvii. 28-29.

METHOD 6. WHEN OMITTED DAYS ELAPSED INCLUDING THE RESIDUAL FRACTION ARE KNOWN

7 (c-d). Or, divide the product of the omitted days, including the residual fraction, elapsed (since the epoch) and the number of civil days (in a *yuga*) by the number of omitted days (in a *yuga*) : the quotient obtained is the *Ahargaṇa*.[1]

METHOD 7. WHEN SUN AND MOON ARE KNOWN

8. Multiply the solar days elapsed (since the epoch) by the revolutions of the Moon and divide the product by the number of solar days in a *yuga* : the result is the (mean) longitude of the Moon in terms of revolutions etc. Subtract 13 times the Sun's (mean) longitude from the Moon's (mean) longitude. Reduce the difference to signs ; halve them and then multiply by 5 ; and then increase the product by the number of solar years elapsed (since the epoch). Then are obtained the *śuddhi* days.

9. Multiply the solar days elapsed (since the epoch) by the number of risings of the asterisms (in a *yuga*) and divide the product by the number of solar days in a *yuga*. Diminish the quotient by the *śuddhi* stated above: the result, or the result increased by *one*, is the *Ahargaṇa*.

Let S_e denote the number of solar days elapsed since the epoch. Also let S and C denote respectively the numbers of solar and civil days in a *yuga*.

Then

$$\textit{Ahargaṇa} = \frac{C \times S_e}{S} - \text{total } \textit{adhimāsa} \text{ fraction in civil days}$$

$$= \frac{(\text{risings of asterisms} - \text{Sun's revolutions}) \times S_e}{S}$$

$$- \text{total } \textit{adhimāsa} \text{ fraction in civil days}$$

$$= \frac{(\text{risings of asterisms}) \times S_e}{S} - \text{solar years elapsed}$$

$$- \text{total } \textit{adhimāsa} \text{ fraction in civil days} \qquad (1)$$

---

1. Same rule occurs in *BrSpSi*, xiii. 17; *SiŚe*, ii. 86.

But (see *infra*, notes on vs. 10 of sec. 4)

total *adhimāsa* fraction in solar (or civil) days

$$= \frac{(\text{Moon's longitude} - 13 \times \text{Sun's longitude}) \text{ in degrees}}{12}$$

$$= \frac{(\text{Moon's long.} - 13 \times \text{Sun's long.}) \text{ in signs} \times 30}{12}$$

$$= \frac{(\text{Moon's long.} - 13 \times \text{Sun's long.}) \text{ in signs} \times 5}{2}. \qquad (2)$$

Hence from (1), we have

$$\textit{Ahargaṇa} = \frac{(\text{risings of asterisms}) \times S_e}{S} - \textit{śuddhi},$$

where

$$\textit{śuddhi} = \text{solar years elapsed} + $$
$$+ \frac{(\text{Moon's long.} - 13 \times \text{Sun's long.}) \text{ in signs} \times 5}{2}.$$

This *Ahargaṇa* might differ by one day. Hence the instruction for the addition of 1 in the rule.

Note. In the above rule, Vaṭeśvara has called the sum of the *adhimāsa* fraction and the solar years elapsed by the term *śuddhi*. In fact, the *adhimāsa* fraction itself is known as *śuddhi*. In what follows, we shall use the term *śuddhi* in the sense of the *adhimāsa* fraction.

METHOD 8. ALTERNATIVE METHOD

10. Or, the *Ahargaṇa* may be obtained from the difference of the risings of the asterisms (in a *yuga*) and the Sun's revolutions (in a *yuga*), in the manner stated above.

That is, the *Ahargaṇa* may be obtained from either of the following formulae :

$$(1)\ \textit{Ahargaṇa} = \frac{(\text{risings of asterisms} - \text{Sun's revolutions}) \times S_e - 30 \times \textit{adhimāsaśeṣa}}{S}$$

(vide Method 3)

(2) $Ahargaṇa = \dfrac{(\text{risings of asterisms} - \text{Sun's revolutions}) \times S_e}{S}$

— total *adhimāsaśeṣa* in civil days,

where total *adhimāsaśeṣa* in civil days is obtained by formula (2) of the previous rule.

LORD OF THE SOLAR YEAR

10(d)-11. Multiply the solar years elapsed (since the epoch) by 66389 and divide by 6000 : (the quotient gives the intercalary days elapsed). When this quotient is divided by 30, the remainder obtained gives the *śuddhi* in terms of days. From these days counted from the first day of the light half of Caitra, one may obtain the lord of the (mean) solar year.

The number of intercalary days in one solar year is equal to

$$\frac{1593336 \times 30}{4320000}$$

$$= \frac{66389}{6000}.$$

Hence the above rule.

The *śuddhi* days lie between the first day of Caitra and the first day of the mean solar year following it. So the lord of the first day of the mean solar year may be easily obtained by counting from the lord of the first day of Caitra. By 'the lord of the (mean) solar year' in the text is meant 'the lord of the first day of the mean solar year'.

METHOD 9. : WHEN *ŚUDDHI* IS KNOWN

12-13. Multiply the solar years elapsed (since the epoch) by 189313 and divide the product by 36000. Diminish the quotient by the *śuddhi*, then add the (lunar) days elapsed since the beginning of the light half of Caitra, then subtract the (corresponding) omitted days, and then add the (solar) days corresponding to the solar years elapsed (since the epoch). Then is obtained the *Ahargaṇa*.

The number of civil days in one solar year is :

$$= \frac{1577917560}{4320000}$$

$$= \frac{131493130}{360000}$$

$$= 360 + \frac{189313}{36000} .$$

Hence the above rule.

METHOD 10 : *AHARGAṆA* FOR THE END OF MEAN SOLAR YEAR

14. Or, multiply the number of solar years elapsed (since the epoch) by 9313 and divide the product by 36000 ; then add the quotient to 365 times the number of solar years elapsed (since the epoch) : the result is the *Ahargaṇa* (for the end of the solar year).[1]

The number of civil days in one solar year is :

$$= 360 + \frac{189313}{36000}$$

$$= 365 + \frac{9313}{36000} .$$

Hence the above rule.

METHOD 11 : ALTERNATIVE METHOD

15. Or, multiply the solar years elapsed (since the epoch) by 45313 and divide the product by 36000 ; then add the quotient to 364 times the solar years elapsed (since the epoch) : the result is the *Ahargaṇa* (for the end of the solar year).

The number of civil days in one solar year is :

$$= 364 + \frac{36000+9313}{36000}$$

$$= 364 + \frac{45313}{36000} .$$

Hence the above rule.

1. *Cf SiŚe*, ii. 4.

SHORTER *AHARGAṆA*

16 (a-b). When the product of the solar years elapsed (since the epoch) and 364 is not added (in the previous rule), the result is the shorter *Ahargaṇa*.

NO END TO METHODS

16 (c-d). Thus, by hundreds of methods, one may determine the larger and the shorter *Ahargaṇa*.

AHARGAṆA SINCE BRAHMĀ'S BIRTH

17-19. In the beginning of the current day of Brahmā, (i.e., in the beginning of the current *kalpa*), the *Ahargaṇa* reckoned from the birth of Brahmā) amounted to

$$6199200 \times (\text{number of civil days in a } yuga)\,;$$

in the beginning of Kṛtayuga, (i.e., in the beginning of the current *yuga*), it increased by

$$459 \times (\text{number of civil days in a } yuga)\,;$$

and in the beginning of the current Kaliyuga, it further increased by

$$\tfrac{3}{4} \times (\text{number of civil days in a } yuga).$$

The sum of these was, thus, the *Ahargaṇa* at the beginning of Kaliyuga since the birth of Brahmā. In other words, it amounted to

$$24798639 \times \frac{\text{civil days in a } yuga}{4}$$

or to 9,78,25,51,98,55,50,210 civil days.

This *Ahargaṇa* for the beginning of Kaliyuga, when added to the days elapsed since the beginning of Kaliyuga, gives the *Ahargaṇa* for the desired day (since the birth of Brahmā).

The above rule follows from the facts that according to Vaṭeśvara :

(1) 6150 *kalpas* or 6150×1008 (=6199200) *yugas* elapsed at the beginning of the current *kalpa*, since the birth of Brahmā ;

(2) 6 *manus* and 27 *yugas* (=459 *yugas*) elapsed at the beginning of the current *yuga*, since the beginning of the current *kalpa* ;

(3) $\frac{3}{4}$ *yugas* elapsed at the beginning of the current Kaliyuga, since the beginning of the current *yuga* ; and

(4) a total of $6199659\frac{3}{4}$ ($=24798639/4$) *yugas* elapsed at the beginning of the current Kaliyuga, since the birth of Brahmā.

Also see *supra*, sec. 1, vs. 10.

LORD OF CURRENT DAY BY BACKWARD COUNTING

20. Subtract the *Ahargaṇa* calculated since the birth of Brahmā, or since the beginning of the current *kalpa*, or since the beginning of the current *yuga*, or since the beginning of the current Kaliyuga, from seven times the number of civil days in the life-span of Brahmā, or in a *kalpa*, or in a *yuga*, or in Kaliyuga, (respectively) ; and divide the difference thus obtained by seven. The residue of the division, being counted backwards from Saturn, Saturn, Sun, or Venus, respectively, gives the lord of the current day.

Since the number of civil days in the life-span of Brahmā and also those in a *kalpa* are already multiples of 7, it is not necessary to multiply them by 7. But since the number of civil days in a *yuga* and the number of civil days in Kaliyuga are not multiples of 7, it is necessary to multiply them by 7 so that they may become multiples of 7.

LUNAR AND SOLAR *AHARGAṆA*

First Method

21. (Set down the *Ahargaṇa* in two places, one below the other). In the lower place, multiply it by the omitted days (in a *yuga*) and divide by the civil days (in a *yuga*) ; and then add the resulting (omitted) days to the *Ahargaṇa* at the upper place. The result obtained is the lunar *Ahargaṇa*.

Set down the lunar *Ahargaṇa* in two places, one below the other. In the lower place, multiply it by the intercalary days (in a *yuga*) and divide by the lunar days (in a *yuga*) ; and then subtract the resulting intercalary days from the lunar *Ahargaṇa* at the upper place : this gives the solar *Ahargaṇa*.

The lunar *Ahargaṇa* means "the number of lunar days elapsed since the epoch" and the solar *Ahargaṇa* means "the number of solar days elapsed since the epoch".

Second Method

22. Or, the omitted days elapsed (since the epoch) and the lunar *Ahargaṇa* being respectively increased and diminished by their difference give the omitted days elapsed (since the epoch) and the lunar *Ahargaṇa* (in the reverse order).

Similarly, the intercalary months elapsed (since the epoch) and the solar *Ahargaṇa* being subtracted from their sum yield the solar *Ahargaṇa* and the intercalary months elapsed (since the epoch), respectively.

That is,

(1) lunar *Ahargaṇa* = omitted days elapsed + (lunar *Ahargaṇa* − omitted days elapsed).

and omitted days elapsed = lunar *Ahargaṇa* − (lunar *Ahargaṇa* − omitted days elapsed).

(2) solar *Ahargaṇa* = (solar *Ahargaṇa* + intercalary months elapsed) − intercalary months elapsed,

and intercalary months elapsed = (solar *Ahargaṇa* + intercalary months elapsed) − solar *Ahargaṇa*.

For other methods on the topic, see *BrSpSi*, xiii. 12-13 and 14.

OTHER METHODS FOR THE *AHARGAṆA*

First Method

23-24. Set down the solar days (elapsed since the epoch in three places). (In the third place) multiply by 271 and divide by 40,50,000; and then subtract (the quotient from the solar days written in the second place). Divide the remainder by 976 and multiply by 30. (This gives the number of intercalary days elapsed since the epoch.) Add this (to the solar days written in the first place). Multiply the sum (thus obtained) by 11 and set down the resulting product in two places (one above the other). Divide the quantity in the upper place by 16,51,030; and add the quotient to the quantity in the lower place. Divide this sum by 703. (This gives the number of omitted days elapsed since the epoch). Subtracting this (from the lunar days elapsed since the epoch), we obtain the civil days elapsed (since the epoch).

The number of intercalary days corresponding to one solar day is :

$$= \frac{1593336 \times 30}{4320000 \times 360}$$

$$= \frac{30}{976} \times \frac{1593336 \times 976}{4320000 \times 360}$$

$$= \frac{30}{976}\left[1 - \frac{271}{4050000}\right],$$

and the number of omitted days corresponding to one lunar day is :

$$= \frac{25082520}{1603000080}$$

$$= \frac{11}{703} \times \frac{703 \times 25082520}{11 \times 1603000080}$$

$$= \frac{11}{703}\left[1 + \frac{1}{1651030}\right] \text{approx.}$$

$$= \frac{1}{703}\left[11 + \frac{11}{1651030}\right] \text{approx.}$$

Hence the above rule.

Second Method

24-26. Or, set down the solar months elapsed (since the epoch) in two places. In one place, multiply them by 66389 and divide the product by 2160000 ; and then add the resulting intercalary months to the elapsed solar months set down at the other place. Multiply the sum by 30 and to the product add the number of lunar days elapsed since the beginning of the current lunar month. Put down the result in two places. In one place, multiply that by 209021 and divide the product by the number of lunar days in a *yuga* as divided by 120. By the resulting omitted lunar days diminish the result placed in the other place. The result thus obtained is the *Ahargaṇa*.

This rule follows from the facts that :

(1) The number of intercalary months in one solar month is :

$$= \frac{1593336}{4320000 \times 12}$$

$$=\frac{66389}{2160000}.$$

(2) The number of omitted days in one lunar day is :

$$=\frac{25082520}{1603000080}$$

$$=\frac{209021}{1603000080/120}.$$

Third Method

27. Or, multiply the solar years elapsed (since the epoch) by 1,31,49,313 and divide the product by 36,000. The quotient increased by the number of days elapsed since the vernal equinox gives the *Ahargaṇa.*

The number of civil days in one solar year is equal to

$$\frac{1577917560}{4320000} \text{ or } \frac{13149313}{36000}.$$

Hence the above rule.

Section 4 : Computation of Mean Planets

1. GENERAL METHOD

1. Multiply the *Ahargaṇa* by the revolution-number (of the planet) and divide (the product) by the number of civil days (in a *yuga*) : then is obtained, in revolutions etc., the (mean) longitude of the planet at (mean) sunrise at Laṅkā.[1] In the case of the apogees and ascending nodes of the planets (other than the Moon), the (mean) longitude is obtained by taking the number of civil days in the life span of Brahmā as divisor.

In the case of the apogees and ascending nodes of the planets other than the Moon, the number of civil days in the life span of Brahmā is taken as the divisor because their revolution-numbers are stated for that period. See *supra*, sec. 1, vss. 16-19.

In the rest of this chapter, the term longitude will be used in the sense of mean longitude. The other quantities used are also mean, although the word mean has not been used with them.

2. MEAN PLANETS FROM SHORTER *AHARGAṆA*

(i) Sun

2-3(a-b). Subtract the years elapsed (since the epoch) from the shorter *Ahargaṇa* ; then multiply by the Sun's revolutions in a *yuga* ; then subtract 11,17,560 as multiplied by the revolutions performed by the Sun (since the epoch) ; and then divide by 13,14,93,130 : the result is the longitude of the Sun in signs etc.

Let Y be the number of years elapsed (since the epoch). Then Sun's

$$\text{longitude} = \frac{\textit{Ahargaṇa} \times \text{Sun's revolution-number}}{\text{civil days in a } \textit{yuga}} \text{ revs.}$$

$$= \frac{(364\,Y + \text{shorter } \textit{Ahargaṇa}) \times (\text{Sun's rev.-no.})}{1577917560} \text{ revs.,} \quad (1)$$

because *Ahargaṇa* $= 364\,Y +$ shorter *Ahargaṇa*. (See *supra*, sec. 3, vs. 15)

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1. *Cf. BrSpSi*, i. 32 ; *SiDVr*, i. 17(a-b) ; *SiŚe*, ii. 14 ; *SiŚi*, I, i (c). 4.

$$= \frac{(364Y + \text{shorter } Ahargaṇa) \times (\text{Sun's rev-no.})}{1577917560} \text{ revs.}$$

$$= \frac{(364Y + \text{shorter } Ahargaṇa) \times (\text{Sun's rev-no.})}{1577917560} - Y \text{ revs.,} \qquad (2)$$

subtracting Y revolutions performed by the Sun in Y years

$$= \left[\frac{[365Y + (\text{shorter } Ahargaṇa - Y)] \times (\text{Sun's rev-no.})}{1577917560} - Y \right] \times$$
$$\times 12, \text{ signs}$$

$$= \frac{(\text{shorter } Ahargaṇa - Y) \times (\text{Sun's rev-no.}) - 1117560Y}{131493130} \text{ signs.}$$

It is to be noted that the subtraction of Y revolutions in step (2) above is meant to get rid of the complete revolutions and to get the Sun's longitude in terms of signs, etc. The complete revolutions, being superfluous, are discarded and the longitude of a planet is always expressed in signs, etc.

(ii) Moon

3 (c-d)-4. Multiply (the shorter *Ahargaṇa*) by the Moon's revolutions in a *yuga*; then add 50,92,86,024 times the years elapsed (since the epoch); then divide by the number of civil days (in a *yuga*) : the result increased by 13 times the solar years elapsed gives the Moon's (mean) longitude.

Let Y be the number of mean solar years elapsed since the epoch.

$$\text{Then Moon's longitude} = \frac{Ahargaṇa \times (\text{Moon's revolutions in a } yuga)}{\text{civil days in a } yuga}$$

$$= \frac{(364\ Y + \text{shorter } Ahargaṇa) \times (\text{Moon's revolutions in a } yuga)}{\text{civil days in a } yuga}$$

$$= \left[\frac{364 \times 57753336Y + \text{shorter } Ahargaṇa \times \text{Moon's revs. in a } yuga}{\text{civil days in a } yuga} - 13Y \right] + 13Y$$

$$= \frac{\text{shorter } Aharga\d{n}a \times \text{Moon's revs. in a } yuga + \\ + (364 \times 57753336 - 13 \times 1577917560)Y}{\text{civil days in a } yuga} + 13Y$$

$$= \frac{\text{shorter } Aharga\d{n}a \times \text{Moon's revs. in a } yuga + 509286024Y}{\text{civil days in a } yuga} + 13Y.$$

(iii) Planets, Mars etc.

5-6. Assuming 364 as the *Ahargaṇa*, multiply it by the revolution-number of the desired planet and divide the product by the civil days (in a *yuga*): multiply whatever is obtained as the remainder by the solar years elapsed and add that product to the product of the shorter *Ahargaṇa* and the revolution-number of the planet. Divide that (sum) by the number of civil days (in a *yuga*). To whatever is obtained, add the (complete) revolutions obtained from the assumed *Ahargaṇa* after multiplying them by the years elapsed. The result is the longitude of the planet in terms of revolutions etc.

Let Y be the number of years elapsed. Then

$$\text{Planet's longitude} = \frac{Aharga\d{n}a \times \text{Planet's rev-number}}{\text{civil days in a } yuga}$$

$$= \frac{(364Y + \text{shorter } Aharga\d{n}a) \times \text{Planet's rev-number}}{\text{civil days in a } yuga}$$

$$= \frac{364 \times \text{Planet's rev-number}}{\text{civil days in a } yuga} \times Y + \\ + \frac{\text{shorter } Aharga\d{n}a \times \text{Planet's rev-number}}{\text{civil days in a } yuga}$$

Let

$$\frac{364 \times \text{Planet's rev-number}}{\text{civil days in a } yuga} = R + \frac{r}{\text{civil days in a } yuga},$$

Then

$$\text{Planet's longitude} = RY + \frac{\text{shorter } Aharga\d{n}a \times \text{Planet's rev-number} + rY}{\text{civil days in a } yuga}.$$

By "the revolution-number of a planet" is meant "the number of revolutions performed by a planet in a *yuga*."

3. ONE PLANET FROM ANOTHER

7. The longitude of the (given) planet, in terms of revolutions etc., multiplied by the revolutions of the desired planet and divided by its own revolutions, gives the longitude of the desired planet in terms of revolutions etc.[1]

From the civil days (in a *yuga*), in the same way, may be obtained the *Ahargaṇa*.

That is,

(1) longitude of desired planet

$$= \frac{\text{Rev-no. of desired planet} \times \text{long. of given planet in revs. etc.}}{\text{rev-no. of given planet}}$$

$$(2)\ Ahargaṇa = \frac{\text{civil days in a } yuga \times \text{ long. of given planet in revs. etc.}}{\text{rev-no. of given planet}}$$

4. SUN AND MOON WITHOUT USING *AHARGAṆA*

(1) Sun and Moon from *Avamaśeṣa* and *Adhimāsaśeṣa*

First Method

8-9. Multiply the intercalary months in a *yuga* by the *Avamaśeṣa* and divide by the civil days (in a *yuga*). Add the quotient obtained to the *Adhimāsaśeṣa* and divide that sum by the lunar months (in a *yuga*) : the result thus obtained is (the total *Adhimāsa* fraction) in terms of days etc.

Now add the days etc. obtained by dividing the *Avamaśeṣa* by the civil days (in a *yuga*) to the months, days, etc. that have elapsed (since the beginning of Caitra) of the current year, and set down the result in two places. In one place, keep it as it is and in the other place multiply it by 13 ; diminish both of them by the result (in days etc.) due to the (total) *Adhimāsa* fraction : then (treating the months, days, etc. as signs, degrees, etc.) are obtained the longitudes of the Sun and the Moon respectively.[2]

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1. *Cf. BrSpSi*, xiii. 27; *ŚiDVr*, i. 26; *MSi*, xvii, 2; *SiŚe*, ii. 26; also 25; *SiŚi*, I, i (c). 14 (c-d).
2. Similar rules are found to occur in *BrSpSi*, xiii. 20-22; *KK* (BC), i. 11-12; *SiŚe*, ii. 21-21; *SiŚi*, I, i (c). 6-7. Also see the problem set in *SiŚe*, xx. 3 (c-d).

The following is the rationale of this rule :

The fraction of the intercalary month (i.e., *Adhimāsa* fraction)

$$= \frac{\textit{Adhimāsaśeṣa}}{\text{lunar days in a } \textit{yuga}}, \text{ in solar months.} \quad (1)$$

The fraction of the omitted day (i.e., *Avama* fraction)

$$= \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}}, \text{ in lunar days.} \quad (2)$$

The fraction of the intercalary month due to (2)

$$= \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \times \frac{\text{intercalary months in a } \textit{yuga}}{\text{lunar days in a } \textit{yuga}}. \quad (3)$$

∴ The total fraction of the intercalary month (i.e., total *Adhimāsa* fraction)

$$= \frac{\textit{Adhimāsaśeṣa} + \dfrac{\textit{Avamaśeṣa} \times \text{intercalary months in a } \textit{yuga}}{\text{civil days in a } \textit{yuga}}}{\text{lunar months in a } \textit{yuga}}, \quad (4)$$

which is obtained in solar days.

Suppose that m lunar months and d lunar days have elapsed since the beginning of Caitra. Then m months and d days denote the time elapsed since the beginning of Caitra up to the beginning of the current lunar day. As (2) is the interval, in lunar days, between the beginning of the current lunar day and sunrise on that day, therefore

$$m \text{ months} + d \text{ days} + (2)$$

denotes the time in lunar months, lunar days, etc., elapsed since the beginning of Caitra up to sunrise on the current lunar day. Likewise

$$m \text{ months} + d \text{ days} + (2) - (4)$$

denotes the time in solar months, solar days, etc., elapsed since the beginning of the mean solar year up to sunrise on the current lunar day.

Let M, D, G and V denote respectively the solar months, solar days, solar *ghaṭīs* and solar *vighaṭīs*, elapsed since the beginning of the current solar year up to sunrise on the current lunar day. Then evidently

Sun's longitude $= M$ signs $+$ D degrees $+$ G minutes $+$ V seconds

$= [$ (m signs and d degrees) $+$ degrees etc. corresponding to (2)] $-$ [degrees etc. corresponding to (4)]

and

Moon's longitude $= 13$ [m signs $+$ d degrees $+$ degrees etc. corresponding to (2)] $-$ [degrees etc. corresponding to (4)],

because

$$\frac{\text{Moon's longitude} - \text{Sun's longitude}}{12} = m \text{ signs} + d \text{ degrees} + $$

$$+ \text{ degrees corresponding to (2).}$$

Corollary 1 : A deduced rule

10. Multiply the result (i.e., days etc., treated as degrees etc.) due to the (total) *Adhimāsa* fraction by 12 ; then subtract that from the degrees of the Moon's longitude ; and then divide that by 13 ; the result thus obtained is the Sun's longitude. The Sun's longitude multiplied by 13 and increased by 12 times the result due to the (total) *Adhimāsa* fraction gives the Moon's longitude.

From the rule stated in vss. 8-9, we have

Moon's longitude $= 13$ (m signs $+$ d degrees $+$ degrees etc. corresponding to *Avama* fraction) $-$ (degrees etc. corresponding to total *Adhimāsa* fraction) (1)

and

$13 \times$ Sun's longitude $= 13$ (m signs $+$ d degrees $+$ degrees etc. corresponding to *Avama* fraction) $-$ 13 (degrees etc. corresponding to total *Adhimāsa* fraction). (2)

Subtracting (2) from (1), we get

Moon's longitude $- 13 \times$ Sun's longitude $= 12$ (degrees etc. corresponding to total *Adhimāsa* fraction).

Hence the above rule.

Corollary 2 : Another deduced rule

11. Add the result (in lunar days etc.) due to the *Avama* fraction to the lunar days elapsed (since Caitra) and multiply the resulting sum by 12 : the resulting lunar days etc. are to be treated as degrees etc. The Moon's longitude diminished by them gives the Sun's longitude and the Sun's longitude increased by them gives the Moon's longitude.[1]

Let δ denote the number of lunar days elapsed since the beginning of Caitra. Then, from the rule stated in vss. 8-9, we have

Moon's longitude = 13 (δ degrees + degrees etc. corresponding to *Avama* fraction) − (degrees etc. corresponding to total *Adhimāsa* fraction)

and

Sun's longitude = δ degrees + degrees etc. corresponding to *Avama* fraction − degrees etc. corresponding to total *Adhimāsa* fraction.

Therefore

Moon's longitude − Sun's longitude = 12 (δ degrees + degrees etc. corresponding to *Avama* fraction).

Hence the above rule.

Alternative rationale.

$$\textit{Tithi} \text{ at sunrise} = \text{number of } \textit{tithis} \text{ elapsed} + \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}}$$

$$\text{Also } \textit{tithi} \text{ at sunrise} = \frac{\text{Moon's longitude at sunrise} - \text{Sun's longitude at sunrise}}{12}$$

Therefore

Moon's longitude − Sun's longitude = 12 (δ degrees + degrees etc. corresponding to *Avama* fraction),

as before.

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1. *Cf. KK* (BC), I, i. 9; *MSi*, xvii. 27; *SiŚe*, ii. 20, also ii. 44.

SUN AND MOON FROM *AVAMAŚEṢA* AND *ADHIMĀSAŚEṢA*

Second Method

12-13. Or, (severally) multiply the *Avamaśeṣa* as divided by the lunar days (in a *yuga*), by the daily motions of the Sun and the Moon: the results are the minutes (of the longitudes of the Sun and the Moon). The months and days (elapsed since the beginning of Caitra) are the signs and degrees of the Sun's longitude whereas those multiplied by 13 are the signs and degrees of the Moon's longitude. The degrees etc. obtained from the *Adhimāsaśeṣa* as divided by the lunar months (in a *yuga*) are to be deducted from both of them (i.e., from the degrees of the Sun's longitude as also from the degrees of the Moon's longitude). Then are obtained the longitudes of the Sun and the Moon.[1]

This rule follows from the rationale of vss. 8-9.

The fraction of the omitted lunar day

$$= \frac{Avama\acute{s}e\d{s}a}{\text{lunar days in a } yuga}, \text{ civil days.} \qquad (1)$$

The fraction of the intercalary month

$$= \frac{Adhim\bar{a}sa\acute{s}e\d{s}a}{\text{lunar months in a } yuga}, \text{ solar days.} \qquad (2)$$

Let m months and d days have elapsed since the beginning of Caitra. Then

Sun's longitude = m signs and d degrees + Sun's motion corresponding to (1) — degrees etc. equivalent to solar days etc. of (2),

and

Moon's longitude = $13m$ signs and $13d$ degrees + Moon's motion corresponding to (1) — degrees etc. equivalent to solar days etc. of (2).

Āryabhaṭa II gives the following formulae for the longitudes of the Sun and the Moon for the beginning of the current lunar day :

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1. A similar rule is stated in *ŚiDVṛ*. i. 21-22.

$$\text{Sun's longitude} = \left\{ \text{no. of } \textit{tithis} \text{ elapsed since Caitrādi} - \frac{\textit{Adhimāsaśeṣa}}{\text{lunar months in a } \textit{yuga}} \right\} \text{degrees}$$

and

$$\text{Moon's longitude} = \left\{ 13 \times (\text{no. of } \textit{tithis} \text{ elapsed since Caitrādi}) - \frac{\textit{Adhimāsaśeṣa}}{\text{lunar months in a } \textit{yuga}} \right\} \text{degrees,}$$

See *MSi*, xvii. 16-17(a-b). Also see *SiŚi*, I, i (c). 7.

Corollary 1 : Sun's and Moon's motions for *Avama* fraction

14. The minutes of the Sun's motion corresponding to the *Avama* fraction[1] may be obtained by dividing 12 times the *Avamaśeṣa* by 13 times the omitted days in a *yuga*; and those of the Moon's motion corresponding to the *Avama* fraction, by dividing 334 times the *Avamaśeṣa* by 27 times the omitted days in a *yuga*.[2]

Since

$$\frac{\text{Sun's daily motion} \times \text{omitted days in a } \textit{yuga}}{\text{lunar days in a } \textit{yuga}} = \frac{59'8'' \times 11}{703} \text{ approx.}$$

$$= \frac{12}{13} \text{ minutes, approx.}$$

and

$$\frac{\text{Moon's daily motion} \times \text{omitted days in a } \textit{yuga}}{\text{lunar days in a } \textit{yuga}} = \frac{790'35'' \times 11}{703} \text{ approx.}$$

$$= \frac{334}{27} \text{ minutes, approx.}$$

---

1. *Avama* fraction $= \dfrac{\textit{Avamaśeṣa}}{\text{lunar days in a } \textit{yuga}}$.

2. *Cf.* *ŚiDVr*. i. 21-22. Lalla takes $(1-1/12)(1+25/2)$ in place of $334/27$.

therefore

$$\text{Sun's daily motion} = \frac{12 \times \text{lunar days in a } yuga}{13 \times \text{omitted days in a } yuga} \text{ minutes}$$

and

$$\text{Moon's daily motion} = \frac{334 \times \text{lunar days in a } yuga}{27 \times \text{omitted days in a } yuga} \text{ minutes.}$$

$$\text{Now } Avama \text{ fraction} = \frac{Avamaśeṣa}{\text{lunar days in a } yuga}, \text{ civil days.}$$

∴ Sun's motion corresponding to *Avama* fraction

$$= \frac{12 \times Avamaśeṣa}{13 \times \text{omitted days in a } yuga} \text{ minutes,}$$

and

Moon's motion corresponding to *Avama* fraction

$$= \frac{334 \times Avamaśeṣa}{27 \times \text{omitted days in a } yuga} \text{ minutes.}$$

Hence the above rule.

Corollary 2 : Alternative formulae for the same

15. Or, divide the *Avamaśeṣa* (severally) by 2,71,08,231 and 20,27,617 : the results are the minutes of the Sun's and Moon's motions corresponding to the *Avama* fraction, respectively.[1]

Apply these to the signs and degrees, equal to the months and days elapsed (since the beginning of Caitra, in the manner stated above in vss. 12-13).

Since

$$\frac{\text{Sun's daily motion}}{\text{lunar days in a } yuga} = \frac{59'8''}{1603000080}$$

$$= \frac{1}{27108231} \text{ minutes.}$$

1. *Cf. MSi*, xvii. 18; *SiŚi*, I, i (c). 6

and

$$\frac{\text{Moon's daily motion}}{\text{lunar days in a } yuga} = \frac{790'35''}{1603000080}$$

$$= \frac{1}{2027617} \text{ minutes.}$$

$$\therefore \text{ Sun's daily motion} = \frac{\text{lunar days in a } yuga}{27108231} \text{ minutes.}$$

and

$$\text{Moon's daily motion} = \frac{\text{lunar days in a } yuga}{2027617} \text{ minutes.}$$

$$\text{Now } Avama \text{ fraction} = \frac{Avamaśeṣa}{\text{lunar days in a } yuga}, \text{ civil days.}$$

∴ Sun's motion corresponding to *Avama* fraction

$$= \frac{Avamaśeṣa}{27108231} \text{ minutes,}$$

and

Moon's motion corresponding to *Avama* fraction

$$= \frac{Avamaśeṣa}{2027617} \text{ minutes.}$$

Hence the above rule.

(2) SUN AND MOON FROM *AVAMAŚEṢA*

First Method

16. Multiply the Sun's minutes (i.e., the minutes of the Sun's motion corresponding to the *Avama* fraction) by 62 and divide by 5 : add it to degrees equal to 12 times the (elapsed) *tithis*. Subtract the result from the Moon's longitude : this gives the Sun's longitude. The same result added to the Sun's longitude gives the Moon's longitude.

The rationale of this rule is as follows :

Moon's daily motion = 790′ 35″

Sun's daily motion = 59′ 8″

∴ motion-difference of Sun and Moon = 731′ 27″

∴ motion-difference of Sun and Moon corresponding to Sun's minutes

$$= \frac{\text{Sun's minutes} \times 731'27''}{59'8''}$$

$$= \frac{\text{Sun's minutes} \times (5 \times 731'27'')}{5 \times 59'8''}$$

$$= \frac{\text{Sun's minutes} \times 62}{5} \text{ minutes.}$$

Let M and S be the Moon's and Sun's longitudes at sunrise on the day in question. Then

$$\frac{M-S}{12} = (\textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{\text{Sun's minutes} \times 62}{5 \times 12} \text{ minutes.}$$

$$\therefore \quad M-S = 12 \times (\textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{62 \times \text{Sun's minutes}}{5} \text{ mins.}$$

$$\therefore \quad S = M - \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{62 \times \text{Sun's minutes}}{5} \text{ minutes} \right\}$$

and

$$M = S + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{62 \times \text{Sun's minutes}}{5} \text{ minutes} \right\}.$$

Śrīpati gives the following formulae : [1]

$$S = M - \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \right.$$
$$\left. + \frac{(\text{motion-diff. of Sun and Moon}) \times \textit{Avamaśeṣa}}{\text{lunar days in a } \textit{yuga}} \text{ mins.} \right\}$$

and

1. See *SiŚe*, ii. 23.

$$M = S + \left\{ (12 \times \textit{tithis} \text{ elapsed) degrees} + \frac{\text{(motion-diff. of Sun and Moon)} \times \textit{Avamaśeṣa}}{\text{lunar days in a } \textit{yuga}} \text{ mins.} \right\}$$

Second Method

17. **Diminish 3/40th fraction of the Moon's minutes (i. e., the minutes of the Moon's motion corresponding to the *Avama* fraction) from themselves, and add whatever is obtained to degrees equivalent to 12 times the *tithis* elapsed. The resulting quantity added to the Sun's longitude gives the Moon's longitude ; and the same quantity subtracted from the Moon's longitude gives the Sun's longitude.**

As shown above

motion-difference of Sun and Moon = 731′27″.

Therefore,

motion-difference of Sun and Moon corresponding to Moon's minutes

$$= \frac{\text{Moon's minutes} \times 731'27''}{790'35''}$$

$$\doteq \text{Moon's minutes} \left(1 - \frac{59'8''}{790'35''}\right)$$

$$= \text{Moon's minutes} \left(1 - \frac{3}{40}\right), \quad \text{minutes.}$$

Let M and S be the longitudes of the Moon and the Sun at sunrise on the day in question. Then, as before,

$$\frac{M-S}{12} = (\textit{tithis} \text{ elapsed) degrees} + \frac{\text{Moon's minutes}}{12} \left(1 - \frac{3}{40}\right) \text{mins.}$$

Therefore

$$M = S + \left\{ (12 \times \textit{tithis} \text{ elapsed) degrees} + \text{Moon's minutes} \left(1 - \frac{3}{40}\right) \right\}$$

and

$$S = M - \left\{ (12 \times \textit{tithis} \text{ elapsed) degrees} + \text{Moon's minutes} \left(1 - \frac{3}{40}\right) \right\},$$

the second term within the curly brackets being in minutes.

Third Method

18. Divide 103 times the *Avamaśeṣa* by 9 times the omitted lunar days (in a *yuga*) and add the resulting minutes to degrees equal to 12 times the elapsed *tithis*. The resulting quantity added to the Sun's longitude gives the Moon's longitude, and the same quantity subtracted from the Moon's longitude gives the Sun's longitude.

From vs. 14 we have that :

Sun's minutes (i.e., the minutes of the Sun's motion corresponding to the *Avama* fraction)

$$= \frac{12 \times \textit{Avamaśeṣa}}{13 \times \text{omitted days in a } \textit{yuga}}.$$

Therefore, from vs. 16,

$$\text{Moon's longitude} = \text{Sun's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{62 \times \text{Sun's minutes}}{5} \text{ minutes} \right\}$$

$$= \text{Sun's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{62 \times 12 \times \textit{Avamaśeṣa}}{13 \times 5 \times \text{omitted days in a } \textit{yuga}} \right\}$$

$$= \text{Sun's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{103 \times \textit{Avamaśeṣa}}{9 \times \text{omitted days in a } \textit{yuga}} \text{ minutes} \right\}$$

Likewise

$$\text{Sun's longitude} = \text{Moon's longitude} - \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{103 \times \textit{Avamaśeṣa}}{9 \times \text{omitted days in a } \textit{yuga}} \text{ minutes} \right\}$$

Fourth Method

19. The Sun's longitude when increased by the *kalāvivara*, i. e., the minutes obtained by dividing the *Avamaśeṣa* by 2191537, and also by

the degrees equal to 12 times the *tithis* elapsed, becomes the Moon's longitude; and the Moon's longitude when diminished by the same amount becomes the Sun's longitude.[1]

The term *kalāvivara* means "the minutes of the difference between he Sun's and Moon's motion corresponding to the *Avama* fraction".

The rationale of the rule is as follows :

Rationale 1. Since motion-difference of the Sun and Moon corresponding to the *Avama* fraction

$$= \frac{\textit{Avamaśeṣa} \times 731'27''}{\text{lunar days in a } \textit{yuga}}$$

$$= \frac{\textit{Avamaśeṣa}}{2191537} \text{ or } \textit{kalāvivara},$$

because (lunar days in a *yuga*)/731′27″ = 1603000080/731′27″

$$= 96180004800/43887$$

$$= 2191537 \quad \text{approx.}$$

therefore, as before,

$$\text{Moon's longitude} = \text{Sun's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{\textit{Avamaśeṣa}}{2191537} \text{ minutes} \right\},$$

and

$$\text{Sun's longitude} = \text{Moon's longitude} - \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{\textit{Avamaśeṣa}}{2191537} \text{ minutes} \right\}.$$

Rationale 2. *Tithi* corresponding to the *Avama* fraction

$$= \frac{\textit{Avamaśeṣa}}{\text{lunar days in a } \textit{yuga}} \times \frac{\text{lunar days in a } \textit{yuga}}{\text{civil days in a } \textit{yuga}}$$

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1. *Cf.* *SiŚe*, ii. 23; *SiŚi*, I, i (c). 5. Rules similar to those stated in vss. 18 and 19 occur also in *ŚiDVr*, i. 34.

$$= \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}}$$

$$= \frac{\textit{Avamaśeṣa}}{1577917560}.$$

The corresponding motion-difference of Sun and Moon

$$= \frac{\textit{Avamaśeṣa} \times 12}{1577917560} \text{ degrees, because 1 } \textit{tithi} = 12 \text{ degrees.}$$

$$= \frac{\textit{Avamaśeṣa}}{131493130} \text{ degrees}$$

$$= \frac{\textit{Avamaśeṣa}}{2191552} \text{ minutes.}$$

Hence, as before,

$$\text{Moon's longitude} = \text{Sun's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{\textit{Avamaśeṣa}}{2191552} \text{ minutes} \right\}$$

and

$$\text{Sun's longitude} = \text{Moon's longitude} + \left\{ (12 \times \textit{tithis} \text{ elapsed}) \text{ degrees} + \frac{\textit{Avamaśeṣa}}{2191552} \text{ minutes} \right\}.$$

Rationale 3. *Tithi* from *tithyanta* to sunrise

$$= \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}}.$$

[The right hand side being a fraction of a lunar day is evidently a fraction of *tithi*.]

∴ If t be the number of *tithis* elapsed at *tithyanta*, then at sunrise

$$\frac{\text{Moon} - \text{Sun}}{12} = t + \frac{\textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}} = t + \frac{\textit{Avamaśeṣa}}{1577917560}$$

$$\therefore \text{ Moon} = \text{Sun} + 12t \text{ degrees} + \frac{12 \times 60 \times Avamaśeṣa}{1577917560} \text{mins.}$$

$$= \text{Sun} + 12t \text{ degrees} + \frac{Avamaśeṣa}{2191552} \text{mins.}$$

The number 2191552 is better than 2191537.

NĀḌIS OF *TITHI* CORRESPONDING TO *AVAMA* FRACTION

20. The *kalāvivara* (i. e., the difference between the minutes of the Sun's and Moon's motions corresponding to the *Avama* fraction) is the mean *tithi* (corresponding to the *Avama* fraction) in terms of minutes. That divided by 12 gives the corresponding *nāḍīs*.

Or, alternatively, the *Avamaśeṣa-tithi-nāḍīs* (i. e., the *nāḍīs* of the *tithi* corresponding to the *Avama* fraction) may be obtained by multiplying the *Avamaśeṣa* by 60 and dividing (the resulting product) by the civil days in a *yuga*.

For,

$$kalāvivara = \frac{Avamaśeṣa \times (\text{motion-diff. of Sun and moon in minutes})}{\text{lunar days in a } yuga}$$

= mean *tithi* corresponding to *Avama* fraction, in terms of minutes.

This divided by 720′ gives the corresponding *tithi*, and the same divided by 12 gives the corresponding *tithi-nāḍīs*.

We also have :

$$Tithi \text{ corresponding to } Avama \text{ fraction} = \frac{Avamaśeṣa}{\text{civil days in a } yuga}$$

$$\text{Corresponding } nāḍīs = \frac{Avamaśeṣa \times 60}{\text{civil days in a } yuga},$$

because one *tithi* = 60 *nāḍīs*.

Fifth Method

21. Multiply the *tithi-liptās* (i. e., the minutes of the *tithi* corresponding to the *Avama* fraction) by 2 and divide by 67. Add that to 12 times the intercalary years (elapsed). This added to 13 times the longitude of the Sun gives the longitude of the Moon and the same subtracted from the longitude of the Moon and then divided by 13 gives the longitude of the Sun.

That is,

$$\text{Moon's longitude} = 12\times(\text{intercalary years elapsed})\text{ revs.} + 13\times(\text{Sun's longitude}) + \frac{2\times \textit{tithi-liptās}}{67}\text{ mins.,} \quad (1)$$

and

$$\text{Sun's longitude} = \frac{1}{13}\left[\text{Moon's longitude} - \left(12\times(\text{intercalary years elapsed})\text{ revs.} + \frac{2\times \textit{tithi-liptās}}{67}\text{mins.}\right)\right] \quad (2)$$

where *tithi-liptās* $= \dfrac{720 \times \textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}}$.

The rationale of these formulae is as follows :

Since

intercalary months in a *yuga* = lunar months in a *yuga*—solar months in a *yuga*.

$$= \left(\text{Moon's rev-no.} - \text{Sun's rev-no.}\right) - 12\times\text{Sun's rev-no.}$$

$$= \text{Moon's rev-no.} - 13\times(\text{Sun's rev-no.})$$

∴ Intercalary years in a *yuga*

$$= \frac{1}{12}\left[\text{Moon's rev-no.} - 13\times\text{Sun's rev-no.}\right]$$

Therefore (see *infra*, vs. 24)

$$\text{Moon's longitude} = 12\,(\text{intercalary years elapsed} + \text{intercalary years corresponding to } \textit{Avama}\text{ fraction})\text{ revs.} + 13\times(\text{Sun's longitude})$$

and

$$\text{Sun's longitude} = \frac{1}{13}\left[\text{Moon's longitude} - 12\left(\text{intercalary years elapsed} + \text{intercalary years corresponding to } \textit{Avama}\text{ fraction}\right)\text{ revs.}\right]$$

But

(12 × intercalary years corresponding to *Avama* fraction) revs.

$$= \frac{12 \times \textit{Avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \times \frac{\text{intercalary years in a } \textit{yuga}}{\text{lunar days in a } \textit{yuga}} \text{ revs.}$$

$$= \frac{\textit{tithi-liptās} \times 1593336}{720 \times 1603000080} \text{ revs.}$$

$$= \frac{\textit{tithi-liptās} \times 1593336 \times 21600}{720 \times 1603000080} \text{ mins.}$$

$$= \frac{\textit{tithi-liptās} \times 1593336}{53433336} \text{ mins.}$$

$$= \frac{\textit{tithi-liptās} \times 2}{67} \text{ mins.}$$

Hence the formulae (1) and (2).

5. OTHER METHODS FOR SUN AND MOON

Method 1. Sun and Moon from lunar and intercalary years.

22. Multiply the *Ahargaṇa* by the intercalary years (in a *yuga*) and divide by the civil days (in a *yuga*): the result is in terms of revolutions etc. Similarly, find the result due to the lunar years (in a *yuga*) (i.e., multiply the lunar years in a *yuga* by the *Ahargaṇa* and divide by the civil days in a *yuga*). The difference of the two results gives the Sun's longitude (in revolutions etc.).

Method 2. Sun and Moon from lunar years.

23. The Moon's longitude diminished by 12 times the result derived from the lunar years (in a *yuga*) gives the Sun's longitude; and the Sun's longitude increased by the same gives the Moon's longitude.

Since

Moon's revolution-number − 13 × (Sun's revolution-number)

= intercalary months in a *yuga*

= 12 × (intercalary years in a *yuga*)

---

1. *Cf.* *ŚiDVṛ*, i. 24 (c-d).

therefore

$$\text{Moon's longitude} - 13\times(\text{Sun's longitude})$$

$$= \frac{12\times(\text{intercalary years in a } yuga)\times \textit{Ahargaṇa}}{\text{civil days in a } yuga} \quad (1)$$

Again, since

$$\text{Moon's revolution-number} - \text{Sun's revolution-number}$$

$$= \text{lunar months in a } yuga$$

$$= 12\times(\text{lunar years in a } yuga),$$

therefore

$$\text{Moon's longitude} - \text{Sun's longitude}$$

$$= \frac{12\times(\text{lunar years in a } yuga)\times \textit{Ahargaṇa}}{\text{civil days in a } yuga}. \quad (2)$$

Subtracting (1) from (2), we get

$$\text{Sun's longitude} = \frac{(\text{lunar years in a } yuga)\times \textit{Ahargaṇa}}{\text{civil days in a } yuga}$$

$$- \frac{(\text{intercalary years in a } yuga)\times \textit{Ahargaṇa}}{\text{civil days in a } yuga}. \quad (3)$$

Formula (3) gives the rule stated in vs. 22, and formula (2) the rule stated in verse 23. The rule stated in vs. 22 also follows directly from the relation between solar, lunar and intercalary years in a *yuga*.

Method 3. Sun and Moon from intercalary years,

24. The result derived from the intercalary years (in a *yuga*) multiplied by 12 and increased by 13 times the Sun's longitude gives the Moon's longitude; and the Moon's longitude diminished by the same amount and divided by 13 gives the Sun's longitude.[1]

That is,

$$\text{Moon's longitude} = \frac{(\text{intercalary years in a } yuga)\times \textit{Ahargaṇa}}{\text{civil days in a } yuga}\times 12$$

$$+ 13\times(\text{Sun's longitude}).$$

---

1. *Cf. BrSpSi*, xiii. 33; *SiŚe*, ii. 19.

$$\text{Sun's longitude} = \frac{1}{13}\left[\text{Moon's longitude} - \frac{(\text{intercalary years in a } yuga) \times Aharga\d{n}a}{\text{civil days in a } yuga} \times 12\right]$$

These follow from result (1) of p. 48 above.

Method 4. Sun from risings of asterisms.

25. The *Ahargaṇa* being multiplied by the number of risings of the asterisms (in a *yuga*) and then divided by the number of civil days in a *yuga*, the result is in terms of revolutions, etc. The signs, etc., give the Sun's longitude, whereas the (complete) revolutions denote the revolutions performed by the asterisms.[1]

This is so, because

$$\frac{(\text{risings of asterisms in a } yuga) \times Aharga\d{n}a}{\text{civil days in a } yuga}$$

$$= \frac{(\text{civil days in a } yuga + \text{Sun's revolutions in a } yuga) \times Aharga\d{n}a}{\text{civil days in a } yuga}$$

$$= \left\{ Aharga\d{n}a + \frac{\text{Sun's revolutions in a } yuga \times Aharga\d{n}a}{\text{civil days in a } yuga} \right\} \text{revs.}$$

Method 5. Sun and Moon from intercalary months.

26. The *Ahargaṇa* multiplied by the number of intercalary months (in a *yuga*) and then divided by the number of civil days (in a *yuga*), gives revolutions, etc. This added to 13 times the Sun's longitude gives the Moon's longitude, and the same subtracted from the Moon's longitude and then divided by 13 gives the Sun's longitude.

This is so, because

$$\text{intercalary months in a } yuga = \text{Moon's revolutions in a } yuga - 13 \times (\text{Sun's revolutions in a } yuga).$$

1. *Cf. ŚiDVṛ*, i. 24(a-b); *SiŚe*, ii. 17.

Method 6. Sun and Moon from lunar months.

27. The *Ahargaṇa* multiplied by the number of lunar months (in a *yuga*) and then divided by the number of civil days in a *yuga* gives the result in revolutions, etc. The Sun's longitude increased by that becomes the Moon's longitude, and the Moon's longitude diminished by that becomes the Sun's longitude.[1]

This is so, because :

lunar months in a *yuga* = Moon's revolutions in a *yuga*
— Sun's revolutions in a *yuga*.

This rule is equivalent to that stated in vs. 23 above.

Method 7. Sun and Moon from *Vyatīpātas*.

28. Multiply the *Ahargaṇa* by the number of *Vyatīpātas* in a *yuga* and then divide by the number of civil days in a *yuga*; divide the result by 2. The final result, in revolutions etc., diminished by the Moon's longitude gives the Sun's longitude, and the same (final) result diminished by the Sun's longitude gives the Moon's longitude.

This is so, because :

number of *Vyatīpātas* in a *yuga* = 2 × (Sun's revolutions in a *yuga*
+ Moon's revolutions in a *yuga*),[2]

Method 8. Sun and Moon from *Vyatīpātas* and lunar months.

29(a-b). (The same final result) being severally diminished and increased by the result (in terms of revolutions etc.) derived from the lunar months (in a *yuga*) and then divided by 2, gives the longitudes of the Sun and the Moon, respectively.

This is so, because :

the so called final result = Moon's longitude + Sun's longitude

and

---

1. *Cf. ŚiDVṛ*, i. 24.
2. See *supra*, sec. 2, vs. 5.

the result derived from lunar months in a *yuga*

=Moon's longitude—Sun's longitude.

Method 9. Sun from *Vyatīpātas* and intercalary months.

29(c-d). (The same final result) diminished by the result derived from the intercalary months in a *yuga* and then divided by 14 gives the longitude of the Sun.

This is so, because :

$$\text{Sun's long.} = \frac{(\text{Moon's long.} + \text{Sun's long.}) - \text{Moon's long.} + 13(\text{Sun's long.})}{14}$$

$$= \frac{(\text{so called final result}) - (\text{result derived from intercalary months in a } yuga)}{14}.$$

Method 10. Sun and Moon from omitted days.

30. Multiply the *Ahargaṇa* by the number of omitted days in a *yuga* and divide by the number of civil days (in a *yuga*). Whatever is obtained should be increased by the *Ahargaṇa* and the sum obtained should be divided by 30. The result, which is in revolutions etc., when added to the Sun's longitude, gives the Moon's longitude.

Since

Omitted days in a *yuga* = lunar days in a *yuga*—civil days in a *yuga*

=30 (Moon's revolution-number—Sun's revolution-number)—civil days in a *yuga*,

therefore, multiplying both sides by the *Ahargaṇa* and dividing by civil days in a *yuga*, we get

$$\frac{\text{Omitted days in a } yuga \times Ahargaṇa}{\text{civil days in a } yuga} = 30 \times (\text{Moon's long.} - \text{Sun's long.}) - Ahargaṇa$$

Hence the above rule.

6. OTHER METHODS FOR PLANETS

Method 1. Two planets from their conjunctions

31. The number of conjunctions of two planets (in a *yuga*) being multiplied by the *Ahargaṇa* and then divided by the number of civil days (in a *yuga*) gives the (difference between the longitudes of the two planets in terms of) revolutions, etc. The longitude of the slower planet increased by that becomes the longitude of the faster planet, and the longitude of the faster planet diminished by that becomes the longitude of the slower planet.[1]

This is so, because :

no. of conjunctions of two planets = revolution-number of faster planet— revolution-number of slower planet.

Method 2. Two planets from sum and difference of their revolutions.

32. Multiply the *Ahargaṇa* by the sum of the revolution-numbers of the two planets and divide by the number of civil days (in a *yuga*) : the result is in terms of revolutions, etc. Set it down in two places. In one place diminish it by the result derived from the conjunctions of the two planets (in a *yuga*), and in the other place increase it by that ; then divide the difference and the sum thus obtained by 2. Then are obtained the longitudes of the two planets (slower and faster, respectively).[2]

That is,

long. of planet P_2 = $\frac{1}{2}$ [(longitude of planet P_1 + long. of planet P_2) —(long. of planet P_1 − long. of planet P_2)]

and

long. of planet P_1 = $\frac{1}{2}$ [(long. of planet P_1 + long. of planet P_2) +(long. of planet P_1 − long. of planet P_2)].

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1. *Cf. ŚiDVr*, i. 25; *MSi*, i. 29; *SiŚi*, I, i(c). 13.
2. *Cf. MSi*, i. 28; *SiŚi*, ii. 29; *SiŚi*, I, i(c). 12. Rules stated in vss. 31 and 32 have been mentioned by Bhāskara I in his comm. on *Ā*, ii. 3(a-b).

Method 3. Two planets from the sum of their revolutions

33. The result (derived in vs. 32) from the sum of the revolutions of the two planets, when diminished by the longitude of the slower planet, gives the longitude of the faster planet, and when diminished by the longitude of the faster planet gives the longitude of the slower planet.[1]

Method 4. Planet from its risings

34. Multiply the *Ahargaṇa* by the number of risings of the planet (in a *yuga*) and divide by the number of civil days (in a *yuga*): the quotient denotes the number of the planet's risings gone by. The residue, in signs etc., being subtracted from or added to the longitude of the planet, according as the planet is faster or slower than the Sun, gives the longitude of the Sun. The longitude of the planet may also be obtained from that of the Sun, similarly.

Since

risings of a planet in a *yuga* = rev-no. of asterisms − planet's rev-no.

= (civil days in a *yuga* + Sun's rev-no.) − planet's rev-no.,

therefore,

$$\frac{\text{risings of the planet} \times \textit{Ahargaṇa}}{\text{civil days in a } \textit{yuga}}$$

= (*Ahargaṇa* + Sun's long. − planet's long.), in revs. etc.

= complete revolutions + difference between Sun's and planet's longitudes in signs etc.

Hence the above rule.

Method 5. Planet for the time of its rising

35. Just as the past risings of the asterisms and the Sun's longitude at sunrise are derived from the Sun's risings in a *yuga* (*vide* vs. 25), in the same way the past risings of the asterisms as also the longitude of the planet for the time of planet-rise may be derived from the risings of the planet in a *yuga*.

1. *Cf. SiŚe*, ii. 28.

The longitudes of the Sun and the Moon, too, may be derived in many ways from the *Avamaśeṣa* following the methods stated heretofore.

Method 6. Planet from civil days *minus Ahargaṇa*

36. Multiply the tabulated revolutions of the planet by the number of civil days (in a *yuga*) as diminished by the *Ahargaṇa* and divide by the number of civil days (in a *yuga*) : the result in revolutions etc. gives the longitude of the planet if the planet has retrograde (i.e., westward) motion. If the planet has direct (i.e., eastward) motion, the same result subtracted from a circle (i.e., 360°) gives the longitude of the planet.[1]

Method 7. Planet from civil days minus planet's revolutions

37. Multiply the *Ahargaṇa* by the civil days (in a *yuga*) as diminished by the revolutions of a planet (in a *yuga*) and divide by the civil days in a *yuga* : the result, in revolutions etc., gives the longitude of the planet if the planet has retrograde (i.e., westward) motion. If the planet has direct motion, the same result subtracted from a complete revolution (i.e., 360°) gives the longitude of the planet.[2]

Method 8. Planet from risings of planet and revolutions of asterisms

38. The difference of the results derived from the revolutions of the asterisms and from the risings of a planet, gives the longitude of the planet. The planet whose past risings give the result derived from the revolutions of the asterisms is the requisite planet.[3]

This is so, because :

revolutions of a planet = revolutions (or risings) of the asterisms − risings of the planet

and the result derived from the revolutions of the asterisms

$$= \frac{\text{(risings of asterisms)} \times \text{(planet's past risings)}}{\text{planet's risings in a } yuga}$$

1. *Cf. SiŚe*, ii. 30(d).
2. *Cf. SiŚe*, ii. 30(a-c).
3. *Cf. SiŚe*, ii. 31.

Method 9. Two planets from risings of asterisms and their own

39. From the sum of the risings of the two planets (in a *yuga*) subtract the risings of the asterisms (in a *yuga*); severally subtract that difference from the risings of those planets; and from the remainders obtained derive the usual results in revolutions etc. In case the subtraction is made from the risings of the slower planet, the result obtained (in revolutions etc.) gives the longitude of the faster planet; otherwise, that gives the longitude of the slower planet.

This is so, because :

revolutions of faster planet = risings of asterisms—risings of faster planet

= (risings of asterisms—risings of faster planet—risings of slower planet) + risings of slower planet

= risings of slower planet—(sum of risings of slower and faster planets—risings of asterisms)

and, similarly,

revolutions of slower planet

= risings of faster planet—(sum of risings of slower and faster planets—risings of asterisms).

Method 10. Alternative process

40. Or, (the sum of) the difference between the risings of the planet and the risings of the asterisms (for the two planets) should be severally increased and diminished by the difference between the risings of the two planets; and the two results thus obtained should be halved. These give (the revolution-numbers of the faster and the slower of) the two planets (respectively).

The sum of the two planets diminished by the slower planet gives the faster planet, and the same diminished by the faster planet gives the slower planet.

Let the risings in a *yuga* of the asterisms and those of the faster and slower planets be A, R and r, respectively. Then

rev-no. of the faster planet

$$= \tfrac{1}{2}\,[\,(A-R)+(A-r)+(r-R)\,],\ \text{i.e., } A-R$$

and rev-no. of the slower planet

$$= \tfrac{1}{2}\,[\,(A-R)+(A-r)-(r-R)\,],\ \text{i.e., } A-r.$$

It should be noted that the number of risings in a *yuga* for the faster planet is smaller than that for the slower planet. Thus, the number of risings of the Sun in a *yuga*

$$= 1582237560 - 4320000$$
$$= 1577917560$$

whereas the number of risings of the planet Jupiter (which moves slower than the Sun) in a *yuga*

$$= 1582237560 - 364220$$
$$= 1581873340.$$

7. MISCELLANEOUS TOPICS

(1) Risings of asterisms and planets

41. The sum of a planet's own revolutions and risings (in a *yuga*) gives the revolutions (or risings) of the asterisms (in a *yuga*). From the risings of the slower and faster planets diminished and increased by the conjunction-revolutions of those two planets are obtained the risings of the faster and slower planets (in a *yuga*).

In other words :

(1) revolutions (or risings) of the asterisms

= revolutions of a planet + risings of that planet.

(2) risings of the faster planet

= risings of the slower planet − (revolutions of the faster planet − revolutions of the slower planet).

(3) risings of the slower planet

= risings of the faster planet + (revolutions of the faster planet − revolutions of the slower planet).

(2) Motion of a planet for its own day (i. e., from one rising of the planet to the next)

42. Multiply the revolutions of the planet (in a *yuga*) by the minutes in a circle (i.e., by 21600) and divide by the number of risings of the planet (in a *yuga*) ; then is obtained the motion of that planet from its one rising to the next (in terms of minutes).

(3) One planet from another. (Alternative method)

43. Multiply the civil days (in a *yuga*) by the revolutions of the planet other than the desired one and divide by the revolutions of the desired planet : this is the so called "divisor". By this divisor, divide the product of the *Ahargaṇa* and the revolutions of the other planet : then is obtained the longitude of the desired planet.[1]

That is, if P be the known planet and Q the desired planet, the

$$\text{longitude of } Q = \frac{\text{revolutions of } P \times \textit{Ahargaṇa}}{D},$$

where

$$D = \frac{\text{revolutions of } P \times \text{civil days in a } \textit{yuga}}{\text{revolutions of } Q}.$$

(4) Special method for finding a planet

44. Multiply the sum or difference of the longitudes, taken along with the revolutions performed, of two or more planets, each multiplied or divided by arbitrary numbers, by the revolutions of the desired planet and divide by the revolutions of those two or more planets operated upon in the same way (as their longitudes are) : the result is the longitude of the desired planet.[2]

Let $R_1, R_2, R_3, \ldots, R_n$ be the revolution-numbers and $L_1, L_2, L_3, \ldots, L_n$ the longitudes in revolutions etc., of n planets. Given the multipliers $m_1, m_2, m_3, \ldots, m_n$ and

$$m_1L_1 \pm m_2L_2 \pm m_3L_3 \pm \ldots \pm m_nL_n,$$

1. *Cf. SiŚe*, ii. 73.
2. *Cf. BrSpSi*, xiii. 28; *SiŚe*, ii. 81.

the problem is to find the longitude of the planet P whose revolution-number is R.

The above rule gives the following formula for the longitude of P:

$$\text{longitude of } P = \frac{(m_1L_1 \pm m_2L_2 \pm \dots \pm m_nL_n) \times R.}{m_1R_1 \pm m_2R_2 \pm \dots \pm m_nR_n},$$

which is true, because

$$\frac{L_1}{R_1} = \frac{L_2}{R_2} = \dots = \frac{L_n}{R_n}.$$

Problems 13 and 14 set in section 9 of the present chapter are based on the above rule.

(5) Another special method for finding a planet

45-46. When the sum of the longitudes of two or more planets severally increased or diminished by the longitudes of those planets as multiplied by a given multiplier, are given, find their sum and divide that by the number of the planets, two or more, as increased or diminished by the given multiplier : this gives the sum of the longitudes of those planets. Divide that by the sum of the revolution-numbers of those planets and multiply that severally by the revolution-numbers of those planets : then are obtained the longitudes of those planets, respectively.[1] When multiplication is made by the revolutions of any other desired planet, then is obtained the longitude of that desired planet.

Let the longitudes of n planets be $L_1, L_2, L_3, \dots, L_n$ and let S be their sum. Given the multiplier m, and the values of

$$S \pm mL_1,\ S \pm mL_2,\ S \pm mL_3, \dots,\ S \pm mL_n,$$

the problem is to find $L_1, L_2, L_3, \dots, L_n$.

The above rule gives the following formula for S:

$$S = \frac{(S \pm mL_1) + (S \pm mL_2) + \dots + (S \pm mL_n)}{n \pm m}, \quad (1)$$

which is exactly the same as given by Brahmagupta and Śrīpati.

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1. *Cf. BrSpSi*, xiii. 47; *SiŚe*, ii. 71-72. In place of formula (2) (see p. 59) Śrīpati gives

$$L_r = \frac{S \sim (S \pm mL_r)}{m}.$$

See *SiŚe*, ii. 72.

S being determined in this way, L_r is obtained by the formula :

$$L_r = \frac{S \times \text{rev-no. of the } r\text{th planet}}{\text{sum of revolutions of all } n \text{ planets}}.$$

Problems 15 and 16 of section 9 of the present chapter are based on the above rule.

(6) A generalisation of the previous rule

47. Severally divide the aggregate of the partial sums (*padasvam*) as increased or diminished by the partial sum multiplied by the multiplier (given for it), by its own multiplier ; then take the sum of all these results, and divide that by (the sum of) as many units as there are partial sums (successively) divided by the given divisors, increased or diminished by 1 ; this gives the aggregate of the partial sums. From that obtain what remains to obtain.

Let there be n partial sums $s_1, s_2, s_3, \ldots\ldots, s_n$ and let $m_1, m_2, m_3, \ldots,$ m_n be the respective multipliers of the n partial sums. Given $m_1, m_2,$ $m_3, \ldots, m_n$ and the values of

$$S \pm m_1 s_1,\ S \pm m_2 s_2, \ldots, S \pm m_n s_n$$

where $S = s_1 + s_2 + \ldots + s_n$, the problem is to find the partial sums, $s_1, s_2, s_3, \ldots, s_n$.

The above rule gives the following formula for S:

$$S = \frac{\dfrac{S \pm m_1 s_1}{m_1} + \dfrac{S \pm m_2 s_2}{m_2} + \ldots + \dfrac{S \pm m_n s_n}{m_n}}{\left(\dfrac{1}{m_1} + \dfrac{1}{m_2} + \ldots + \dfrac{1}{m_n}\right) \pm 1}.$$

S being thus determined, the partial sum s_r can be easily obtained by the formula

$$s_r = \frac{(S \pm m_r s_r) \sim S}{m_r}.$$

It may be noted that the above formula of Vaṭeśvara is a generalisation of Brahmagupta's formula for equal multipliers, viz.

$$S = \frac{(S \pm m s_1) + (S \pm m s_2) + \ldots + (S \pm m s_n)}{n \pm m}.$$

See *BrSpSi*, xiii. 47.

(7) A special rule for solving a linear equation involving one unknown and several known planets

48-49. Find the sum or difference of the revolution-numbers of the planets, as multiplied or divided by the given multipliers or divisors (in the problem); or (when the subtraction is not possible), add the number of civil days in a *yuga* (to the minuend) and then subtract the revolution-number of the subtractive planet (as multiplied or divided by the given multiplier or divisor). (If the sum or difference thus obtained exceeds the number of civil days in a *yuga*), divide the sum or difference by the number of civil days in a *yuga*. (Discard the quotient and retain the remainder). Now depending upon whether the result due to the "other" (i.e., desired) planet is additive or subtractive (in the given problem), subtract the remainder from or add that to the number of civil days in a *yuga*; then increase or diminish that by the revolutions of the given planet; and finally (if the sum or difference exceeds the number of civil days in a *yuga*) divide it out by the number of civil days in a *yuga*. (Discard the quotient and take the remainder.) This gives the revolutions of the other planet.[1]

The following solved examples shall illustrate the above method.

Example 1. If

$$10.\ \text{Moon} \pm 3.\ \text{Mercury} + \text{Unknown planet} = \text{Saturn},$$

find the revolution-number of the unknown planet.[2]

Let the revolution-numbers of Moon, Mercury, Saturn and the unknown planet be M, M', S, and x respectively. Also Let C be the number of civil days in a *yuga*. Then we have to solve the equation

$$10M \pm 3M' + x = S,$$

giving

$$x = S - (10M \pm 3M')$$

$$\text{or}\quad C + S - (10M \pm 3M'), \qquad (1)$$

---

1. *Cf. BrSpSi*, xiii. 34-35; *SiŚe*, ii. 76-77; *SiŚi*, II, xiii. 8-9. In *SiŚe* the rule is explained very clearly and is illustrated by means of examples. A similar rule is given in *MSi*, xvii. 3-5.

2. In this problem, Saturn is the 'given planet' of the text.

because addition of any multiple of C does not make any difference. Since x has to be less than C, the right hand side should be divided out by C, if necessary.

The process stated in the text is as follows :

Find the value of $10M \pm 3M'$; if it exceeds C, divide it out by C. Since x is additive, subtract $10M \pm 3M'$ from C, and then add S to it. Thus one gets $C-(10M \pm 3M')+S$. This is the value of x, i. e.,

$$x=C-(10M \pm 3M')+S,$$

which is the same as (1). If this value of x exceeds C, divide it by C and take the remainder.

Example 2. If

10. Moon ± 3. Mercury – Unknown planet = Saturn,

find the revolutions of the unknown planet.

We have to solve

$$10M \pm 3M'-x=S,$$

giving $$x=(10M \pm 3M')-S, \text{ or } C+(10M \pm 3M')-S. \qquad (2)$$

The process stated in the text is as follows :

Find the value of $10M \pm 3M'$; if it exceeds C, divide it out by C. Since x is subtractive, add it to C and then subtract S from it. Thus we get

$$x=C+(10M \pm 3M')-S,$$

which is the same as (2) above. If this value of x exceeds C, divide it by C and take the remainder.

It is to be noted that the multiplier of the unknown planet in the problems has necessarily to be unity.

(8) Planet for the end of lunar day

50. Multiply the revolutions of the planet (in a *yuga*) by the elapsed lunar days and divide by the lunar days in a *yuga* : then is obtained, in revolutions etc., the longitude of the planet for the end of the lunar day (elapsed).

(9) Planet for the end of solar day

51. Multiply the revolutions of the planet by the (elapsed) solar days and divide by the solar days in a *yuga* : the result, in revolutions etc., gives the longitude of the planet for the end of the elapsed solar day.

(10) Planet for sunrise of the gods or sunset of the demons

52. Multiply the revolutions of the planet by the elapsed solar years and divide by the solar years in a *yuga* : the result is the longitude of the planet at sunrise of the gods or sunset of the demons.

(11) Planet for the beginning of Jovian year

53(a-b). In a similar manner, one should calculate the planets for the commencement of the Jovian year with the help of the elapsed Jovian years.

8. POSITIONS OF PLANETS AT THE BEGINNINGS OF BRAHMĀ'S DAY, CURRENT KALPA AND KALIYUGA

53 (c-d). At the commencement of Brahmā's day, the planets as well as their *mandoccas, śīghroccas,* and ascending nodes were situated at the junction of (the signs) Pisces and Aries.

54. The revolutions (for the duration of Brahmā's life) being increased by one-fortieth of themselves give the signs etc. of the longitude in the beginning of the current *kalpa*. These being increased by the minutes obtained by multiplying the (same) revolutions by 613 and dividing by 4480 give the longitude in the beginning of Kali*yuga*.

The number of *kalpas* in the life-span of Brahmā is 72000, and the number of *kalpas* elapsed at the beginning of the current *kalpa* since the birth of Brahmā is 6150. See *supra*, sec. 2, vs. 8; and sec. 3, vs. 17.

Therefore the longitude of a planet's apogee or ascending node at the beginning of the current *kalpa*

$$= \frac{6150 \times R}{72000} \text{ revolutions}$$

$$= (R + R/40) \text{ signs,}$$

where R is the revolution-number of the planet's apogee or ascending node in the life-span of Brahmā.

Again, the number of *yugas* elapsed since the beginning of the current *kalpa* up to the beginning of Kaliyuga is equal to $459\frac{3}{4}$. Therefore, the motion of the planet's apogee or ascending node for this period

$$= \frac{459\frac{3}{4}R}{1008\times 72000}\text{ revolutions}$$

$$= \frac{613R}{4480}\text{ minutes.}$$

Hence the longitude of the planet's apogee or ascending node (whose revolution-number in the life-span of Brahmā is R) at the beginning of

Kaliyuga $=(R+R/40)$ signs $+\ \frac{613R}{4480}$ minutes.

A PASSING REFERENCE TO THE RULE OF THREE AND REDUCTION OF NUMERATOR AND DENOMINATOR

55. All unknown quantities should be determined from the known ones by the rule of three. Everything stated above is in most cases simplified by using the reduced numerator and denominator. For this purpose one should divide both of them by the last non-zero remainder of their mutual division. The reduced quantities are called *dṛḍha* or coprime.

ACTUAL POSITIONS OF PLANETS' APOGEES AND ASCENDING NODES IN THE BEGINNING OF KALIYUGA

56-62. (When the longitudes of the planets are calculated by taking the beginning of Kaliyuga as the epoch, then to account for the positions of the planets) in the beginning of Kaliyuga one should add 3 signs to the Moon's apogee; 6 signs to the Moon's ascending node; 2 signs 18° 51′ 37″ to the Sun's apogee; 4 signs 8° 50′ 50″ to Mars' apogee; 7 signs 16° 42′ 54″ to Mercury's apogee; 5 signs 22° 48′ 31″ to Jupiter's apogee; 2 signs 20° 3′ 26″ to Venus' apogee; 7 signs 25° 56′ 53″ to Saturn's apogee; 10 signs 20° 10′ 12″ to Mars' ascending node; 11 signs 10° 19′ 54″ to Mercury's ascending node; 9 signs 10° 10′ 14″ to Jupiter's ascending node; 10 signs 0° 7′ 17″ to Venus' ascending node; and 8 signs 20° 1′ 0″ to Saturn's ascending node. These positions have been obtained by dividing the product of the days elapsed in the beginning of Kaliyuga and the revolutions (of the apogees and ascending nodes) by the civil days in the life-span of Brahmā.[1]

1. Positions of planets' apogees and ascending nodes are also stated in *Ā*, i. 8(a-b); *BrSpSi*, i. 52-57; *ŚiDVr*, iii. 1(a-b); xi. 5(c-d); *SiŚe*, ii. 54; *SiŚi*, I, i (c). 19-20.

The motion of the planets' apogees is direct whereas that of the planets' ascending nodes is retrograde. Therefore, the longitudes of the planets' apogees and ascending nodes, measured eastwards as usual, are as follows :

Table 8. Longitudes of planets' apogees in the beginning of Kaliyuga.

	signs	degrees	minutes	seconds
Sun	2	18	51	37
Moon	3	0	0	0
Mars	4	8	50	50
Mercury	7	16	42	54
Jupiter	5	22	48	31
Venus	2	20	3	26
Saturn	7	25	56	53

Table 9. Longitudes of planets' ascending nodes in the beginning of Kaliyuga

	signs	degrees	minutes	seconds
Moon	6	0	0	0
Mars	1	9	49	48
Mercury	0	19	40	6
Jupiter	2	19	49	46
Venus	1	29	52	43
Saturn	3	9	59	0

Āryabhaṭa I[1] and Lalla[2] have stated the following positions of the planets' apogees and ascending nodes :

1. See *Ā*, i. 9.
2. See *ŚiDVṛ*, iii. 1 (a); xi. 5 (c-d).

	Position of apogee	Position of ascending node
Sun	2 signs 18°	
Moon	3 signs	6 signs
Mars	3 signs 28°	1 sign 10°
Mercury	7 signs	20°
Jupiter	6 signs	2 signs 20°
Venus	3 signs	2 signs
Saturn	7 signs 26°	3 signs 10°

Śrīpati[1] and Bhāskara II[2] have given the following positions for the beginning of Kaliyuga :

	Śrīpati	Bhāskara II
Sun's apogee	2 signs 17° 45′ 36″	2 signs 17° 45′ 36″
Moon's apogee	4 signs 5° 29″ 45″ 36‴	4 signs 5° 29′ 46″
Moon's ascending node	5 signs 3° 12′ 57″ 36‴	5 signs 3° 12′ 58″

1. See *SiŚe*, ii. 54.
2. See *SiŚi*, I, i (c). 19-20.

Section 5

Śuddhi or intercalary fraction, for solar year etc.

1. *Śuddhi for solar year*

RESIDUAL INTERCALARY, RESIDUAL CIVIL AND RESIDUAL OMITTED DAYS

1. Severally multiply the elapsed years by 2334, 9313 and 29021 and divide each product by 36000: the quotients thus obtained are the elapsed (residual) intercalary days, (residual) civil days, and (residual) omitted days respectively.[1] Then divide the remainders by 600: the quotients obtained are the corresponding *ghaṭīs*. Then divide the new remainders by 10: the quotients obtained are the corresponding *palas* (or *vighaṭīs*).

According to Vaṭeśvara :

(1) intercalary days in a year $= \frac{1593336 \times 30}{4320000} = 11 + \frac{2334}{36000}$

(2) civil days in a year $= \frac{1577917560}{4320000} = 365 + \frac{9313}{36000}$

(3) omitted days in a year $= \frac{25082520}{4320000} = 5 + \frac{29021}{36000}$.

Thus the residual intercalary, civil and omitted days in a year are respectively equal to

$$\frac{2334}{36000}, \ \frac{9313}{36000} \text{ and } \frac{29021}{36000}.$$

These residues will go on accumulating year to year. The above rule tells us how to find the accumulated amounts of these residues, in terms of days, *ghaṭīs* and *vighaṭīs*, corresponding to the years elapsed since the epoch.

ABRIDGED RULE FOR RESIDUAL INTERCALARY DAYS AND PARTICULAR CASES

2. Alternatively, multiply the years elapsed by 389 and divide by 6000: the result is the elapsed (residual) intercalary days. Thus in 200

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1. Similar rules are stated in *BrSpSi*, i. 40; *MBh*, i. 27-28; *SiDVr*, i. 27, 28; *SiŚe*, ii. 34, 35 (a-b).

years there are 13 (residual) intercalary days as diminished by 1/30 of a day; in 108 years there are 7 (residual) intercalary days increased by 1 *pala* for every 15 years; and in 16 years, there is 1 (residual) intercalary day increased by 7 *nāḍīs* for every 50 years.

As shown above (in vs. 1), the number of residual intercalary days in 1 year

$$= \frac{2334}{36000}$$

$$= \frac{389}{6000},$$

dividing numerator and denominator by 6.

Thus :

(1) In 200 years, the number of residual intercalary days

$$= \frac{389 \times 200}{6000} = \frac{389}{30} = 13 - \frac{1}{30}.$$

(2) In 108 years, the number of residual intercalary days

$$= \frac{389 \times 108}{6000} \text{ days}$$

$$= 7 \text{ days} + \frac{2}{1000} \text{ of a day}$$

$$= 7 \text{ days} + (1 \textit{ pala} \text{ in every 15 years}),$$

because $\frac{2}{1000}$ of a day $= \frac{36}{5}$ *palas* and $\frac{36}{5}$ *palas* in 108 years is equivalent to 1 *pala* in 15 years.

(3) In 16 years, the number of residual intercalary days

$$= \frac{389 \times 16}{6000} \text{ days}$$

$$= 1 \text{ day} + \frac{224}{6000} \text{ of a day}$$

$$= 1 \text{ day} + (7 \textit{ nāḍīs} \text{ in every 50 years}).$$

Hence the rule stated above.

PARTICULAR CASES OF RESIDUAL CIVIL DAYS

3. In 375 years there are 97 (residual) civil days increased by 1 *pala* for every 10 years; in 120 years there are 31 (residual) civil days increased by 13 *palas* for every 10 years; in 144 years there are 37 (residual) civil days increased by 21 *nāḍīs* for every 200 years; and in 96 years, there are 25 (residual) civil days diminished by 31 *palas* for every 5 years.

This can be easily seen to be true.

PARTICULAR CASES OF RESIDUAL OMITTED DAYS

4. In 36 years there are 29 (residual) omitted days increased by 7 *nāḍīs* for every 200 years; in 96 years there are 77 (residual) omitted days increased by 73 *nāḍīs* for every 300 years; in 5 years there are 4 (residual) omitted days increased by 221 *vināḍīs* for every 10 years; and in 300 years there are 241 (residual) omitted days increased by 101 *palas* for every 10 years.

This too can be easily seen to be true.

The word "*vāk*" used in the Sanskrit text bears the numerical value 1.

RELATION BETWEEN RESIDUAL INTERCALARY, RESIDUAL CIVIL, AND RESIDUAL OMITTED DAYS

5. The sum of the elapsed years and the (residual) intercalary days when diminished by the (residual) omitted days gives the (residual) civil days and when diminished by the (residual) civil days gives the (residual) omitted days. And the sum of the (residual) civil days and the (residual) omitted days when diminished by the elapsed years gives the (residual) intercalary days.

Let Y be the number of elapsed years. Then

$$\text{residual intercalary days} = \frac{2334}{36000} Y$$

$$\text{residual omitted days} = \frac{29021}{36000} Y$$

$$\text{and residual civil days} = \frac{9313}{36000} Y.$$

Evidently

$$Y + \frac{2334}{36000}Y = \frac{29021}{36000}Y + \frac{9313}{36000}Y.$$

That is,

elapsed years + residual intercalary days = residual omitted days + residual civil days.

Hence the above rule.

PAST INTERCALARY MONTHS AND *ŚUDDHI*

First Method

6. When the sum of the (residual) omitted days and the (residual) civil days is increased by 10 times the elapsed years and the resulting sum is divided by 30, the quotient denotes the elapsed intercalary months, and the remaining fraction (of the intercalary month) in terms of days etc. is the *śuddhi*.[1]

Let Y be the number of the years elapsed. Then

$$\text{elapsed intercalary days} = 11\frac{2334}{36000}Y$$

$$= 10Y + \frac{29021}{36000}Y + \frac{9313}{36000}Y$$

= 10 × elapsed years + residual omitted days + residual civil days.

$$\therefore \text{elapsed intercalary months} = \frac{1}{30}\Big[10 \times \text{elapsed years} + \text{residual omitted days} + \text{residual civil days}\Big]$$

The quotient gives the complete intercalary months elapsed, and the residue, in days etc., is the *Adhimāsaśeṣa* known as *Śuddhi*.

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1. *Cf. BrSpSi*, i. 41; *ŚiDVr*, i. 29; *SiŚe*, ii. 36.

Second Method

7. Or, add the (residual) intercalary days to 11 times the elapsed years and divide the resulting sum by 30: the quotient denotes the number of intercalary months elapsed and the residue, in days etc., the so called ***śuddhi*.**[1]

Let Y be the number of years elapsed. Then the elapsed intercalary days are equal to

$$\left(11 + \frac{2334}{36000}\right)Y \quad \text{or} \quad 11Y + \frac{2334}{36000}Y$$

$$= 11 \times \text{ elapsed years} + \text{residual intercalary days.}$$

Therefore, the elapsed intercalary months are equal to

$$\frac{1}{30}\left\{11 \times \text{elapsed years} + \text{residual intercalary days}\right\}.$$

The quotient denotes the complete intercalary months elapsed, and the residue, in days etc., denotes the *Adhimāsaśeṣa*, known as *śuddhi*.

Third Method

8. Or, multiply the number of years elapsed by 66389 and divide the product by 180000: the quotient denotes the number of intercalary months elapsed and the (residue in) days etc., the so called ***śuddhi*.**[2]

Let Y be the number of years elapsed. Then the intercalary months elapsed in Y years

$$= \frac{1593336}{4320000}\,Y \quad \text{(see } \textit{supra}\text{, sec. 2, vss. 3-4)}$$

$$\doteq \frac{66389}{180000}\,Y.$$

When $66389Y$ is divided by 180000, the quotient will give the complete intercalary months elapsed, and the remainder reduced to days etc. will give the *śuddhi*.

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1. Similar rules occur in *MBh*, i. 22; *SiŚe*, ii. 38; *SiŚi*, I, i (e). 6.
2. *Cf. SiŚe*, ii. 37. Śrīpati subtracts *Avamaghaṭīs* also.

Fourth Method

9. **Or, adding 11 times the denominator (of the residual intercalary days in a year) to the numerator thereof, calculate the intercalary days elapsed. When they are divided by 30, the quotient gives the number of complete intercalary months elapsed, and the residue, in days etc., gives the** *śuddhi*.

This is so, because the number of intercalary days in Y years

$$= \left(11 + \frac{2334}{36000}\right)Y = \frac{11 \times 36000 + 2334}{36000} Y.$$

LORD OF SOLAR YEAR

First Method

10 (a-b). **The remainder obtained on dividing the sum of the number of years elapsed and the (residual) civil days by 7 yields the lord of the (solar) year.**[1]

This is so, because the number of civil days elapsed at the end of Y solar years

$$= \left(365 + \frac{9313}{36000}\right)Y \equiv Y + \frac{9313}{36000} Y \pmod 7.$$

Second Method

10 (c-d). **Nine times the number of years elapsed diminished by the (residual) omitted days and increased by the (residual) intercalary days, when divided by 7, the remainder yields the lord of the (solar) year.**

This is so, because the number of civil days elapsed at the end of Y solar years may be written as

$$357Y + 9Y - \frac{29021}{36000} Y + \frac{2334}{36000} Y$$

$$\equiv 9Y - \frac{29021}{36000} Y + \frac{2334}{36000} Y \pmod 7)$$

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1. Same rule is stated in *BrSpSi*. i. 42 (a-b); *SiŚe*. ii. 35 (c-d),

Third Method

11. The product of 5 and the number of elapsed years, increased by the (residual) omitted days and diminished by the (residual) intercalary days, when divided by 7, the difference of the remainder obtained and 7 gives the lord of the (solar) year, which is indeed identical with the lord of the first day of the (solar) year.

The number of civil days elapsed at the end of Y years

$$= 365\,Y + \frac{9313}{36000}Y$$

$$= 366\,Y - \frac{29021}{36000}Y + \frac{2334}{36000}\,Y \text{ (\textit{vide} vs. 5)}$$

$$\equiv -\left[5Y + \frac{29021}{36000}\,Y - \frac{2334}{36000}\,Y\right] (\text{mod}\,7)$$

$$\equiv 7 - \left[5Y + \frac{29021}{36000}\,Y - \frac{2334}{36000}\,Y\right] (\text{mod } 7).$$

Hence the above rule.

The word *pañcaka* in the Sanskrit text is used in the sense of "year", because a year was sometimes taken to be an aggregate of 5 seasons. The *Aitareya-Brāhmaṇa*, for example, reads;

पंचर्तवो हेमन्तशिशिरयोः समासेन

Fourth Method

12(a-b). Twice the (residual) civil days diminished by the (residual) intercalary days and increased by the (residual) omitted days (when divided by seven, the remainder) gives the lord of the (solar) year.

This is so, because the number of civil days elapsed at the end of Y solar years

$$= 365\,Y + \frac{9313}{36000}\,Y$$

$$= 364Y + Y + \frac{9313}{36000}\,Y$$

$$= 364\,Y + 2 \times \frac{9313}{36000}\,Y - \frac{2334}{36000}\,Y + \frac{29021}{36000}\,Y,$$

because $Y = \frac{9313}{36000}\,Y - \frac{2334}{36000}\,Y + \frac{29021}{36000}\,Y$ (*vide* vs. 5)

$$\equiv 2 \times \frac{9313}{36000}\,Y - \frac{2334}{36000}\,Y + \frac{29021}{36000}\,Y \pmod 7$$

Fifth Method

12 (c-d). (In the fraction denoting the residual civil days for a year) add the denominator to the numerator, and therefrom calculate the (residual) civil days, as before, and divide them by 7: the remainder obtained gives the lord of the (solar) year.

This is so, because the number of civil days elapsed at the end of Y solar years

$$= \left(365 + \frac{9313}{36000}\right) Y$$

$$\equiv \left(1 + \frac{9313}{36000}\right) Y \pmod 7$$

$$\equiv \frac{36000+9313}{36000}\,Y \pmod 7$$

LORD OF LUNAR YEAR

13 (a-b). The methods for finding the lord of the solar year have been stated above. Now shall be described the methods for finding the lord of the lunar year, i.e., the lord of the first day of the light half of Caitra.

First Method

13 (c-d). The lord of the lúnar year may be obtained as before by calculating the *Ahargaṇa* (for the beginning of Caitra) from the solar years elapsed.

Second Method

14 (a-b). Or, determine the lord of the (lunar) year from 5 times the solar years elapsed diminished by the difference of the intercalary (months elapsed reduced to) days and the sum of the omitted days elapsed and twice the solar years elapsed.

Let Y be the number of solar years elapsed. Then the number of civil days elapsed up to the beginning of Caitra

$$= 360\,Y + \text{intercalary months elapsed reduced to days} \\ -\text{omitted days elapsed}$$

$$\equiv 3Y + \text{intercalary months elapsed reduced to days} \\ -\text{omitted days elapsed} \pmod 7$$

$$\equiv 5Y - (2Y + \text{omitted days elapsed} - \text{intercalary months} \\ \text{elapsed reduced to days}) \pmod 7.$$

Hence the rule.

Third Method

14 (c-d). Or, the lord of the (lunar) year may be obtained by adding the omitted *ghaṭīs* (corresponding to the *śuddhi*) and the solar years elapsed to the (residual) civil days minus the *śuddhi*.

Let Y be the number of solar years elapsed. Then the civil days elapsed in the beginning of Caitra

$$= \left(365 + \frac{9313}{36000}\right)Y - (\textit{śuddhi} - \text{omitted } \textit{ghaṭīs} \text{ corresponding} \\ \text{to } \textit{śuddhi})$$

$$= 364Y + (Y + \text{residual civil days}) - (\textit{śuddhi} - \\ -\text{omitted } \textit{ghaṭīs} \text{ corresponding to } \textit{śuddhi})$$

$$\equiv (Y + \text{omitted } \textit{ghaṭīs} \text{ corresponding to } \textit{śuddhi}) \\ + (\text{residual civil days} - \textit{śuddhi}) \pmod 7.$$

Hence the rule.

15. This is how one bases his calculations on lunar elements corresponding to solar revolutions or solar years elapsed.

LORDS OF SOLAR AND LUNAR YEARS DERIVED FROM EACH OTHER

16. The lord of the lunar year increased by (the days, etc., of) the *śuddhi* and diminished by the (corresponding) omitted *nāḍīs* gives the lord of the solar year; and the lord of the solar year diminished by (the days, etc., of) the *śuddhi* and increased by the (corresponding) omitted *nāḍīs* gives the lord of the lunar year.

The lord of the first day of the light half of Caitra is the lord of the lunar year, and the lord of the first day of the solar year is the lord of the solar year. The first day of the light half of Caitra falls earlier than the first day of the solar year. The number of days between the two is equal to

śuddhi— corresponding omitted *nāḍīs* etc.

Hence the above rule.

PLANETS FOR THE END OF SOLAR YEAR

Moon

First Method

17. The longitudes of the planets may by obtained from the (elapsed) solar years in the manner stated earlier.

Alternatively, twelve times the *śuddhi* gives the Moon's longitude in terms of degrees etc.[1]

This is so, because the Sun's longitude is zero at the begining or end of a solar year.

Second Method

18. Set down the number of elapsed solar years in three places. In the first place multiply it by 132 degrees, in the middle place by 46 (minutes), and in the last place by 34/50 (minutes): then is obtained the Moon's longitude (at the end of the elapsed solar year).[2]

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1. Same method occurs in *MSi*, xvii. 26; *SiŚi*, I, i (e). 10 (a-b).
2. This rule agrees with that quoted by Albīrūnī from the *Karaṇasāra* of the author. See Albīrūnī's *India*, II, p. 54.

Moon's motion for one solar year (according to Vaṭeśvara)

$$= \frac{57753336}{4320000} \text{ revs.}$$

$$= 13 \text{ revs. } 132^\circ \left(46\frac{34}{50}\right)'$$

Hence the above rule.

Moon's apogee and Moon's ascending node

19. The longitudes of the Moon's apogee and the Moon's ascending node are obtained on multiplying the number of years elapsed (in the first place) by 40 and 19 (degrees), (in the middle place) by 41 and 21 (minutes), and (in the last place) by 11/200 and 34/200 (minutes), respectively.

The yearly motions of the Moon's apogee and the Moon's ascending node (according to Vaṭeśvara) are :

	Yearly motion
Moon's apogee	$40^\circ \left(41\frac{11}{200}\right)'$
Moon's ascending node	$19^\circ \left(21\frac{34}{200}\right)'$

Hence the above rule.

Mars and other planets

20-21. The longitude of Mars is obtained on multiplying the number of years elapsed (in the first place) by 191 (degrees); the longitude of the *Śīghrocca* of Mercury, by 54 (degrees); the longitude of Jupiter, by 30 (degrees); the longitude of the *Śīghrocca* of Venus, by 225 (degrees) and the longitude of Saturn, by 12 (degrees); in the middle place, by 24, 45, 21, 11 and 12 minutes (respectively); and in the last place, by 7, 14, 5, 44 and 42 (minutes, respectively) each divided (by 50) as in the case of the Moon.

The yearly motions of the planets, according to Vaṭeśvara, are as follows :

Planet	Yearly motion
Mars	$191^\circ\left(24\frac{7}{50}\right)'$
Śīghrocca of Mercury	$54^\circ\left(45\frac{14}{50}\right)'$
Jupiter	$30^\circ\left(21\frac{5}{50}\right)'$
Śīghrocca of Venus	$225^\circ\left(11\frac{44}{50}\right)'$
Saturn	$12^\circ\left(12\frac{42}{50}\right)'$

Hence the above rule.

PLANETS DERIVED FROM THE SUN (FIRST METHOD)

Moon

22. Add 13 times the Sun's longitude to the result obtained by multiplying the Sun's longitude by 66389 and dividing the product by 18×10000 : the result is the Moon's longitude.

This is so, because :

$$\text{Moon's revolution-number} = 13\times\text{Sun's revolution-number} + \text{intercalary months}$$

$$= 13\times\text{Sun's revolution-number}+1593336$$

$$=\left(13+\frac{1593336}{4320000}\right)\times\text{Sun's revolution-number}$$

$$=\left(13+\frac{66389}{180000}\right)\times\text{Sun's revolution-number.}$$

Since the longitudes of the planets are proportional to their revolution-numbers, therefore

$$\text{Moon's longitude} = \left(13 + \frac{66389}{180000}\right) \times \text{Sun's longitude.}$$

Mars

23. Multiply the Sun's longitude by 34207 and divide the product by 1080000; add that to one-half of the Sun's longitude: the result is Mars' longitude.

According to Vaṭeśvara :

Mars' revolution-number = 2296828.

Therefore,

$$\frac{\text{Mars' longitude}}{\text{Sun's longitude}} = \frac{\text{Mars' revolution-number}}{\text{Sun's revolution-number}}$$

$$= \frac{2296828}{4320000}$$

$$= \frac{1}{2} + \frac{34207}{1080000}$$

Hence $$\text{Mars' longitude} = \left(\frac{1}{2} + \frac{34207}{1080000}\right) \times \text{Sun's longitude.}$$

Śīghrocca of Mercury

24. Whatever is obtained by dividing 20533 times the Sun's longitude by 135000, should be added to 4 times the Sun's longitude: thus is obtained the longitude of the *Śīghrocca* of Mercury.

According to Vaṭeśvara :

Revolution-number of *Śīghrocca* of Mercury = 17937056.

Therefore

$$\frac{\text{longitude of } \textit{Śīghrocca} \text{ of Mercury}}{\text{Sun's longitude}} = \frac{17937056}{4320000}$$

$$= 4 + \frac{20533}{135000}.$$

Hence the rule.

Jupiter

25. To one-twelfth of the Sun's longitude, add whatever is obtained by multiplying the Sun's longitude by 211 and dividing by 216000: then is obtained Jupiter's longitude.

According to Vaṭeśvara :

Jupiter's revolution-number = 364220

Therefore,

$$\text{Jupiter's longitude} = \frac{364220}{4320000} \times \text{Sun's longitude}$$

$$= \left(\frac{1}{12} + \frac{211}{216000}\right) \times \text{Sun's longitude}.$$

The rule for Jupiter's longitude, stated above, does not occur in the manuscripts used. The verse containing this rule seems to have been left out by the scribe due to oversight. It has been inserted there to complete the text.

Śīghrocca of Venus

26. Whatever is obtained by multiplying the Sun's longitude by 32511 and dividing the product by 20000 is the longitude of the *Śīghrocca* of Venus, as stated by the sages.

According to Vaṭeśvara :

Revolution-number of *Śīghrocca* of Venus = 7022376.

Therefore,

$$\text{longitude of } \textit{Śīghrocca} \text{ of Venus} = \frac{7022376}{4320000} \times \text{Sun's longitude}$$

$$= \frac{32511}{20000} \times \text{Sun's longitude.}$$

Saturn

27. To one-thirtieth of the Sun's longitude add the result obtained by multiplying the Sun's longitude by 107 and dividing the product by 180000: thus is obtained Saturn's longitude.

According to Vaṭeśvara :

Saturn's revolution-number = 146568.

Therefore,

$$\text{Saturn's longitude} = \frac{146568}{4320000} \times \text{Sun's longitude}$$

$$= \left(\frac{1}{30} + \frac{107}{180000}\right) \times \text{Sun's longitude.}$$

Moon's apogee

28. To one-ninth of the Sun's longitude, add the Sun's longitude multiplied by 2737 and divided by 1440000: thus is obtained the longitude of the Moon's apogee.

According to Vaṭeśvara :

Revolution-number of Moon's apogee = 488211.

Therefore,

$$\text{longitude of Moon's apogee} = \frac{488211}{4320000} \times \text{Sun's longitude}$$

$$= \left(\frac{1}{9} + \frac{2737}{1440000}\right) \times \text{Sun's longitude.}$$

Moon's ascending node

29. To one-twentieth of the Sun's longitude, add the Sun's longitude multiplied by 8117 and divided by 2160000: the result is the longitude of the Moon's ascending node.

According to Vaṭeśvara :

Revolution-number of Moon's ascending node = 232234.

Therefore,

longitude of Moon's ascending node

$$= \frac{232234}{4320000} \times \text{Sun's longitude}$$

$$= \left(\frac{1}{20} + \frac{8117}{2160000}\right) \times \text{Sun's longitude.}$$

Rules similar to those stated in stanzas 22 to 29 are also found to occur in *BrSpSi*, xxv. 33-36; in *ŚiDVṛ*, i. 50-52 (see S. Dvivedi's edition); and in *LMā* (ASS), i. 8-10.

CALCULATION OF SHORTER *AHARGAṆA*

First Method

30-31(a-b). Diminish the lunar days (*tithis*) elapsed since the beginning of Caitra by the *śuddhi* and set down the result in two places. In one place multiply that by 11 and to the resulting product add the quotient obtained by dividing 703 times the *Avama-ghaṭīs* (i.e., residual omitted *ghaṭīs*, corresponding to the beginning of the current year) by 60. Divide what is obtained by 703 and subtract the resulting omitted days, from the result at the other place. That increased by the *Avama-ghaṭīs* for (the beginning of) the (current) year gives the *Ahargaṇa* (reckoned from the beginning of the current solar year).[1]

That is,

$$\text{Shorter } Ahargaṇa = (L-S) - \frac{11(L-S)+703\ A_g/60}{703} + A_g, \quad (1)$$

1. Similar rules are stated in *BrSpSi*, i. 42-44; *ŚiDVṛ*. i. 31; *SiŚe*, ii. 40-41 (a-b); *SiŚe*, I. i (e). 12(c-d)-13.

where $L=$ lunar days (*tithis*) elapsed since the beginning of Caitra,

$S=$ *śuddhi* for the beginning of the current solar year,

and $A_g=$ *Avama-ghaṭīs* for the beginning of the current solar year.

The above rule gives the so called Shorter *Ahargaṇa*, i.e., the number of civil days elapsed at sunrise on the current lunar day since the commencement of the current solar year. It can be easily derived by subtracting the *Ahargaṇa* for the beginning of the current solar year from the *Ahargaṇa* for sunrise on the current day. For details, see Bina Chatterjee's edition of Lalla's *Śiṣya-dhī-vṛddhida*, Part II, pp. 21-22, note to vs. 31.

Second Method

31(c-d)-32. Or, add the *Avama-nāḍīs* (=*Avama-ghaṭīs*) to the lunar days elapsed since the beginning of Caitra and diminish that by the *śuddhi*. Set down the result in two places In one place multiply that by 11 and to the product add the result obtained by multiplying the *Avama-ghaṭīs* by 173 and dividing (the product) by 15. Divide that by 703 and subtract the resulting *Avama* from the result at the other place. Then is obtained the *Ahargaṇa* (reckoned from the beginning of the current solar year).

That is,

$$\text{Shorter } Ahargaṇa = (L+A_g-S) - \frac{11(L+A_g-S)+173A_g/15}{703}, \qquad (2)$$

where, as before,

$L=$ lunar days elapsed since the beginning of Caitra,

$A_g=$ *Avama-ghaṭīs* for the beginning of the current solar year,

and $S=$ *śuddhi* for the beginning of the current solar year.

Formula (2) is equivalent to formula (1) above, as can be seen by replacing A_g *ghaṭīs* by $A_g/60$ days in the second bracket.

Third Method

33-34 (a-b). Or, subtract the *tithis* elapsed since (the beginning of) Caitra by the *śuddhi* and set down the result in three places. In the lowest place, divide that by 703 and add the quotient obtained to the

result in the middle. To that add the quotient obtained by dividing 16 times the *Avama-ghaṭīs* by 15. Divide that by 64 and subtract the resulting *Avama* (days etc.) from the result in the other (uppermost) place. That increased by the *Avama-ghaṭīs* gives the *Ahargaṇa* (reckoned from the beginning of the current solar year).

That is

$$\text{Shorter } Ahargaṇa = (L-S) - \frac{(L-S)(1+1/703)+16A_g/15}{64} + A_g. \quad (3)$$

One can easily see that this formula is equivalent to formula (1). The difference is in form only.

Fourth Method

34(c-d)-35. **Subtract the *śuddhi* from the lunar days elapsed since (the beginning of) Caitra as increased by the *Avama-nāḍīs* and set down the result in three places (one below the other). Divide the result in the lowest place by 703 and add that to the result in the middle. To that add 21 times the *Avama-ghaṭīs* as divided by 20. Divide that by 64 and subtract the resulting *Avama* (from the result in the uppermost place): the result is the *Ahargaṇa* (reckoned from the beginning of the current solar year).**

That is,

$$\text{Shorter } Ahargaṇa = (L+A_g-S) - \frac{(L+A_g-S)(1+1/703)+21A_g/20}{64}. \quad (4)$$

This formula is equivalent to formula (2), because

$$\frac{11}{703} = \frac{1+1/703}{64} \text{ and } \frac{173}{15\times703} = \frac{21}{20\times64}, \text{ approx.}$$

Fifth Method

36-37(a-b) **Subtract the *śuddhi* from the lunar days elapsed since (the beginning of) Caitra, and set down the result in two places. In one place, multiply that by 10 and to that add the quotient obtained by dividing 213 times the *Avama-ghaṭīs* by 20. Divide that by 639 and subtract the resulting *Avama* from the result in the other place. That increased by the *Avama-ghaṭīs* gives the *Ahargaṇa* (reckoned from the beginning of the current solar year).**

That is,

$$\text{Shorter } Ahargaṇa = (L-S) - \frac{10(L-S)+213A_g/20}{639} + A_g. \quad (5)$$

This formula is equivalent to formula (1), because

$$\frac{11}{703} = \frac{10}{636}, \text{ approx.}$$

Sixth Method

37(c-d)-38. Or, add the *Avama-ghaṭikās* to the lunar days (elapsed since the beginning of Caitra) and diminish that sum by the *śuddhi*. Set down the result in two places (one below the other). In the lower place, multiply that by 10 and to the product obtained add 629 times the *Avama-ghaṭikās* as divided by 60. Divide that by 639 and subtraet the resulting *Avama* (from the result in the upper place): then is obtained the *Ahargaṇa* (reckoned from the beginning of the current solar year).

That is,

$$\text{Shorter } Ahargaṇa = (L+A_g-S) - \frac{10(L+A_g-S)+629A_g/60}{639}. \quad (6)$$

This formula is equivalent to formula (2), because

$$\frac{11}{703} = \frac{10}{639} \text{ and } \frac{173}{15\times703} = \frac{629}{60\times639}, \text{ approx.}$$

PLANETS FOR THE END OF SOLAR MONTH

39-43. (Set down the elapsed solar months in three places.) Multiplication (of the solar months in the first place) by 41 gives (the degrees of) the Moon; by 15, (the degrees of) Mars; by 124, (the degrees of) the *Śīghrocca* of Mercury; by 2, (the degrees of) Jupiter; by 48, (the degrees of) the *Śīghrocca* of Venus; multiplication by 1, (the degrees of) Saturn; by 3, (the degrees of) the Moon's apogee; and by 1, (the degrees of) the Moon's ascending node. Then, of the solar months set down in three places, multiply those in the middle (severally) by 3, 57, 33, 31,45, 1, 23, and 36: then are obtained the minutes for the same planets in their respective order. The last heap (of solar months) should then be (severally)

multiplied by 2136, 28, 1856, 1820, 2376, 168, 1011, and 1834 and each product should be severally divided by 2400: (these are also the minutes of the same planets in their respective order). Thus are obtained the mean longitudes of the planets at the end of the (elapsed) solar month (in terms of degrees and minutes). The (solar) months (elapsed) themselves, treated as signs, constitute the mean longitude of the Sun.

According to Vaṭeśvara, the mean motions of the planets for one solar month are as given below :

Planet	Motion for one solar month
Moon	$41° \left(3\ \frac{2136}{2400}\right)'$
Mars	$15° \left(57\frac{28}{2400}\right)'$
Śīghrocca of Mercury	$124° \left(33\frac{1856}{2400}\right)'$
Jupiter	$2° \left(31\frac{1820}{2400}\right)'$
Śīghrocca of Venus	$48° \left(45\frac{2376}{2400}\right)'$
Saturn	$1° \left(1\ \frac{168}{2400}\right)'$
Moon's apogee	$3° \left(23\frac{1011}{2400}\right)'$
Moon's ascending node	$1° \left(36\frac{1834}{2400}\right)'$

Hence the above rule.

2. *Śuddhi for solar month*

RESIDUAL CIVIL, RESIDUAL OMITTED, AND RESIDUAL INTERCALARY DAYS

44-45. (Severally) multiply the elapsed solar months by 189313 and 209021, and divide (each product) by 432000: then are obtained the (residual) civil days and (residual) omitted days (respectively). Their

sum divided by 30 gives the intercalary months. The remainder (of the division) gives the days of the *śuddhi*, as well as the fraction of the residual (intercalary) day.

According to Vaṭeśvara :

$$\text{no. of civil days in 1 solar month} = 30 + \frac{189313}{432000}$$

$$\text{no. of omitted days in 1 solar month} = \frac{209021}{432000}$$

$$\text{and no. of intercalary days in 1 solar month} = \frac{398334}{432000}$$

$$= \frac{189313}{432000} + \frac{209021}{432000}.$$

Hence the rule.

LORD OF SOLAR MONTH

46(a-b). From the sum of twice the (solar) months elapsed and the (residual) civil days is obtained the true lord of the (current) solar month.

Since the number of civil days in one month

$$= 30 + \frac{189313}{432000}$$

$$\equiv 2 + \frac{189313}{432000} \pmod 7,$$

therefore, the number of civil days in S solar months

$$\equiv 2S + \frac{189313}{432000} S \pmod 7$$

$$\equiv 2S + \text{residual civil days} \pmod 7.$$

Hence the above rule.

SHORTER *AHARGAṆA* AND LORD OF CURRENT DAY

46(c-d)-47. Diminish the (lunar) days elapsed (since the beginning of the current lunar month) by the *śuddhi* (for the beginning of the current solar month) and set down the result in two places. In one place, multiply that by 11 and to the product add 692 times the *Avamaśeṣa* accompanied by its dlvisor and divide (the resulting sum) by 703. The

quotient subtracted from the result at the other place gives the *Ahargaṇa* (reckoned from the beginning of the current solar month). (The lord of) the (current) day is ascertained from the lord of the (current) solar month.

That is,

$$\text{Shorter } Ahargaṇa = (L-S) - \frac{11(L-S) + 692 \times (Avama \text{ fraction})}{703},$$

where L = no. of lunar days elapsed since the beginning of the current lunar month, and

S = *śuddhi* for the beginning of the current solar month.

The above formula is analogous to that of Brahmagupta. See *BrSpSi*, i. 43-44.

PLANETS FOR THE END OF SOLAR DAY

48-51. (Set down the solar *Ahargaṇa* in two places.) Multiply the solar *Ahargaṇa* (written in one place) (severally) by 802, 31, 249, 5, 97, and 2: the results are the (longitudes of the planets, in terms of) minutes, beginning with Moon. Again multiply the solar *Ahargaṇa* written in the other place (severally) by 2334, 16207, 2264, 1055, 9594 and 642, each divided by 18000: (these are the residual minutes of the longitudes of the same planets).

The longitudes of the Moon's apogee and the Moon's ascending node, in terms of minutes, are obtained by multiplying the solar *Ahargaṇa* in one place by 6 and 3 respectively and in the other place by

$\frac{56211}{72000}$ and $\frac{16234}{72000}$ respectively.

Thus are obtained the longitudes of the planets for the end of the elapsed solar day.

The mean motions of the planets for one solar day, according to Vaṭeśvara, are as given below.

Planet	Motion for 1 solar day in minutes	Planet	Motion for 1 solar day in minutes
Moon	$802\frac{2334}{10080}$	*Śighrocca* of Venus	$97\frac{9594}{18000}$
Mars	$31\frac{16207}{18000}$	Saturn	$2\frac{642}{18000}$
Śighrocca of Mercury	$249\frac{2264}{18000}$	Moon's apogee	$6\frac{56211}{72000}$
Jupiter	$5\frac{1055}{18000}$	Moon's ascending node	$3\frac{16234}{72000}$

Hence the above rule.

3. *Śuddhi for solar day*

RESIDUAL CIVIL AND RESIDUAL OMITTED DAYS, *ŚUDDHI*, *AHARGAṆA* AND LORD OF DAY

52. Using the multipliers and 30 times the divisors (prescribed in the case of the solar months) one can find, from the solar days (elapsed), the (residual) civil days, the (residual) omitted days, the *śuddhi*, the *Ahargaṇa*, and the lord of the (current) day, as before.

SOLAR YEARS, SOLAR MONTHS AND SOLAR DAYS

53. Dividing the (solar) days (elapsed) by 360 are obtained the (solar) years (elapsed); then dividing the remainder by 30 are obtained the (solar) months (elapsed) since the beginning of the current solar year; the remainder obtained then gives the (solar) days elapsed since the beginning of the current (solar) month (lit. the desired days).

AVAMAŚEṢA FOR THE CURRENT DAY

54-56. The days elapsed since the beginning of Caitra should be diminished by the *śuddhi* : (then will be obtained the days elapsed since the beginning of the current solar year). From them should be obtained the *Avamaśeṣa* for the current day.

This is the process when the civil *śuddhi* for the end of the (elapsed) solar year is less (than the days elapsed). When the days (elapsed) are less, the difference (obtained by subtracting the days elapsed from the *śuddhi*) is called "negative *Ahargaṇa*". (This denotes the number of days to elapse before the beginning of the solar year.). Multiply it by 11 and subtract the resulting product from the *Varṣānta-avamaśeṣa* (i.e., *Avamaśeṣa* for the end of the solar year) multiplied by 692 and divided by its own divisor.

In case the *Varṣānta-avamaśeṣa* multiplied by 692 and divided by its own divisor is less than the other and can be subtracted therefrom, the negative *Ahargaṇa* should be diminished by 1 (and that should be treated as the correct negative *Ahargaṇa*). This having been done, subtraction should be made from the minuend increased by 703 (i.e., 11 times the corrected negative *Ahargaṇa* should be subtracted from 703 plus *Avamaśeṣa* × 692/divisor). The remainder obtained should be taken as the *Avamaśeṣa* for the current day.

In Fig. 1 below, let C denote the beginning of Caitra, V the beginning of the current solar year, and S the beginning of the current civil day.

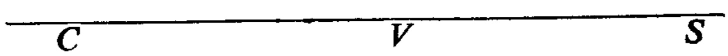


Fig. 1

Then

lunar days between C and V = *śuddhi*,

civil days between C and V = civil *śuddhi* = $\dfrac{\text{śuddhi} \times 11}{703}$,

and

civil days between C and S = civil days elapsed since C, the beginning of Caitra.

Now the *Ahargaṇa* reckoned from V, i.e.,

Varṣāntādi Ahargaṇa = civil days between V and S

= civil days between C and S

— civil days between C and V

$= \text{civil days elapsed since } C - \text{civil } \acute{s}uddhi$

$= \text{lunar days elapsed since } C - \acute{s}uddhi, \text{ (roughly).}$

In case S lies between C and V (see Fig. 2), the

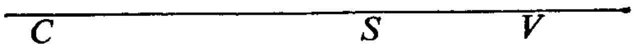


Fig. 2

Varṣāntādi Ahargaṇa is negative, and we have

negative *Varṣāntādi Ahargaṇa* = civil days between S and V

= civil days between C and V

− civil days between C and S

= civil *śuddhi*—civil days elapsed since C

= *śuddhi* − lunar days elapsed since C, (roughly)

= A, say.

Also, in this case,

Avama fraction at S = *Avama* fraction at V — *Avama* corresponding to civil days between S and V

$$= \frac{\textit{Varṣānta Avamaśeṣa}}{\text{divisor}} \text{ (in lunar reckoning)}$$

$$- \frac{A \times 11}{703} \text{ (in civil reckoning)}$$

$$= \left\{ \frac{\textit{Varṣānta Avamaśeṣa} \times 692}{\text{divisor} \times 703} - \frac{A \times 11}{703} \right\},$$

(in civil reckoning)

where divisor = civil days in a *yuga*.

$$\therefore \textit{Avamaśeṣa} \text{ at } S = \frac{\textit{Varṣānta Avamaśeṣa} \times 692}{\text{divisor}} - A \times 11.$$

In case the *Avama* fraction at V is less than the *Avama* corresponding to civil days between S and V, it means that one omitted day has fallen between S and V, so that

$$\text{correct negative } Aharga\d{n}a = A-1.$$

Therefore, in this case, we should take

$$Avama \text{ fraction at } V = 1+\frac{Var\d{s}\bar{a}nta\ Avama\acute{s}e\d{s}a \times 692}{\text{divisor} \times 703},$$

so that

$$Avama\acute{s}e\d{s}a \text{ at } S = 703+\frac{Var\d{s}\bar{a}nta\ Avama\acute{s}e\d{s}a \times 692}{\text{divisor}}-(A-1)\times 11.$$

MOON FOR THE END OF SOLAR YEAR OR MONTH

57. The Sun's longitude at the end of the solar year or solar month, when increased by the degrees corresponding to 12 times the *śuddhi* and 12 times the *Avamaśeṣa* (corresponding to the end of the solar year or solar month) as divided by its own divisor, becomes the Moon's longitude (at sunrise occurring just after the end of the solar year or solar month).

See Fig. 3. Let A denote the end of the lunar year or lunar month, B the end of the solar year or solar month, and C the point where sunrise occurs just after the end of the solar year or month. Then

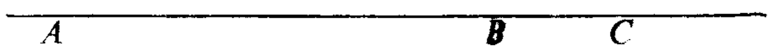


Fig. 3.

tithi at $A = 0$, A being the beginning of the lunar month.

$$tithi \text{ at } C = \text{lunar days between } A \text{ and } B + Avama \text{ fraction}$$

$$= \acute{s}uddhi + \frac{Avama\acute{s}e\d{s}a}{\text{divisor}}. \qquad (1)$$

We also have

$$tithi \text{ at } C = \frac{\text{Moon's longitude at } C - \text{Sun's longitude at } C}{12}, \qquad (2)$$

both longitudes being reduced to degrees.

Therefore, from (1) and (2),

$$\text{Moon's longitude at } C = \text{Sun's longitude at } C + 12\ \textit{śuddhi} + 12 \times \textit{Avamaśeṣa}/\text{divisor},$$

days etc. of *śuddhi* and *Avamaśeṣa* being treated as degrees etc.

Note. When the Sun's longitude is known for the end of the solar year or solar month, the Moon's longitude for that time is actually equal to

$$\text{Sun's longitude} + 12\ \textit{śuddhi},$$

the *Avamaśeṣa* is not needed at all. See *supra*, vs. 17.

SUN FOR THE END OF LUNAR MONTH

58. Multiply the solar months elapsed by the intercalary months in a *yuga* and divide by the solar months (in a *yuga*): the quotient added to the solar months elapsed gives the complete lunar months elapsed. (The remainder is the *Adhimāsaśeṣa*, i.e., the residue of intercalary months.) Multiply the remainder by 5 and divide the resulting product by 8905556: the quotient treated as degrees subtracted from the (complete) lunar months (treated as signs) elapsed since the beginning of the light half of Caitra gives the mean longitude of the Sun in signs etc. (at the end of the lunar month).

The rationale of this rule is as follows :

Suppose that S solar years have elapsed since the epoch and m lunar months since the beginning of Caitra. Then taking $12S+m$ $(= S'$, say) as the number of solar months elapsed, the number of intercalary months elapsed

$$= \frac{S' \times \text{intercalary months in a } \textit{yuga}}{\text{solar months in a } \textit{yuga}}$$

$$= I + \frac{R}{\text{solar months in a } \textit{yuga}}.$$

Then $S'+I$ denotes the number of complete lunar months elapsed,

and

residual fraction of the intercalary months

$$= \frac{R}{\text{solar months in a } yuga}, \text{ in lunar reckoning}$$

$$= \frac{R}{\text{lunar months in a } yuga} \text{ solar months}$$

$$= \frac{R \times 30}{53433336} \text{ solar days}$$

$$= \frac{R \times 5}{8905556} \text{ solar days.}$$

Since m lunar months have elapsed since the beginning of Caitra, therefore the number of solar months and solar days elapsed since the beginning of the solar year up to the end of the mth lunar month

$$= m \text{ solar months} - \frac{R \times 5}{8905556} \text{ solar days.}$$

Hence the Sun's longitude at the end of the mth lunar month

$$= m \text{ signs} - \frac{R \times 5}{8905556} \text{ degrees.}$$

Also see *supra*, notes on vss. 8-9 of sec. 4.

The word *svacchedena* of the Sanskrit text should be read with the previous verse.

LORD OF LUNAR MONTH

59. Multiply the lunar months elapsed by 3407673 and divide by 2226389: the quotient gives the lord of the (current) lunar month. (This is how one may determine the lord of the current lunar month) from the elapsed lunar months.

The number of civil days in one lunar month

$$= \frac{1577917560}{53433336}$$

$$= 28\frac{3407673}{2226389}$$

$$\equiv \frac{3407673}{2226389} \pmod{7}$$

Hence the above rule.

MONTHLY MOTION OF THE PLANETS

60-64(a-c). Add every lunar month 15° 28′ 28″ to the longitude of Mars; 120° 50′ 55″ to the longitude of the *Śīghrocca* of Mercury; 2° 27′ 14″ to the longitude of Jupiter; 47° 18′ 44″ to the longitude of the *Śīghrocca* of Venus; 59′ 15″ to the longitude of Saturn; 3° 17′ 21″ to the longitude of the Moon's apogee; 30° 40′ 12″ to the longitude of the Sun increased by that of the Moon's ascending node; and 1° 33′ 53″ to the longitude of the Moon's ascending node and diminish it by $\frac{1}{218}$ of a minute. Also apply every month $\frac{1}{129}$, $\frac{1}{208}$, $\frac{1}{3304}$, $\frac{1}{162}$, $\frac{1}{948}$, $\frac{1}{185}$, and $\frac{1}{233}$ of a minute to the planets Mars etc., as a negative correction in the case of Jupiter, the *Śīghrocca* of Mercury and Saturn and as a positive correction in the case of other planets.

The mean motions of the planets for one lunar month, according to Vaṭeśvara, are as exhibited below ;

Planet	Monthly motion
Mars	$15° 28' 28'' + \frac{1'}{129}$
Śīghrocca of Mercury	$120° 50' 55'' - \frac{1'}{208}$
Jupiter	$2° 27' 14'' - \frac{1'}{3304}$
Śīghrocca of Venus	$47° 18' 44'' + \frac{1'}{162}$
Saturn	$59' 15'' - \frac{1'}{948}$
Moon's apogee	$3° 17' 21'' + \frac{1'}{185}$
Sun + Moon's ascending node	$30° 40' 12'' + \frac{1'}{233}$
Moon's ascending node	$1° 33' 53'' - \frac{1'}{218}$

DAILY MOTION FROM MONTHLY MOTION

64(c-d). The monthly motion (in terms of minutes) multiplied by 2 and increased by $\frac{1}{63}$ of itself gives the daily motion, (in terms of seconds).

Let the monthly motion of a planet be n mins. Then the daily motion of that planet

$$= \frac{\text{lunar months in a } yuga \times n}{\text{civil days in a } yuga} \text{ minutes}$$

$$= \frac{53433336 \times n}{1577917560} \text{ minutes}$$

$$= \frac{53433336 \times 60 \times n}{1577917560} \text{ seconds}$$

$$= 2\left(1 + \frac{1}{63}\right) n \text{ seconds, approx.}$$

CONVERSION OF LUNAR DAYS INTO CIVIL DAYS

65-66. Multiply the elapsed lunar days by 209021 and divide by 13358334: the quotient subtracted from the (elapsed lunar) days gives the corresponding civil days. The remainder should be multiplied by 60 and divided by its own divisor (i.e., by 13358334): the resulting quotient gives the elapsed *ghaṭikās*.

Next (is described) the calculation of the planets.

This is so, because, according to Vaṭeśvara,

$$1 \text{ lunar day} = \frac{1577917560}{53433336 \times 30} \text{ civil days}$$

$$= \left(1 - \frac{209021}{13358334}\right) \text{ civil days.}$$

PLANETS FOR THE END OF LUNAR DAY

Sun

67. Multiply the elapsed lunar days by 66389 and divide the product by 2226389: the quotient deducted from elapsed lunar days gives the Sun's longitude (in terms of degrees).

Sun's longitude in terms of degrees

$$= \text{solar days elapsed}$$

$$= \text{lunar days elapsed} - \text{intercalary days elapsed}$$

$$= \text{lunar days elapsed} - \frac{\text{lunar days elapsed} \times 1593336 \times 30}{53433336 \times 30}$$

$$= \text{lunar days elapsed} - \frac{\text{lunar days elapsed} \times 66389}{2226389}.$$

Moon

68(a-b). To the Sun's longitude, add degrees equal to 12 times the lunar days elapsed: the result is the Moon's longitude.

Let L denote the elapsed lunar days. Then

$$\text{Moon's longitude} = \frac{57753336\ L}{1603000080} \text{revs.}$$

$$= \frac{(4320000+53433336)\ L}{1603000080} \text{ revs.}$$

$$= \frac{4320000\ L}{1603000080} \text{revs.} + \frac{L}{30} \text{ revs.}$$

$$= \text{Sun's longitude} + 12\ L \text{ degrees.}$$

Mars

68(c-d)-69. Multiply the (elapsed) lunar days by 983 and divide by 26716668; and subtract the resulting degrees etc. from the sum of the (mean) longitudes of the Sun and the Moon. One twenty-seventh of that is the mean longitude of Mars.

$$\text{Mars' longitude} = \frac{2296828\ L}{1603000080} \text{ revs.} = \frac{1}{27} \times \frac{62014356\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{27}\left[\frac{57753336\ L}{1603000080} + \frac{4320000\ L}{1603000080} - \frac{58980\ L}{1603000080}\right] \text{revs.}$$

$$= \frac{1}{27}\left[\text{Moon's longitude} + \text{Sun's longitude} - \frac{983\ L}{26716668} \text{revs.}\right]$$

Śīghrocca of Mercury

70-71 (a). Multiply the (elapsed) lunar days by 164257 and divide by 200375010, and subtract the result (in revolutions etc.) from one-third of the Moon's longitude: then is obtained the longitude of the *Śīghrocca* of Mercury.

$$\text{Longitude of } \acute{S}\bar{\imath}ghrocca \text{ of Mercury} = \frac{17937056\ L}{1603000080} \text{ revs.}$$

$$= \frac{(19251112 - 1314056)\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{3} \times \frac{57753336\ L}{1603000080} - \frac{1314056\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{3} \text{ (Moon's longitude)} - \frac{164257\ L}{200375010} \text{ revs.}$$

Jupiter

71(b-d)-72(a-b). Multiply the (elapsed) lunar days by 383 and divide the product by 200375010, and add the result to one-twentieth of what is obtained on subtracting 22 times Mars' longitude from the Moon's longitude: then is obtained Jupiter's longitude.

$$\text{Jupiter's longitude} = \frac{364220\ L}{1603000080} \text{ revs.}$$

$$= \frac{361156\ L}{1603000080} + \frac{3064\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{20}\Big[\text{Moon's longitude} - 22\text{ (Mars' longitude)}\Big] + \frac{383\ L}{200375010} \text{ revs.}$$

Śīghrocca of Venus

72(c-d)-73(a-b). One-half of the difference between the longitudes of the *Śīghrocca* of Mercury and the Sun, increased by the result obtained on dividing 26731 times the (elapsed lunar) days by 200375010, is the longitude of the *Śīghrocca* of Venus.

Longitude of the *Śīghrocca* of Venus $= \frac{7022376\ L}{1603000080}$ revs.

$$= \frac{6808528\ L}{1603000080} + \frac{213848\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{2} \times \frac{17937056\ L - 4320000\ L}{1603000080} + \frac{213848\ L}{1603000080} \text{ revs.}$$

$$= \frac{1}{2}\Big[\text{longitude of } \textit{Śīghrocca} \text{ of Mercury} - \text{Sun's longitude}\Big] + \frac{26731\ L}{200375010} \text{ revs.}$$

Saturn

73(c-d)-74(a-b). One-eighteenth of the difference between the longitudes of the *Śīghrocca* of Venus and the Sun, diminished by the result obtained on dividing 9 times the (elapsed lunar) days by 4047980, is Saturn's longitude.

Saturn's longitude $= \frac{146568\ L}{1603000080}$ revs.

$$= \left(\frac{150132\ L}{1603000080} - \frac{3564\ L}{1603000080}\right) \text{revs}$$

$$= \frac{1}{18}\Big[\text{long. of } \textit{Śighrocca} \text{ of Venus} - \text{Sun's longitude}\Big] - \frac{9\ L}{4047980} \text{ revs.}$$

Moon's apogee

74(c-d)-75(a-b). One-tenth of the sum of the longitudes of Jupiter and the Sun, increased by the result obtained on dividing 1799 times the (elapsed lunar) days by 145727280, is the longitude of the Moon's apogee.

$$\text{Longitude of Moon's apogee} = \frac{488211\ L}{1603000080} \text{ revs.}$$

$$= \left(\frac{468422\ L}{1603000080} + \frac{19789\ L}{1603000080}\right) \text{revs.}$$

$$= \frac{\text{Jupiter's long.} + \text{Sun's long.}}{10}$$

$$+ \frac{1799\ L}{145727280} \text{ revs.}$$

Moon's ascending node

75(c-d)-76. One-twelfth of the difference between the longitudes of the Sun and the *Śīghrocca* of Venus, increased by the result obtained on dividing 1759 times the (elapsed lunar) days by 400750020, is the longitude of the Moon's ascending node.

The mean longitudes of the planets obtained in this way correspond to the end of the (elapsed) lunar day.

$$\text{Longitude of Moon's ascending node} = \frac{232234\ L}{1603000080} \text{ revs.}$$

$$= \left(\frac{225198\ L}{1603000080} + \frac{7036\ L}{1603000080}\right) \text{revs.}$$

$$= \frac{\text{long. of } \textit{Śīghrocca} \text{ of Venus} - \text{Sun's long.}}{12} + \frac{1759\ L}{400750020} \text{ revs.}$$

4. *Śuddhi for Jovian year*

ŚUDDHI FOR THE BEGINNING OF JOVIAN YEAR

(i) In terms of civil days etc.

77-78. Multiply the (elapsed) Jovian years by 18000 and divide by 18211. Keep the quotient (denoting the solar years elapsed) separately. Multiply the remainder by 13149313 and divide the product by 655596000; the result is in terms of (civil) days, etc. This increased by the *śuddhi*, in civil days etc., for the (beginning of the current) solar year gives the *śuddhi* for the beginning of the current Jovian year in terms of civil days etc.

Let J denote the number of Jovian years elapsed. Then the solar years corresponding to J Jovian years are

$$=\frac{4320000\ J}{364220\times 12}=\frac{18000\ J}{18211}=Q+\frac{R}{18211},$$

where Q denotes the number of complete solar years, and $R/18211$ the fraction of the current solar year elapsed at the beginning of the current Jovian year.

The number of civil days corresponding to $R/18211$ of a solar year

$$=\frac{1577917560\ R}{4320000\times 18211}=\frac{3149313\ R}{655596000}.$$

This gives the civil days etc. elapsed since the beginning of the current solar year up to the beginning of the current Jovian year. This being increased by the *śuddhi*, in civil days etc., for the beginning of the current solar year (i.e., by the civil days etc. lying between the beginning of Caitra and the beginning of the current solar year) gives the *śuddhi* for the beginning of the current Jovian year, i.e., the civil days etc. lying between the beginning of Caitra and the beginning of the current Jovian year.

(ii) In terms of lunar days etc.

79. Or, multiply the (same) remainder by 2226389 and divide by 109266000: the result is in (lunar) days etc. Increase it by the *śuddhi* (for the beginning of the current solar year), in terms of lunar days etc. : (the result is the *śuddhi* for the beginning of the current Jovian year in terms of lunar days etc.)

This is so, because the number of lunar days corresponding to $R/18211$ of a solar year is:

$$=\frac{1603000080\ R}{4320000\times 18211}=\frac{2226389\ R}{109266000}.$$

This gives the lunar days etc. elapsed since the beginning of the current solar year up to the beginning of the current Jovian year. And this being increased by the *śuddhi*, in lunar days etc., for the beginning

of the current solar year (i.e., by the lunar days, etc., lying between the beginning of Caitra and the beginning of the current solar year) gives the *śuddhi* for the beginning of the current Jovian year, i.e., the lunar days, etc., lying between the beginning of Caitra and the beginning of the current Jovian year.

(iii) In terms of lunar days etc. (Alternative method)

80-82. Multiply the (elapsed) Jovian years by 41069 and divide by 2185320. Add the quotient to the (elapsed) Jovian years. (Then are obtained the lunar years corresponding to the elapsed Jovian years.) Multiply whatever is obtained by 66389 and divide by 2160000. (Then are obtained the intercalary years corresponding to the elapsed Jovian years.) These should be diminished by the complete intercalary years and also by the complete intercalary months : the remainder (i.e., the fraction of the intercalary month) (reduced to days) gives the lunar days elapsed since the beginning of Caitra up to the beginning of the current Jovian year.

Calculations with the civil or lunar *śuddhi* in the case of the Jovian year are as before.

Let J be the number of elapsed Jovian years. Then the lunar years corresponding to these Jovian years are :

$$= \frac{53433336\ J}{364220 \times 12 \times 12} = \frac{2226389\ J}{2185320}$$

$$= J + \frac{41069\ J}{2185320} = L, \text{ say;}$$

and the intercalary years corresponding to L lunar years are :

$$= \frac{1593336\ L}{53433336}, \text{ in solar reckoning}$$

$$= \frac{1593336\ L}{4320000 \times 12}, \text{ in lunar reckoning}$$

$$= \frac{66389\ L}{2160000}, \text{ in lunar reckoning.}$$

Hence the above rule.

It must be remembered that the *śuddhi* means the fraction of the intercalary month.

LORD OF JOVIAN YEAR

83. Multiply the (elapsed) Jovian years by 971 and divide by 36422; and add the resulting quotient to 4 times the number of (elapsed) Jovian years : this gives the lord of the (current) Jovian year.

This is so, because the number of civil days in J Jovian years is equal to

$$\frac{1577917560\ J}{364220 \times 12}$$

$$= 361\ J + \frac{971\ J}{36422}$$

$$\equiv 4J + \frac{971\ J}{36422} \pmod 7.$$

AVAMAGHAṬĪS FOR THE BEGINNING OF CURRENT JOVIAN YEAR

84-85(a-b). Multiply the (elapsed) Jovian years by 26911 and divide by 36422. Multiply the remainder (of the division) by 60 and divide by its own divisor (i.e., by 36422) : the quotient gives the *ghaṭīs* of the *Avamaśeṣa* (or *Avama-ghaṭīs*) for the end of the (elapsed) Jovian year.

The omitted days corresponding to J Jovian years

$$= \frac{25082520\ J}{364220 \times 12}$$

$$= 5\,J + \frac{26911\ J}{36422}.$$

Hence the rule.

SHORTER *AHARGAṆA*

First Method

85(c-d)-86. Diminish the (lunar) days elapsed since the beginning of the light half of Caitra by the *śuddhi* (calculated in terms of lunar days etc.). Set down the result in two places (one below the other).

In the lower place, multiply that by 11 and then divide by 703 : the result is in days, etc. Increase that by the *Avamaghaṭīs*, and subtract the sum from the result at the other place: thus is obtained the *Ahargaṇa* (reckoned since the beginning of the current Jovian year).

That is,

$$\text{Shorter } Ahargaṇa = (L-S) - \left\{ \frac{11\,(L-S)}{703} + A_g \right\},$$

where L = lunar days elapsed since the beginning of Caitra,

S = *śuddhi* for the beginning of the current Jovian year,

and A_g = *Avama-ghaṭīs* for the beginning of the current Jovian year.

Second Method

87-88. Or, diminish the (lunar) days (elapsed since the beginning of Caitra) minus the *śuddhi*, by the *hīnadina* (i.e., by the *Avama-ghaṭīs*). Set down the result in two places (one below the other). In the lower place, multiply that by 11 and divide by 703: the result is in days etc. Multiply the *Avama-ghaṭikās* for the end of the (elapsed) Jovian year by 11 and divide by 703, and subtract that from those *Avama-ghaṭikās*. The *ghaṭīs* (thus obtained) and the result (obtained above) in days etc., being subtracted from the result set down at the upper place, one gets the *Ahargaṇa* reckoned from the beginning of the (current) Jovian year.

That is,

$$\text{Shorter } Ahargaṇa = (L-S-A_g) - \left\{ \frac{11(L-S-A_g)}{703} + \left(1 - \frac{11}{703}\right) A_g \right\}$$

$$= \left(L-S-A_g \right) - \frac{11\,(L-S-A_g) + 692A_g}{703},$$

where L = lunar days elapsed since the beginning of Caitra,

S = *śuddhi* for the beginning of the current Jovian year,

and A_g = *Avamaghaṭīs* for the beginning of the current Jovian year.

These rules are similar to those already stated.

PLANETS FOR THE END OF JOVIAN YEAR

Sun

89. Multiply the (elapsed) Jovian years by 211 and divide by 18211 : the resulting years etc. subtracted from the (elapsed) Jovian years give the Sun's mean longitude for the end of the (elapsed) Jovian year (in terms of revolutions etc.).

Number of Jovian years in a *yuga* =(Jupiter's revolution-number) × 12

$$= 362220 \times 12.$$

Therefore, Sun's mean longitude at the end of J Jovian years

$$= \frac{4320000 \times J}{364220 \times 12} \text{revs.} = \frac{18000}{18211} J \text{ revs.}$$

$$= \left(J - \frac{211}{18211} J\right) \text{revs.}$$

Moon

90. Multiply the (elapsed) Jovian years by 41069 and divide by 182110 : the result obtained added to the Sun's longitude gives the Moon's longitude.

Mean longitude of the Moon at the end of J Jovian years

$$= \frac{57753336\, J}{364220 \times 12} \text{ revs.}$$

$$= \left(12\, J + \frac{18000}{18211} J + \frac{41069}{182110} J\right) \text{ revs.}$$

$$= \left(\frac{18000}{18211} J + \frac{41069}{182110} J\right) \text{revs.,}$$

neglecting 12 J complete revolutions

$$= \text{Sun's longitude} + \frac{41069}{182110} J \text{ revs.}$$

Mars

91. **Multiply the (elapsed) Jovian years by 34207 and divide by 1092660 : the resulting quantity added to half the Sun's longitude gives the mean longitude of Mars.**

Mean longitude of Mars at the end of J Jovian years

$$= \frac{2296828\ J}{364220 \times 12} \text{ revs.}$$

$$= \left(\frac{1}{2} \cdot \frac{18000}{18211} J + \frac{34207\ J}{1092660}\right) \text{revs.}$$

$$= \frac{1}{2} (\text{Sun's longitude}) + \frac{34207}{1092660} J \text{ revs.}$$

Śīghrocca of Mercury

92. **The (mean) longitude of the *Śīghrocca* of Mercury is obtained by multiplying the (elapsed) Jovian years by 28406 and dividing the product by 273165.**

Mean longitude of *Śīghrocca* of Mercury at the end of J Jovian years

$$= \frac{17937056\ J}{364220 \times 12} \text{ revs.}$$

$$= \left(4\,J + \frac{28406\,J}{273165}\right) \text{ revs.}$$

$$= \frac{28406\ J}{273165} \text{ revs., neglecting } 4\,J \text{ complete revolutions,}$$

Śīghrocca of Venus

93. **To one-half of the (elapsed) Jovian years add what is obtained on multiplying the (elapsed) Jovian years by 9717 and dividing the resulting product by 91055 : the result is the (mean) longitude of the *Śīghrocca* of Venus.**

Mean longitude of *Śīghrocca* of Venus at the end of J Jovian years

$$= \frac{7022376 \times J}{364220 \times 12} \text{ revs.}$$

$$= \left(\frac{3}{2} J + \frac{9717}{91055} J \right) \text{ revs.}$$

$$= \left(\frac{1}{2} J + \frac{9717}{91055} J \right) \text{ revs., neglecting } J \text{ complete revolutions.}$$

Saturn

94. Multiplying the (elapsed) Jovian years by 6107 and dividing the product obtained by 182110 is obtained the mean longitude of Saturn.

Mean longitude of Saturn at the end of J Jovian years

$$= \frac{146568\ J}{364220 \times 12} \text{ revs.}$$

$$= \frac{6107}{182110} J \text{ revs.}$$

Moon's apogee

95. To one-tenth of the Sun's longitude, add what is obtained by multiplying the (elapsed) Jovian years by 18737 and then dividing by 1456880: then is obtained the (mean) longitude of the Moon's apogee.

Mean longitude of the Moon's apogee at the end of J Jovian years

$$= \frac{488211\ J}{364220 \times 12} \text{ revs.}$$

$$= \left(\frac{1800\ J}{18211} + \frac{18737\ J}{1456880} \right) \text{ revs.}$$

$$= \frac{\text{Sun's longitude}}{10} + \frac{18737\ J}{1456880} \text{ revs.}$$

Moon's ascending node

96. To $\frac{1}{20}$ of the Sun's longitude add the result obtained on multiplying the (elapsed) Jovian years by 8117 and dividing that by 2185320: the result is the (mean) longitude of the Moon's ascending node.

This is how the longitudes of the planets are calculated (for the end of the elapsed Jovian year).

Mean longitude of the Moon's ascending node at the end of J Jovian years

$$= \frac{232234\ J}{364220 \times 12} \text{ revs.}$$

$$= \frac{108000 + 8117}{2185320}\ J \text{ revs.}$$

$$= \left(\frac{18000}{18211 \times 20}\ J + \frac{8117}{2185320}\ J \right) \text{ revs.}$$

$$= \frac{\text{Sun's longitude}}{20} + \frac{8117}{2185320}\ J \text{ revs.}$$

5. *Miscellaneous Topics*

MEAN DAILY MOTIONS OF THE PLANETS

97-105(a-b). (The mean daily motions of the planets are as follows) :

Moon: $\frac{18}{492}$ revolution ; or $\frac{18}{41}$ sign plus $\frac{27}{4802}$ degree ;

Moon's apogee : $\frac{1}{9}$ degree plus $\frac{1}{61}$ minute; or $\left(\frac{20}{3} + \frac{10}{609}\right)$ minutes; or $\frac{1}{269}$ sign minus $\frac{1}{120}$ minute ;

Moon's ascending node : $\frac{1}{19}$ degree plus $\frac{2}{94}$ minute ; or $\frac{1}{566}$ sign minus $\frac{13}{184}$ second ;

Mars : $\frac{11}{21}$ degree plus $\frac{1}{80}$ minute ; or $\frac{4}{229}$ sign;

Śīghrocca of Mercury : 4 degrees plus $\frac{72}{13}$ minutes ; or $\frac{3}{22}$ sign plus $\frac{6}{71}$ minute ;

Jupiter : $\left(5 - \frac{1}{70}\right)$ minutes; or $\frac{1}{361}$ sign minus $\frac{1}{45}$ second;

Śīghrocca of Venus : $\frac{8}{5}$ degrees plus $\frac{4}{31}$ minute; and

Saturn : $\frac{1}{897}$ sign minus $\frac{7}{353}$ second; or $\left(\frac{361}{180} + \frac{1}{1245}\right)$ minutes.

With the help of these, computing the motions for the desired *Ahargaṇa* (in terms of revolutions, signs, degrees, minutes and seconds) and adding the results (obtained from the various fractions) together, one may obtain (the mean longitude of a planet) for the desired day.

Mean daily motions of the planets of the type stated above are also given in *BrSpSi*, i. 45, 47-50; *MSi*, i. 43-47; ii. 12-16; *SiŚe*, ii. 42-50; *KPr*, i. 4-12; *SiŚi*, I, i (e). 15-21; *KKu*, i. 7-12; *GLā*, i. 10-14; *SiSā*, i. 105-113; *KKau*, i. 16-23; *SL*, i. 6-12.

PLANETS DERIVED FROM THE SUN (SECOND METHOD)

105(c-d). The mean longitudes of the planets may also be obtained by adding the results derived from the Sun's longitude in degrees.

106. The Moon's longitude is thus equal to

$$\left(\frac{75^\circ}{2113} + \frac{40^\circ}{3}\right) \times \text{Sun's longitude in degrees ;}$$

and the longitudes of the Moon's apogee and the Moon's ascending node are equal to

$$\frac{61'}{9} \times \text{Sun's longitude in degrees,}$$

$$\text{and } \frac{29'}{9} \times \text{Sun's longitude in degrees,}$$

respectively ;

107. Mars' longitude is equal to

$$\frac{5^\circ 19'}{10} \times \text{Sun's longitude in degrees;}$$

the longitude of the *Śīghrocca* of Mercury is equal to

$$\frac{33°13'}{8} \times \text{Sun's longitude in degrees;}$$

108. Jupiter's longitude is equal to

$$\frac{1°26'}{17} \times \text{Sun's longitude in degrees;}$$

the longitude of the *Śīghrocca* of Venus is equal to

$$\frac{24°23'}{15} \times \text{Sun's longitude in degrees; and}$$

109. Saturn's longitude is equal to

$$\frac{59'}{28} \times \text{Sun's longitude in degrees.}$$[1]

In a similar manner, by abrading the multiplier and the divisor, one may find out in many ways the longitude of the Sun from the longitude of any desired planet.

Since the Sun moves 1° in one solar day, therefore "Sun's longitude in degrees" means the number of solar days elapsed. According to Vaṭeśvara, the mean motions of the planets for one solar day are as follows :

Planet	Motion for 1 solar day
Moon	$\frac{40°}{3} + \frac{75°}{2113}$
Moon's apogee	$\frac{61'}{9}$
Moon's ascending node	$\frac{29'}{9}$
Mars	$\frac{5°19'}{10}$
Śīghrocca of Mercury	$\frac{33°13'}{8}$
Jupiter	$\frac{1°26'}{17}$
Śīghrocca of Venus	$\frac{24°23'}{15}$
Saturn	$\frac{59'}{28}$

1. Similar rules are stated in *MBh*, i. 31-38.

These figures agree with those given in vss. 48-51 above.

Note. In the rules stated below the intercalary months and the omitted days are supposed to have been obtained by using the following sets of reduced parameters :

$$\left.\begin{array}{r} \text{no. of solar days} = 976 \\ \text{no. of lunar days} = 1006 \\ \text{no. of intercalary days} = 30 \end{array}\right\}$$

$$\left.\begin{array}{r} \text{no. of lunar days} = 703 \\ \text{no. of civil days} = 692 \\ \text{no. of omitted days} = 11 \end{array}\right\}$$

DAYS ELAPSED SINCE THE FALL OF OMITTED DAY AND INTERCALARY MONTH

110-111. The *Avama-ghaṭīs* (for the beginning of the current solar year) when multiplied by the divisor prescribed for the *Avama* (i.e., by 703/11) and divided by 60 give the (corresponding) days (i.e., the days elapsed at the beginning of the current solar year since the fall of the omitted day). Those days when diminished by the *śuddhi*, calculated in terms of days, give the days corresponding to the *Avama* fraction for the beginning of Caitra (i.e., the days elapsed at the beginning of Caitra since the fall of the omitted day). When the subtraction of the subtrahend (i.e., *śuddhi* days) is not possible, then subtraction should be made from the minuend after adding *one* day to the *Avama*.

The *śuddhi* (in terms of days) multiplied by 976 and divided by 30 gives the *Adhimāsaśeṣaśeṣa* (i.e., the days elapsed at the beginning of the current solar year since the fall of the intercalary month).

That is,

(1) number of (lunar) days elapsed at the beginning of the current solar year since the fall or occurrence of the omitted day

$$= \frac{A_g}{60} \times \frac{703}{11},$$

where A_g = *Avama-ghaṭīs* for the beginning of the current solar year;

(2) number of (lunar) days elapsed at the beginning of Caitra since the fall or occurrence of the omitted day

$$= \frac{A_g}{60} \times \frac{703}{11} - \textit{śuddhi},$$

$$\text{or } \left(1+\frac{A_g}{60}\right) \times \frac{703}{11} - \textit{śuddhi},$$

where A_g = *Avama-ghaṭīs* for the beginning of the current solar year;

(3) number of (solar) days elapsed at the beginning of the current solar year since the fall of the intercalary month

$$= \frac{976 \times \textit{śuddhi} \text{ days}}{30}.$$

AHARGAṆA RECKONED FROM THE BEGINNING OF CAITRA

112-113. Set down the lunar days elapsed since the beginning of Caitra in two places. In one place, divide them by 976 plus the corresponding intercalary days (i.e., by 976+30=1006). Add the resulting intercalary months, reduced to days, to the quantity in the other place. Set down this result (again) in two places. In one place, calculate the (corresponding) omitted days in the manner stated before; and subtract them from the result in the other place. Thus is obtained the *Ahargaṇa* for the desired day. Here (the *Ahargaṇa* being reckoned from the beginning of Caitra) the lord of the current day should be determined by counting the days from the beginning of the light half of Caitra. The lord of the (lunar) year (i.e., the lord of the first day of Caitra) should be ascertained from the lord of the (solar) year in the manner stated before.

Let t denote the number of lunar days (*tithis*) elapsed since the beginning of Caitra.

Since one intercalary month corresponds to 1006 lunar days, therefore the number of intercalary months corresponding to t lunar days

$$= \frac{t}{1006}.$$

Now treating t as the number of solar days elapsed since the beginning of Caitra, the number of lunar days elapsed since the beginning of Caitra

$$= t + \frac{30t}{1006}, \text{ taking the integral part only.}$$

Let a be the corresponding omitted days. Then the number of civil days elapsed at sunrise on the current lunar day since the beginning of Caitra

$$= t + \frac{30t}{1006} - a.$$

The rule is approximate and the *Ahargaṇa* calculated from it may be in excess or defect by 1.

SUN AND MOON WITHOUT USING *AHARGAṆA*

114-115. Divide the own *Avamaśeṣa* (of the current day) by its divisor minus multiplier (i.e., by 703−11=692): the result is the *Avamaśeṣa*, in (lunar) days etc. (This is to be treated as the first result.) Add it to the *Adhiśeṣa*. Multiply the sum by 30 and divide by 1006 : the result is (the total *Adhiśeṣa*) in days etc. (This is to be treated as the second result.)

Add the first result to the months and days (elapsed since the beginning of Caitra) and set down the resulting sum in two places. In one place, multiply by 1 and in the second place by 13; and from each of them subtract the second result. (Treat the months and days, etc., as signs and degrees, etc.) Then are obtained, in signs etc., the mean longitudes of the Sun and the Moon, respectively. Or, they may be obtained in various other ways in the manner described before.

We have :

$$\textit{Avama} \text{ fraction} = \frac{\textit{Avamaśeṣa}}{703} \text{ civil days}$$

$$= \frac{\textit{Avamaśeṣa}}{703-11} \text{ lunar days.} \qquad (1)$$

Intercalary days corresponding to (1)

$$= \frac{\textit{Avamaśeṣa}}{703-11} \times \frac{30}{1006} \text{ days}$$

$$\therefore \text{ total intercalary days } = \frac{30 \times \textit{Adhiśeṣa}}{1006} + \frac{\textit{Avamaśeṣa}}{703-11} \times \frac{30}{1006}$$

$$= \left(\textit{Adhiśeṣa} + \frac{\textit{Avamaśeṣa}}{703-11}\right) \times \frac{30}{1006}. \qquad (2)$$

Suppose that m lunar months and d lunar days have elapsed since the beginning of Caitra. Then lunar months and lunar days, etc., elapsed at sunrise on the current day since the beginning of Caitra are

m months + d days + (1).

Likewise, solar months, solar days, etc., elapsed at sunrise on the current day since the beginning of the current solar year are

m months + d days + (1) − (2).

Hence at sunrise on the current day,

Sun's longitude = m signs + d degrees + days etc. corresponding to (1) treated as degrees etc. − days etc. corresponding to (2) treated as degrees etc.

and

Moon's longitude = 13 [m signs + d degrees + days etc. corresponding to (1) treated as degrees etc.] − days etc. corresponding to (2) treated as degrees etc.,

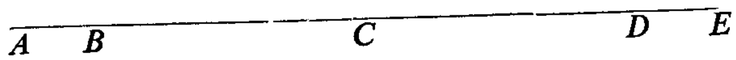
because

Moon's longitude— Sun's longitude

= 12 [m signs + d degrees + degrees etc. corresponding to (1)].

PLANETS FROM *AHARGAṆA* SINCE CAITRĀDI

116. The longitude of the planet for the end of the *śuddhi*, when diminished by the product of the *śuddhi* (in terms of days) and the planet's daily motion, gives the (planet's) longitude for the beginning of the *śuddhi*. That increased by the (planet's) motion corresponding to the *Avamaghaṭīs* (for the beginning of Caitra) and also by the (planet's) motion corresponding to the *Ahargaṇa* (reckoned from the beginning of Caitra) gives the mean longitude (of the planet) for (sunrise on) the desired day.



Let A denote the beginning of Caitra, B the point of sunrise on the first day of Caitra, C the beginning of the current solar year, D the beginning of the current lunar day, and E the point of sunrise on that day.

Evidently,

Longitude at E = longitude at A + motion corresponding to AB
+ motion corresponding to BE
= (longitude at C — motion corresponding to AC) + motion corresponding to AB + motion corresponding to BE
= (longitude at the end of *śuddhi* — motion corresponding to *śuddhi*) + motion corresponding to *Avamaghaṭīs* + motion corresponding to the *Ahargaṇa*.

LORDS OF DEGREES OF ZODIACAL SIGNS

117-120. The following glorious immortals are the lords of the (thirty) degrees of every sign in their respective order :

(1) Brahmā, (2) Prajāpati, (3) Dyauḥ (Heaven), (4) Śastra (Weapon), (5) Taru (Tree), (6) Anna (Food), (7) Vāsa (Residence), (8) Kāla, (9) Agni (Fire), (10) Kha (Sky), (11) Ravi (Sun), (12) Śaśi (Moon), (13) Indra (God of rain), (14) Go (Cow), (15) Niyati (Destiny), (16) Savitṛ, (17) Guha, (18) Aja (Unborn), (19) Pitṛ (Manes), (20) Varuṇa, (21) Hali or Balarāma, (22) Vāyu (Wind), (23) Yama, (24) Vāk (Speech), (25) Śrī (Lakṣmī or Wealth), (26) Dhanada (Kubera), (27) Niraya (Hell), (28) Bhūmi (Earth), (29) Veda, and (30) Parapuruṣa (the Supreme Spirit).

These (gods) are stated by the learned to rule the degree occupied by the Sun on the basis of the Sun's mean motion, and are to be worshipped with devotion on the days ruled over by them by the devotees of the Sun for the sake of prosperity. The days assigned to Yama, Kāla, Niyati, Fire, and Weapon are inauspicious.

The above-mentioned 30 names of the lords of the 30 degrees of the zodiacal signs are given after the names of the 30 gods presiding over the 30 Parsi days of the month, which are also called by the same names. These names are found to occur for the first time in the *Pañca-siddhāntikā* of Varāhamihira. The following table gives the names of the 30 lords of the degrees of the zodiacal signs as given by Varāhamihira and Vaṭeśvara, along with their original Parsi names on which they are based :

Table 10. Lords of the 30 degrees of the signs

Degree	Parsi name	Name according to Varāhamihira	Name according to Vaṭeśvara
1	Ahurmazd (Lord God)	Kamalodbhava (Brahmā)	Brahmā
2	Bahman (Protector of creatures)	Prajeśa (Prajāpati)	Prajāpati
3	Ardibahesht (Holder of keys of Heaven)	Svarga (Heaven)	Dyauḥ (Heaven)
4	Shahrivar (Lord of pure metal)	Śastra (Weapon)	Śastra (Weapon)
5	Spandarmad (Charitable)	Druma (Tree)	Taru (Tree)
6	Khurdad (Lord of festivals)	Anna (Food)	Anna (Food)
7	Amardad	Vāsa (Residence)	Vāsa (Residence
8	Depadar	Kāla	Kāla
9	Adar (Fire)	Anala (Fire)	Agni (Fire)
10	Avan (Water)	Abhra (Cloud)	Kha (Sky)
11	Khurshed (Sun)	Ravi (Sun)	Ravi (Sun),
12	Mah (Moon)	Śaśi (Moon)	Śaśi (Moon)
13	Tir (Distributor of water)	Indra (God of rain)	Indra (God of rain)
14	Gosh	Go (Cow)	Go (Cow)

Degree	Parsi name	Name according to Varāhamihira	Name according to Vaṭeśvara
15	Depmeher	Niyati (Destiny)	Niyati (Destiny)
16	Meher (Light)	Hara (=Meher)	Savitṛ
17	Sarosh (Protector of living and dead)	Bhava (Śiva)	Guha
18	Rashna	Guha	Aja (Unborn)
19	Farvardin (Spirits of the Dead)	Pitṛ (Manes)	Pitṛ (Manes)
20	Behram (or Varenes)	Varuṇa	Varuṇa
21	Ram	Baladeva (Balarāma)	Hali (Balarāma)
22	Govad (Wind)	Samīraṇa (Wind)	Vāyu (Wind)
23	Depdin	Yama	Yama
24	Din	Vāk (Speech)	Vāk (Speech)
25	Ashisvang (Right-eous)	Śrī (Lakṣmī or wealth)	Śrī (Lakṣmī or Wealth)
26	Ashtad	Dhanada (Kubera)	Dhanada (Kubera)
27	Asman (Sky)	Niraya (Hell)	Niraya (Hell)
28	Zamyad (Earth)	Dhātrī (Earth)	Bhūmi (Earth)
29	Marespand (Zarath-ustrian law and religion)	Veda	Veda
30	Aneran (Endless lights of shining Heaven)	Paraḥ Puruṣaḥ (the Supreme Spirit)	Parapuruṣa (the Supreme Spirit)

For details, see K. S. Shukla, "The *Pañca-siddhāntikā* of Varāhamihira (2)", *Gaṇita*, vol. 28, 1977, pp. 106-116.

Section 6 : Methods of a *Karaṇa* work

In *Karaṇa* works, calculations are generally made by means of specially devised multipliers and divisors. Our author, in the present chapter, defines six such parameters which have been designated by him as (1) multiplier, (2) divisor, (3) additive, (4) subtractive, (5) *dhanagaṇa*, and (6) *kṣayagaṇa*, and makes use of them in finding the longitudes of the planets. In the end he gives general instruction for the benefit of *Karaṇa* writers.

DAYS CORRESPONDING TO RESIDUAL FRACTIONS OF INTERCALARY MONTHS, OMITTED DAYS AND PLANET'S REVOLUTIONS

1. Divide the residue of the intercalary months (*adhimāsa-śeṣa*), the residue of the omitted days (*avamaśeṣa*), and the residue of the planet's revolutions each corresponding to the beginning of Caitra, respectively by the number of intercalary months in a *yuga*, the number of omitted days in a *yuga*, and the number of planet's revolutions in a *yuga*, stated heretofore; the results obtained are the solar days corresponding to the fraction of the intercalary month, the lunar days corresponding to the fraction of the omitted day, and the civil days corresponding to the fraction of the planet's revolution, (respectively).

The following is the rationale of the above rule :

Since

(1) fraction of the intercalary month $= \dfrac{\text{residue of the intercalary months}}{\text{solar days in a } yuga}$,

(2) fraction of the omitted day $= \dfrac{\text{residue of the omitted days}}{\text{lunar days in a } yuga}$,

(3) fraction of the planet's revolution $= \dfrac{\text{residue of the planet's revolutions}}{\text{civil days in a } yuga}$,

therefore, we immediately get

(4) solar days corresponding to the fraction of the intercalary month

$$= \frac{\text{residue of the intercalary months}}{\text{intercalary months in a } yuga},$$

(5) lunar days corresponding to the fraction of the omitted day

$$= \frac{\text{residue of the omitted days}}{\text{omitted days in a } yuga},$$

(6) civil days corresponding to the fraction of the planet's revolution

$$= \frac{\text{residue of the planet's revolutions}}{\text{planet's revolutions in a } yuga}.$$

The solar, lunar, and civil days obtained in the above stanza have been referred to as "additive" (*kṣepa*) in stanza 3 below. The significance of the term additive will be clear subsequently. We shall refer to the above additives as a_1, a_2 and a_3 respectively.

MULTIPLIERS

2. The solar days (in a *yuga*), the lunar days (in a *yuga*), and the civil days (in a *yuga*) divided by the intercalary months (in a *yuga*), the omitted days (in a *yuga*), and the planet's revolutions (in a *yuga*), (respectively), yield the (corresponding) divisors (i. e., divisor for the intercalary months, divisor for the omitted days, and divisor for the planet's revolutions respectively). When the remainder of the division is large, one should imagine an optional multiplier by one's own intellect and then one should perform the above division after multiplying (the dividend) by that (optional) multiplier.

One can easily see that the above-mentioned divisors denote the solar days corresponding to one intercalary month, the lunar days corresponding to one omitted day, and the civil days corresponding to one revolution of the planet, respectively.

Let d_1 denote the divisor corresponding to the intercalary months, d_2 the divisor corresponding to the omitted days, and d_3 the divisor corresponding to the planet's revolutions. Then

$$d_1 = \frac{\text{solar days in a } yuga}{\text{intercalary months in a } yuga}$$

$$d_2 = \frac{\text{lunar days in a } yuga}{\text{omitted days in a } yuga}$$

$$d_3 = \frac{\text{civil days in a } yuga}{\text{planet's revolutions in a } yuga}.$$

Since the numerator divided by the denominator will in each case leave a significant remainder, it would be desirable to find out the multipliers M_1, M_2 and M_3 in such a way that

$$M_1 \times \text{solar days in a } yuga$$

divided by intercalary months in a *yuga*,

$$M_2 \times \text{lunar days in a } yuga$$

divided by omitted days in a *yuga*, and

$$M_3 \times \text{civil days in a } yuga$$

divided by planet's revolutions in a *yuga* may leave small remainders, preferably 1, in each case.

Then, the fraction of the intercalary month, i.e.,

$$\frac{\text{residue of the intercalary months}}{\text{solar days in a } yuga}$$

$$= \frac{\dfrac{M_1 \times \text{residue of the intercalary months}}{\text{intercalary months in a } yuga}}{\dfrac{M_1 \times \text{solar days in a } yuga}{\text{intercalary months in a } yuga}} = \frac{M_1 a_1}{M_1 d_1},$$

the fraction of the omitted day, i. e.,

$$\frac{\text{residue of the omitted days}}{\text{lunar days in a } yuga}$$

$$= \frac{\dfrac{M_2 \times \text{residue of the omitted days}}{\text{omitted days in a } yuga}}{\dfrac{M_2 \times \text{lunar days in a } yuga}{\text{omitted days in a } yuga}} = \frac{M_2 a_2}{M_2 d_2}$$

and the fraction of the planet's revolution, i.e.,

$$\frac{\text{residue of the planet's revolutions}}{\text{civil days in a } yuga}$$

$$= \frac{\dfrac{M_3 \times \text{residue of the planet's revolutions}}{\text{planet's revolutions in a } yuga}}{\dfrac{M_3 \times \text{civil days in a } yuga}{\text{planet's revolutions in a } yuga}} = \frac{M_3 a_3}{M_3 d_3}.$$

See the next stanza.

DIVISORS AND ADDITIVES

3. The divisor multiplied by the (corresponding) multiplier should be taken as the divisor, and the additive multiplied by the (corresponding) multiplier should be taken as the additive. In case the additive is greater than one-half of the divisor, the additive should be subtracted from the divisor.

What is meant is that d_1M_1, d_2M_2 and d_3M_3 should be taken as the divisors D_1, D_2 and D_3 corresponding to the intercalary months, omitted days and planet's revolutions respectively; and a_1M_1, a_2M_2, and a_3M_3 should be taken as the additives A_1, A_2 and A_3 corresponding to the intercalary months, omitted days, and planet's revolutions respectively.

That is,

$$d_1 M_1 = D_1 \qquad a_1 M_1 = A_1$$
$$d_2 M_2 = D_2 \qquad a_2 M_2 = A_2$$
$$d_3 M_3 = D_3 \qquad a_3 M_3 = A_3$$

so that,

$$\frac{\text{residue of the intercalary months}}{\text{solar days in a } yuga} = \frac{A_1}{D_1}$$

$$\frac{\text{residue of the omitted days}}{\text{lunar days in a } yuga} = \frac{A_2}{D_2}$$

$$\frac{\text{residue of the planet's revolutions}}{\text{civil days in a } yuga} = \frac{A_3}{D_3};$$

and

$$\frac{\text{intercalary months in a } yuga}{\text{solar days in a } yuga} = \frac{M_1}{D_1}$$

$$\frac{\text{omitted days in a } yuga}{\text{lunar days in a } yuga} = \frac{M_2}{D_2}$$

$$\frac{\text{planet's revolutions in a } yuga}{\text{civil days in a } yuga} = \frac{M_3}{D_3}.$$

These results will be used later.

If $A_1 > \frac{1}{2}D_1$, $A_2 > \frac{1}{2}D_2$, and $A_3 > \frac{1}{2}D_3$, then we should take

$$\frac{\text{negative residue of the intercalary months}}{\text{solar days in a } yuga} = \frac{D_1 - A_1}{D_1}$$

$$\frac{\text{negative residue of the omitted days}}{\text{lunar days in a } yuga} = \frac{D_2 - A_2}{D_2}$$

$$\frac{\text{negative residue of the planet's revolutions}}{\text{civil days in a } yuga} = \frac{D_3 - A_3}{D_3}.$$

$D_1 - A_1$, $D_2 - A_2$, and $D_3 - A_3$, being negative residues, are called subtractives. See below.

ADDITIVE, SUBTRACTIVE, *DHANAGAṆA*, AND *KṢAYAGAṆA* FOR PLANET'S REVOLUTIONS

4. The same (additive), and, if the divisor has been diminished by that, the remainder (obtained) are called the additive (in the former case) and the subtractive (in the latter). The civil days (elapsed since the beginning of Caitra) divided by that (subtractive or additive) and multiplied by the corresponding divisor yield the *kṣayagaṇa* or the *dhanagaṇa* (in the case of the planet's revolutions).

That is, the A's are the additives, $(D-A)$'s are the subtractives and, if C denote the civil days elapsed since the beginning of Caitra, C multiplied by (D/A)'s are the *dhanagaṇas* and C multiplied by $\left(\frac{D}{D-A}\right)$'s are the *kṣayagaṇas*.

Thus in the case of the planet's revolutions,

$$dhanagaṇa = \frac{C \times D_3}{A_3} = \frac{C \times (\text{civil days in a } yuga)}{\text{residue of the planet's revolutions}}$$

$$\text{and } kṣayagaṇa = \frac{C \times D_3}{D_3 - A_3} = \frac{C \times (\text{civil days in a } yuga)}{(\text{civil days in a } yuga - \text{residue of the planet's revolutions})}$$

ADDITIVES AND SUBTRACTIVES FOR INTERCALARY MONTHS AND OMITTED DAYS

5. Similarly, the solar days corresponding to the fraction of the intercalary month and the lunar days corresponding to the fraction of the omitted day, multiplied by the (corresponding) multipliers, (i.e., M_1a_1 and M_2a_2, or A_1 and A_2), are the additives, and the same subtracted from the (corresponding) divisors (i.e., $D_1 - A_1$ and $D_2 - A_2$) are the so called subtractives (in the case of intercalary months and omitted days.)

CALCULATION OF A PLANET'S LONGITUDE

6. Set down the civil days elapsed since the beginning of Caitra in two places. In one place, multiply them by the multiplier and divide by the divisor. In the other place, divide by the *kṣayagaṇa* or the *dhanagaṇa*. Subtract the second result from or add that to the first result (in the respective order). Thus are obtained the mean longitudes of the planets.

The rationale of this rule is as follows :

Let C denote the number of civil days elapsed since the beginning of Caitra. Then

$$\text{planet's mean longitude} = \frac{C\times \text{planet's revolution-number}}{\text{civil days in a } yuga} \text{ revs.}$$

$$\pm \frac{\text{residue of planet's revolutions at Caitrādi}}{\text{civil days in a } yuga}$$

$+$ or $-$ being taken according as the residue is positive or negative

$$= \frac{C\times M_3}{D_3} + \frac{A_3}{D_3}, \text{ or } \frac{C\times M_3}{D_3} - \frac{D_3-A_3}{D_3}$$

$$= \frac{C\times M_3}{D_3} + \frac{C}{C\times D_3/A_3}$$

$$\text{or } \frac{C\times M_3}{D_3} - \frac{C}{C\times D_3/(D_3-A_3)}$$

$$= \frac{C\times M_3}{D_3} + \frac{C}{dhanagaṇa}$$

$$\text{or } \frac{C\times M_3}{D_3} - \frac{C}{kṣayagaṇa} \text{ revs.}$$

CALCULATION OF *AHARGAṆA* SINCE CAITRĀDI

7. (The solar days elapsed since the beginning of the current solar year multiplied by the multiplier M_1 for the intercalary months) increased by the (corresponding) additive (A_1) or diminished by the (corresponding) subtractive (D_1-A_1) and (the sum or difference thus obtained) divided by the (corresponding) divisor (D_1) gives the intercalary

months (elapsed since the beginning of Caitra). In the same way, proceeding with the lunar days (elapsed since the beginning of Caitra), one may obtain the omitted days (elapsed since the beginning of Caitra). These omitted days subtracted from the lunar days (elapsed since the beginning of Caitra) gives the *Ahargaṇa*.

The rationale of this rule is as follows:

Let S denote the solar days elapsed since the beginning of the current solar year. Then

intercalary months elapsed since the beginning of Caitra

$$= \frac{S \times \text{intercalary months in a } yuga}{\text{solar days in a } yuga}$$

$$\pm \frac{\text{residue of the intercalary months at Caitrādi}}{\text{solar days in a } yuga},$$

$+$ or $-$ being taken according as the residue is positive or negative

$$= \frac{S \times M_1 + A_1}{D_1} \text{ or } \frac{S \times M_1 - (D_1 - A_1)}{D_1}.$$

Again, let L denote the lunar days elapsed since the beginning of Caitra. Then

omitted days elapsed since the beginning of Caitra

$$= \frac{L \times \text{omitted days in a } yuga}{\text{lunar days in a } yuga}$$

$$\pm \frac{\text{residue of the omitted days at Caitrādi}}{\text{lunar days in a } yuga},$$

$+$ or $-$ being taken according as the residue is positive or negative

$$= \frac{L \times M_2 + A_2}{D_2} \text{ or } \frac{L \times M_2 - (D_2 - A_2)}{D_2}.$$

PLANET'S LONGITUDE BY ALTERNATIVE METHOD

8. The *Ahargaṇa* multiplied by the multiplier (for the planet's revolutions), then increased by the additive (for the planet's revolutions), or diminished by the subtractive (for the planet's revolutions), and then

divided by the divisor (for the planet's revolutions) gives the (mean) longitude of the planet in terms of revolutions etc. When the *Ahargaṇa* is negative, the longitude thus obtained should be increased by one revolution.

The following is the rationale of this rule :

$$\text{planet's longitude} = \frac{Ahargaṇa \times \text{planet's revolution-number}}{\text{civil days in a } yuga}$$

$$\pm \frac{\text{residue of planet's revolutions at Caitrādi}}{\text{civil days in a } yuga},$$

+or−being taken according as the residue is positive or negative

$$= \frac{Ahargaṇa \times M_3 + A_3}{D_3}$$

$$\text{or } \frac{Ahargaṇa \times M_3 - (D_3 - A_3)}{D_3} \text{ revs.,}$$

the *Ahargaṇa* being reckoned since Caitrādi, and one revolution being added to the result where necessary.

TRUE LONGITUDE

9. Thus, (where necessary), by adding one revolution or subtracting from one revolution, one may, in this way, obtain the mean longitude of a planet. So also are obtained the longitudes of the ascending nodes of the planets. Further, by calculating the corrections due to *manda* and *śīghra* anomalies, by the rule of three, and applying them to the mean longitude, in the manner prescribed, one may obtain the true longitude of a planet.

TECHNIQUE OF WRITING A *KARAṆA*

10. One should write a *Karaṇa* work in such a way that its rules may be well abridged, (hitherto) unknown to others, easy of application by the dull-witted, and such that the integral character of the intercalary months, omitted lunar days, and the revolutions of the planets in a *yuga* may be preserved.

Section 7

Mean planets by the orbital method

LINEAR MEASURES

1-3. Eight *aṇus* or minute particles (of dust) seen in a beam of sunlight (entering through an aperture) in the interior of a house, make one *kacāgra* (or bālāgra); eight of them make one *likṣā*; and eight of them are said to make one *yūkā*. Eight *yūkās* make one *yava*; eight of them make one *aṅgula* (finger-breadth or digit); twelve *aṅgulas* make one *vitasti*; two *vitastis* make one *kara* (hand or cubit); four *karas* make one *nṛ*; 1000 *nṛ* are said to make one *krośa*; 8 of them make one *yojana*; and 1,24,74,72,05,76,000 of them, say the learned, make (the circumference of) the circle of the sky.[1]

The above linear measures may be expressed in the tabular form as follows :

Table 11. Linear measures

8 *aṇus*	=	1 *kacāgra*
8 *kacāgras*	=	1 *likṣā*
8 *likṣās*	=	1 *yūkā*
8 *yūkās*	=	1 *yava*
8 *yavas*	=	1 *aṅgula* (digit)
12 *aṅgulas*	=	1 *vitasti*
2 *vitastis*	=	1 *kara* (*cubit*)
4 *karas*	=	1 *nṛ*
1000 *nṛ*	=	1 *krośa*
8 *krośas*	=	1 *yojana*
1,24,74,72,05,76,000 *yojanas*	=	1 circle of sky

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1. *Cf. SiŚe*, ii. 69-70 ; Śrīpati differs in some cases.

SKY LIGHTED BY THE SUN

4. As far as the lights of the sky do not dissolve themselves, so far is this sky lighted by the rays of the Sun.[1]

Bhāskara I (629 A. D.) says :

"One hears being said that the sky is of innumerable number of *yojanas*, how can then its orbit be of a finite number of *yojanas* in length ? The explanation is as follows : The sky, for us, extends as far as the Sun's rays illumine it; beyond that the sky is unlimited. By assigning a length to the orbit of the sky it is shown that it is this distance that is reached by the rays of the Sun."[2]

ORBIT OF THE SKY

5(a-c). The product of the revolution-numbers of the Sun and Moon in a *yuga*, divided by 20, is (the length of) the orbit of the sky (in terms of *yojanas*). Or, it is equal to 10 times the minutes in a circle multiplied by the revolution-number of the Moon.[3] **That (orbit of the sky) divided by the revolution-number of a planet gives the orbit of that planet.**[4]

One can easily see that (according to Vaṭeśvara)

(1) $\dfrac{\text{Sun's rev-no.} \times \text{Moon's rev.-no.}}{20}$,

(2) $10 \times$ (minutes in a circle) $\times$ (Moon's rev-no.), and

(3) (orbit of a planet) $\times$ (rev.-no. of that planet)

are each equal to 1,24,74,72,05,76,000 *yojanas*, which has been stated to be the length of the circumference of the sky. (See vs. 3 above)

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1. *Cf. ŚiDVr*, v. 2 (c-d) ; *SūSi*, xii. 90(d) ; *SiŚe*, ii. 59(c-d) ; *SiŚi*, I, i (d). 2.
2. See Bhāskara I's comm. on *Ā*, iii. 12.
3. *Cf. BrSpSi*, xxi. 11(a-b); *ŚiDVr*, v. 2; *SiŚe*, ii. 59 (a-b).
4. *Cf. BrSpSi*, xxi. 11(c-d); *ŚiDVr*, v. 3(b); *SiŚe*, ii. 61(a-b) ; *MSi*, i. 32(c-d); *SiŚi*, I, i (d). 4 (a-b).

ORBIT OF THE ASTERISMS

5(d)-6(a-b). The orbit of the Sun, multiplied by 60, is the orbit of the asterisms;[1] or the orbit of the sky, divided by 72,000, is the orbit of the asterisms. Thrice the revolution-number of the Moon is also declared to be the circle of the asterisms.

It can be easily seen that (according to Vaṭeśvara)

(1) (Sun's orbit) $\times$ 60,

(2) (Orbit of the sky)/72000, and

(3) 3 $\times$ (Moon's rev-no.)

are each equal to 17,32,60,008 *yojanas*, which has been stated to be the length of the orbit of the asterisms. (See vs. 12 below)

ORBITS OF SUN AND MOON

6(c-d). The solar years in a *yuga*, divided by 20, is the orbit of the Moon; and the revolution-number of the Moon, divided by 20, is the orbit of the Sun.

The first rule follows from the relation : (vide vs. 5(a-c))

$$\text{Orbit of the Moon} \times \text{Moon's rev-no.}$$
$$= \frac{\text{solar years in a } yuga \times \text{Moon's rev-no.}}{20},$$

and the second rule follows from the relation : (vide vs. 5(a-c))

$$\text{Orbit of the Sun} \times \text{Sun's rev-no.} = \frac{\text{Sun's rev-no.} \times \text{Moon's rev-no.}}{20}$$

ORBITS OF PLANETS IN *YOJANAS*

7-12. The length of Sun's orbit is $2887667 - \frac{1}{5}$ *yojanas*; of Moon's orbit, 21600×10 (*yojanas*); of Mars' orbit, $5431282 \frac{626}{574207}$ (*yojanas*); of the orbit of *Śīghroçca* of Mercury, $695472 \frac{11424}{560533}$

1. *Cf. Ā*, i. 6 (d); *MSi*, i. 32 (c-d).

(*yojanas*); of Jupiter's orbit, $34250509\frac{9401}{11211}$ *yojanas*; of the orbit of *Śīghrocca* of Venus, $1776424\frac{5112}{10837}$ (*yojanas*); of Saturn's orbit, $85112170\frac{1810}{6107}$ *yojanas*; and of the entire circle of the asterisms, 173260008 (*yojanas*).[1]

The orbits of the apogees and nodes of the planets have not been stated because they are the same as those of the corresponding planets. (See *BrSpSi*, xiv. 45). The apogees and nodes of a planet move on the orbit of the planet itself.

PLANETS' DAILY MOTION IN *YOJANAS*

13. The orbit of the sky, divided by the civil days in a *yuga*, gives the (linear) daily motion of the planets (in terms of *yojanas*). The product of that and the desired *Ahargaṇa* gives the *yojanas* traversed by the planets.[2] The product of the planet's (linear) daily motion (in terms of *yojanas*) in its own orbit and the *Ahargaṇa* is also equal to the same thing.

YOJANAS OF PLANET'S DAILY MOTION

14. $7905 + \frac{10685535}{(\text{civil days in a } yuga)/120}$ **is the (linear) daily motion of every planet in terms of *yojanas*.**[3]

This immediately follows from the formula :

$$\text{linear daily motion of a planet} = \frac{\text{orbit of the sky}}{\text{civil days in a } yuga}$$

stated in vs. 13.

1. For the lengths of orbits given by other astronomers, see *MSi*, i. 34 (c-d); *SiŚe*, ii. 63-65; *SiŚi*, I, i (d). 5.
2. *Cf. MSi*, i. 33 (c-d) ; *SiŚe*, ii. 66 ; *SiŚi*, I, i (d). 6 (a-b).
3. For the linear daily motion of the planets according to other astronomers, see *MSi*, i. 34(a-b) ; *SiŚe*, ii. 62 (c-d) ; *SiŚi*, I, i (d). 6 (c-d).

PLANETS' LONGITUDE BY ORBITAL METHOD

First Method

15. Multiply these (i.e., the *yojanas* traversed by the desired planet) by the revolution-number of the desired planet and divide (the resulting product) by the orbit of the sky in terms of *yojanas* : the result is the (mean) longitude of the same planet in terms of revolutions etc.[1]

That is,

$$\text{planet's mean longitude} = \frac{\text{planet's rev-no.} \times \textit{yojanas} \text{ traversed by the planet}}{\text{orbit of the sky}}$$

This is equivalent to the usual formula :

$$\text{planet's mean longitude} = \frac{\text{planet's rev-no.} \times \textit{Ahargaṇa}}{\text{civil days in a } \textit{yuga}},$$

because (*vide* vs. 13 above)

$$\frac{\textit{yojanas} \text{ traversed by the planet}}{\text{orbit of the sky}} = \frac{\textit{Ahargaṇa}}{\text{civil days in a } \textit{yuga}}.$$

Second Method

16(a-b). Or, divide the *yojanas* (traversed by the planet) by the planet's own orbit (in *yojanas*) : the result is the planet's (mean) longitude in revolutions etc.[2]

That is,

$$\text{planet's mean longitude} = \frac{\textit{yojanas} \text{ traversed by the planet}}{\text{orbit of the planet}}.$$

This is equivalent to the previous formula, because (*vide* vs. 5(a-c))

$$\text{orbit of the planet} = \frac{\text{orbit of the sky}}{\text{planet's rev-no.}}.$$

Third Method

16(c-d). Or, the planet's mean longitude in terms of revolutions etc. may be obtained by dividing the product of the circle of the sky and the *Ahargaṇa* by the planet's orbit multiplied by the civil days (in a *yuga*).

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1. *Cf. SiŚe*, ii. 67(c-d) ; *SiŚi*, I, i (d). 7-8(a-b).
2. *Cf. MSi*, i. 37(a-b); *SiŚe*, ii. 68(a-b).

That is,

$$\text{planet's mean longitude} = \frac{\text{circle of the sky} \times \textit{Ahargaṇa}}{\text{planet's orbit} \times \text{civil days in a } \textit{yuga}} .$$

This is equivalent to the usual formula, because

$$\text{planet's rev-no.} = \frac{\text{circle of the sky}}{\text{planet's orbit}}$$

TIMES TAKEN IN TRAVERSING THE CIRCLE OF ASTERISMS AND THE CIRCLE OF THE SKY

17. These planets, moving eastwards in their orbits with a uniform motion, in a period of 60 solar years, traverse as many *yojanas* as there are in the circle of the asterisms; while, during the years of a *yuga*, they traverse as many *yojanas* as there are in the circle of the sky.[1]

EQUALITY OF ORBITS OF MERCURY AND VENUS WITH THAT OF THE SUN EXPLAINED

18. As the orbit of Mercury or Venus multiplied by the revolution-number of the Sun gives the *yojanas* described (by the planets) during the years of a *yuga* (i.e., the *yojanas* of the circle of the sky), so the orbit of Mercury or Venus is equivalent to that of the Sun. This is also evident from their daily motion in minutes (which is the same as that of the Sun).[2]

In this connection Bhāskara II's statement is noteworthy He says :

"The orbits of Venus and Mercury have been contemplated as being equal to that of the Sun. But this is done simply to obtain their (mean) longitudes (by the orbital method). Actually they move in the stated orbits of their *Śīghroccas*."[3]

His statement regarding the orbits of the apogees (or apsides) and nodes of the planets is also noteworthy. He says :

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1. *Cf. SiŚi*, I, i (d). 4(c-d).
2. *Cf. SiŚi*, I, i (d). 9 (a-b).
3. *SiŚi*, I, i (d). 9,

"The orbits of the apogees (or apsides) and nodes of the planets have been contemplated as being different from those of the planets. But this is done simply to obtain their (mean) longitudes (by the orbital method). (Actually their orbits are the same as those of the planets)."[1]

IDENTITY OF THE *ŚĪGHROCCAS* OF MARS, JUPITER AND SATURN AND THE SUN EXPLAINED

19. Also, since the orbit of the body that moves in the orbit of the *Śīghroccas* of Mars, Jupiter and Saturn, when multiplied by the revolution-number of the Sun, gives the orbit of the sky, therefore it is the Sun that lies at their *Śīghroccas*.

The arguments given in the above two stanzas are based on the Hindu conception that, during the course of a *yuga*, all planets traverse a distance equal to the circumference of the circle of the sky.

ORDER OF PLANETS AND THEIR MOTION AT LAṄKĀ AND ELSEWHERE

20. The Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn, and the asterisms, situated away from the Earth in the order stated, revolve at right angles to the horizon at Laṅkā and inclined to the horizon at other places on the Earth (not lying on the equator).

Actually the order of the planets in the sequence of increasing distance from the Earth is :

Moon, Venus, Mercury, Sun, Mars, Jupiter and Saturn

but the Hindu astronomers have stated them in the following order :

Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn.

The order in which the planets have been arranged by the Hindus is based on the criterion that the greater is the periodic time of a planet the greater is its distance from the Earth, but they have missed to see that the periodic times of the *śīghroccas* of Mercury and Venus are heliocentric and not geocentric. This explains the discrepancy in the order of those planets in the list given by the Hindu astronomers.

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1. *SiŚi*, I, i (d). 8 (c-d).

Table 12. Periodic times of the planets according to Vaṭeśvara

Planet	Periodic time (in days)
Sun	365.258694444
Moon	27.32166952
Mars	686.998573685
Śīghrocca of Mercury	87.96970695
Jupiter	4332.3199165339
Śīghrocca of Venus	224.6985293
Saturn	10765.771246111

SUCCESSION OF LORDS OF HOURS, ETC.
(counted from Saturn in the order of increasing velocity)

21. The seven planets, Saturn etc., in the order of increasing velocity, are the lords of the successive hours; those occurring fourth in order are the lords of the successive days; those occurring seventh in order are the lords of the successive civil months; and those occurring third in order are the lords of the successive civil years.

SUCCESSION OF LORDS OF DAYS ETC.
(counted from Moon in the order of increasing distance)

22. (Of the same seven planets), beginning with Moon, in the order of increasing distance, those occurring fifth in order are the lords of the successive days; those occurring sixth in order are the lords of the successive civil years; those occurring in the successive order are the lords of the successive civil months; and those occurring seventh in order are the lords of the successive hours.

The planets in the order of increasing velocity are :

Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon

and the same in the order of increasing distance are :

Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn.

These planets appear as lords of the successive hours in the following order :

(1) Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon ;
as lords of the successive days in the following order :

(2) Saturn, Sun, Moon, Mars, Mercury, Jupiter, Venus ;
as lords of the successive civil months in the following order :

(3) Saturn, Moon, Mercury, Venus, Sun, Mars, Jupiter ;
and as lords of the successive civil years in the following order :

(4) Saturn, Mars, Venus, Moon, Jupiter, Sun, Mercury.

This explains the above rules.

DIFFERENCE IN REVOLUTION-PERIODS OF PLANETS EXPLAINED

23. In a smaller orbit, the (arcs denoting) signs, degrees and minutes are smaller in length, whereas in a larger orbit they are larger in length. That is why the Moon traverses its smaller orbit in shorter time, and Saturn its larger orbit in longer time.[1]

DIFFERENCE IN SIDEREAL AND CIVIL DAYS EXPLAINED

24. The circle of asterisms rises on the eastern horizon and sets on the western horizon at the rate of one minute of arc in one respiration. A planet, on the other hand, on account of its own (eastward) motion, has its diurnal motion equal to the minutes in a circle plus the minutes of its own (eastward) motion, and so it rises after so many respirations have passed.

A star rises after every 24 sidereal hours. The Sun rises after every 24 sidereal hours plus $\frac{59^m 8^s}{15}$ approx.

The prose order of the second half of the Sanskrit text is as follows :
ग्रहश्च, यतो जवत्वं, स्वभुक्तिलिप्तायुतचक्रलिप्ताभोगः, अतएव तेन समं कालेन उदयं व्रजति।

1. Same is stated in *Ā*, iii. 13-14; *BrSpSi*, xxi. 14; *SiŚi*, I, i (e). 27.

Section 8 : The longitude correction

PRIME MERIDIAN

1-2. Laṅkā, (then northwards), Kumārī, then Kāñcī, Mānāṭa, Aśvetapurī, then northwards, the Śveta mountain, thereafter Vātsyagulma, the city of Avantī, then Gargarāṭa, Āśramapattana, Mālavanagara, Paṭṭaśiva, Rohitaka, Sthāṇvīśvara, the Himalaya mountain, and lastly Meru—(these are situated on the prime meridian). For these places correction for longitude does not exist.

Laṅkā is the place where the Hindu prime meridian intersects the equator. Kumāri is Kanyākumārī. Kāñcī is the famous city of Kāñcīpuram in Madras State. Mānāṭa is called Pāṇāṭa by Bhāskara I (629 A.D.),[1] and Pānāṭa (or Pannāṭa) by Śrīpati (1039 A. D.). Aśvetapurī is called Misitapura by Bhāskara I,[1] and Asitapurī in a verse mentioned in Bina Chatterjee's edition of the *Khaṇḍa-khādyaka* [2] The Śveta mountain (or the white mountain) is the Śveta-śaila of Lalla, the Sitādri of Śrīpati, and the Sita-parvata of Bhāskara II. According to Śrīpati, it is the seat of the six-faced god Svāmikārtikeya. It can be identified with Krauñca-giri or Kumāra-parvata, situated at a distance of 3 *yojanas* from Śrīśaila. Vātsyagulma is modern Basim in Akola District, Mahārāṣṭra, about 70 km. south of Akola. The city of Avantī is modern Ujjain. Gargarāṭa has been mentioned as a place on the prime meridian by Śrīpati, Bhoja, Bhāskara II, and others, but its exact identification is uncertain. It might be Karkarāja, one of the 28 sacred places of Ujjain. Āśramapattana has been called a pleasant place by Śrīpati. It is probably Aṅkapāda or Sāndīpaniāśrama, one of the 12 important places of Ujjain. It is about 3 km. from Gopālamandira on way to Maṅgaleśvara. Mālavanagara is Mehidapuranagara or Malva on the river Sipra, north of Ujjain. Paṭṭaśiva, or principal Śiva, remains unidentified. Rohitaka is modern Rohtak in Haryana. Sthāṇvīśvara is a sacred place near Thanesar in Haryana. There is a sacred lake on the bank of which there is an ancient temple of Sthāṇu Śiva. Meru is the north pole.

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1. and 2. See *MBh*, ii. 1.
2. See Vol. II, p. 9, footnote.

EARTH'S DIAMETER AND CIRCUMFERENCE

3. The Earth's diameter is comprised of 1054 *yojanas*.[1] That multiplied by 3927 and divided by 1250 gives the (Earth's) circumference. The value (of the Earth's circumference), thus obtained, is more accurate than the one obtained on multiplying (the Earth's diameter) by $\sqrt{10}$.

That is,

Earth's diameter = 1054 *yojanas*.

and Earth's circumference = 1054π *yojanas*.

Rationale. According to the Hindu astronomers, horizontal parallax of a planet = 4 *ghaṭīs*, and also = Earth's semi-diameter in *yojanas*. So the linear motion of a planet for 4 *ghaṭīs* = Earth's semi-diameter in *yojanas*. Therefore,

$$\text{Earth's semi-diameter} = \frac{\text{planets' mean daily motion in } yojanas}{15}$$

$$= \frac{7905}{15} \; yojanas \text{ (vide } supra\text{, sec. 7, vs. 14)}$$

$$= 527 \; yojanas.$$

$$\therefore \text{ Earth's diameter} = 1054 \; yojanas.$$

The author now states an approximate rule for the longitude-correction and points out its defects.

ANCIENT RULE FOR LONGITUDE AND LONGITUDE-CORRECTION

Base of the longitude triangle

4. By the degrees of the difference between the latitude of a town on the vertically situated prime meridian and the latitude of the local

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1. The same value has been taken by Brahmagupta, Śrīpati and Bhāskara II. See *BrSpSi*, i. 37; *SiŚe*, ii, 94; *SiŚi*. I, i (g). 1(a-b). According to these astronomers Earth's diameter = 1581 *yojanas*

$$\text{or } \frac{1581\times 2}{3} = 1054 \text{ in terms of Vaṭeśvara's } yojanas.$$

It may be mentioned that Brahmagupta and Śrīpati derived the value of the Earth's circumference as 5000 *yojanas* by taking $\pi = \sqrt{10}$. Bhāskara II. however, took π = 3.1416 and derived the value of the Earth's circumference as 4967 *yojanas*.

town, multiply the circumference of the Earth and divide (the resulting product) by the degrees in a circle (i.e., by 360). Then is obtained the base (of the longitude triangle).

The base denotes the distance of the arbitrarily chosen town on the prime meridian from the local circle of latitude.

Upright and Hypotenuse

5. The number of *yojanas* between the town on the prime meridian and the local town, comprising the hypotenuse (of the longitude triangle), is known from the common talk of the people. The square root of the difference between the squares of that (hypotenuse) and the base is the upright (of the longitude triangle) known as *deśāntara* (i.e., longitude in *yojanas*).

Let A be the local town, C the town on the prime meridian, and B the place where the prime meridian intersects the local circle of latitude. Then BC denotes the base, AB the upright, and AC the hypotenuse of the longitude triangle ABC. The above rule is based on the assumption that ABC is a plane triangle right-angled at B. For a criticism of this rule, see vs. 7(a-b) below.

Longitude-correction

6. Whatever is obtained, in minutes etc., on dividing the product of *deśāntara* (i. e., longitude in *yojanas*) and a planet's daily motion (in minutes) by the Earth's circumference (in *yojanas*) should be subtracted from or added to the mean longitude of the planet, according as the local place is to the east or to the west of the prime meridian.[1]

That is,

longitude correction

$$= \frac{\text{longitude in } \textit{yojanas} \times \text{planet's daily motion in minutes}}{\text{Earth's circumference in } \textit{yojanas}} \text{ minutes}$$

This formula is erroneous in so far as "Earth's circumference in *yojanas*" has been used in place of "circumference, in *yojanas*, of the local circle of latitude".

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1. Rules similar to those stated in vss. 4-6 occur also in *MBh*, ii. 3-4; *LBh*, i, 25-26; *BrSpSi*, i. 36-37; *KK* (BC), i. 15; *SiDVr*, i. 44; *SiŚe*, ii. 99-100; *SiSā*, i. 143-144.

CRITICISM OF THE ABOVE RULE

7(a-b). The *yojanas* of the hypotenuse being inaccurate and the Earth's surface being spherical (lit. Earth's circumference being curved), this (rule) is incorrect and unacceptable.

CRITICISM OF OTHER VIEWS

7(c-d)-8. Some say that the *deśāntara* should be obtained after reducing the hypotenuse by one-fourth of itself, while others that the *yojanas* of the latitude-difference should be determined with the help of two shadows of the Sun (cast by a gnomon, one at the local place and the other at the place on the prime meridian). Both are incorrect, the former because of the upright having become too small, and the latter because of inappreciable difference between the two shadows.

In what follows, the author gives the correct rules.

LONGITUDE IN TIME

9. Having calculated the time of lunar eclipse for the local place (without applying longitude-correction to the Sun and Moon), find the difference of this time from the time of actual observation (of the eclipse at the same place) : this is the correct value of longitude (in time) (for the local place).[1]

LONGITUDE-CORRECTION

First Method

10. The *nāḍīs* of the longitude should be multiplied by the daily motion of the planet (in minutes) and then divided by 60: the resulting quotient, in minutes, should be subtracted from or added to (the mean longitude of the planet) as before. Then is obtained the mean longitude of the planet for the local place.[2] **The longitude (in terms of *ghaṭīs*) should also be applied similarly to the *nāḍīs* of the mean *tithi*.**

Second Method

11. The length (in *yojanas*) of the (corrected) circumference of the Earth divided by 60 and multiplied by the *nāḍīs* of the longitude, gives

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1. *Cf. SiŚi*, I, i(g). 4.
2. *Cf. SiŚe*, ii, 106; *SiŚi*, I, i(g). 5(c-d).

(the correct value of) the number of *yojanas* (of the distance of the local place from the prime meridian) measured along the circumference (of the local circle of latitude). The corresponding correction should be calculated and applied to (the mean longitude of) a planet, as before.

By "the corrected circumference of the Earth" is meant "the circumference of the local circle of latitude".

COMMENCEMENT OF DAY (*VĀRAPRAVṚTTI*)

12. In a place situated towards the east of the prime meridian, the day is said to commence when *ghaṭīs* amounting to longitude in *ghaṭīs* elapse after sunrise at that place; and in a place situated towards the west of the prime meridian, the day is said to commence so many *ghaṭīs* before the local sunrise.[1]

13. When the Sun is in the southern hemisphere, the day commences before sunrise by an amount equal to the Sun's ascensional difference; and when the Sun is in the northern hemisphere, the day commences after sunrise by an amount again equal to the Sun's ascensional difference.[2]

The day is supposed to begin at Laṅkā. The object of the two rules stated above is to find the time of sunrise at Laṅkā in relation to the time of sunrise at the local place If *A* be the local place and *B* the place where the local circle of latitude intersects the prime meridian, then vs. 12 is meant to give the time of sunrise at *B* in relation to the time of sunrise at *A* and vs. 13 to give the time of sunrise at Laṅkā in relation to the time of sunrise at *B*.

DEŚĀNTARA, *BHUJĀNTARA* AND *CARA* CORRECTIONS

14. From the revolutions (in a *yuga*) divided by the civil days (in a *yuga*) is obtained the daily motion in minutes etc. With the help of this, one should calculate the *bhujāntara* correction as also the corrections due to longitude and (Sun's) ascensional difference, as in the case of a planet.

The *bhujāntara* correction is the correction due to the Sun's equation of the centre.

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1. *Cf. BrSpSi*, i. 36, *SūSi*, i. 77; *SiŚi*, I, i(g). 6.
2. The contents of vss. 12 and 13 have been condensed in one vs. by Āryabhaṭa II. See *MSi*, ii. 41.

LORDS OF CIVIL YEAR AND CIVIL MONTH

15-16. Divide the *Ahargaṇa* elapsed since the birth of Brahmā, since the beginning of the current *kalpa*, or since the beginning of Kṛtayuga, by 360; multiply the quotient by 3 and then divide by 7: the remainder increased by 1 and counted (in the sequence of the lords of the weekdays) beginning with Saturn, Saturn, or Sun, respectively (depending on the epoch chosen) gives the lord of the current civil year.[1]

Divide the *Ahargaṇa* by 30, multiply the quotient by 2, add 1, and then divide by 7: the remainder (counted in the sequence of the lords of the week days beginning with Saturn etc. as before) gives the lord of the current civil month.[2]

Both the rules are obvious. In the former case, the quotient of the division is multiplied by 3 because $360 \equiv 3 \pmod 7$; and in the latter case, the quotient of the division is multiplied by 2 because $30 \equiv 2 \pmod 7$.

LORDS OF HOUR ETC.

17. Multiply the *ghaṭīs* elapsed since the commencement of the current day (see vss. 12-13) by 2 and divide by 5 : (the quotient denotes the number of hours elapsed). Add 1 to the quotient and divide the sum by 7: the remainder counted with the lord of the current day succeeded by the sixth ones (in the order of the lords of weekdays) gives the lord of the current hour.

Or, multiply the quotient (denoting the number of the hours elapsed) by 5 and add 1 to that. (If the sum obtained is greater than 7, divide by 7, and take the remainder). The sum (or the remainder) counted with the lord of the current day (in the order of the lords of the week-days) gives the lord of the current hour.

The lords of the days occurring third (in the sequence of the lords of weekdays) are the lords of the successive civil months; those occurring fourth in order are the lords of the successive civil years; and those occurring second (i.e., occurring next) in order are the lords of the successive days.[3]

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1. *Cf. BrSpSi*, xiii. 44; *SiŚe*, ii. 87; *SiDa*, part I, ch. 1, sec. 3, vs. 16.
2. *Cf. BrSpSi*, xiii. 43; *SiŚe*, ii. 88; *SiDa*, part I, ch. 1, sec. 3, vss. 14 (c-d)-15 (a-b).
3. *Cf. SiŚe*, ii. 90.

The lords of the successive hours occur in the following order : (vide supra, sec. 7, vs. 22, notes)

Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon

but these planets successively occur at the sixth place in the following order of the lords of the weekdays :

Saturn, Sun, Moon, Mars, Mercury, Jupiter, Venus.

Hence the rule for the lords of hours.

In the alternative rule for the lord of the current hour, the quotient has been multiplied by 5 because after every 5 lords of the weekdays occur the lords of the successive hours.

The rule for the lords of the civil months, civil years and civil days follows from the fact that

$$30 \equiv 2 \pmod 7$$

$$360 \equiv 3 \pmod 7$$

and $$7 \equiv 0 \pmod 7.$$

LORD OF HOUR : ANOTHER RULE

18. Multiply the (sidereal) hours intervening between the rising point of the ecliptic and the Sun, by 5 and divide by 7 : the remainder increased by 1 and counted from the lord of the current day in the order of the lords of the weekdays gives the lord of the current hour, because the lord of the hour occurs at the sixth place in the sequence of the lords of weekdays.[1]

The sidereal hours intervening between the rising point of the ecliptic and the Sun (i.e., the civil hours elapsed since sunrise) are obtained by the following rule :

Subtract the Sun's longitude at the moment from the longitude of the rising point of the ecliptic, reduce the difference to signs and multiply them by 2.

The remaining part of the rule agrees with the alternative rule given in vs. 17 above.

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1. *Cf. BrSpSi*, xiii. 45; *SiŚe*, ii, 89.

Section 9 : Examples on Chapter I

1. Since those who are ill-versed in astronomy become mute and feel ashamed when faced with (an volley of) questions, therefore I now proceed to give out a chapter on astronomical problems according to my intellect.

2. One who finds the *Ahargaṇa* from the solar days (elapsed), without making use of the intercalary months and the omitted days, and the mean longitude of a planet from that (*Ahargaṇa*), is an astronomer.

3. One who finds the *Ahargaṇa* from the solar days (elapsed), without making use of the intercalary months, the lunar months, the omitted days, and the civil days, knows the mean motion clearly.

4. One who finds the lunar days from the civil days, therefrom the solar days, and therefrom the sidereal days, and also finds the omitted days from the intercalary days and the intercalary days from the omitted days, is an astronomer.

5. One who finds the longitudes of the Sun and Moon without making use of the *Ahargaṇa*, and therefrom obtains the longitude of a different desired planet, and also derives the Sun's longitude from the Moon's longitude and *vice versa*, in a variety of ways, is an astronomer.

6. One who finds the longitudes of the planets for the time of rising of the asterism Aśvinī or for the time of rising of the desired planet, is the foremost amongst the astronomers.[1]

7. One who finds briefly (the lord of) the current day from the seventh day of the succession of weekdays by the inverse process, and calculates the retrograde (or westward) longitudes of the planets, knows the mean motion clearly.

8. One who finds the longitudes of the faster and slower planets from each other in a variety of ways, and also derives the Sun's longitude from the planet's longitude and *vice versa*, is an astronomer.

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1. Same as *BrSpSi*, xiii. 7.

9. One who, having obtained the risings of a planet (in a *yuga*), finds the daily motion of a planet from those risings of the planet, and from them (i.e., from those risings of the planet) derives the revolutions of the asterisms (in a *yuga*), in a variety of ways, is an astronomer.

10. One who finds the longitude of the desired planet from the *Ahargaṇa* multiplied by the revolution-number of another planet, or multiplied by the multiplier given in the problem, is proficient in the reduction of fractions.

11. One who finds the longitude of the desired planet from the omitted days, the mean *tithi* from the risings of that planet, and the longitudes of the Sun and Moon in a variety of ways, knows the mean motion.

12. One who, by using the abraded multipliers and divisors, briefly obtains the *Ahargaṇa* since the commencement of the current *kalpa*, since the birth of Brahmā, since the beginning of Kṛtayuga, or since the beginning of Kaliyuga, is an astronomer.

13. One who finds the longitude of the desired planet from

$$\frac{2\times \text{Sun's long.}+3\times \text{Moon's long.}}{8} - \text{Mercury's long.}$$

knows the mean motion (of the planets) like an Emblic Myrobalan placed on (the palm of) the hand.

14. One who obtains the longitude of the desired planet from

$$9\times(\text{Mars' long.})+8\times(\text{Jupiter's long.})+9\times(\text{Saturn's long.})$$
$$+10\times(\text{Mercury's long.})+11\times(\text{Venus' long.})$$

is an astronomer.

15. The sum of the longitudes of the Sun, Moon, Mars and Mercury is severally diminished or increased by three times the individual longitudes of those planets. One who, from those sums (and differences), obtains the individual longitudes of those planets is an astronomer.

Given

$$S-3\times(\text{Sun's long.}),\ S-3\times(\text{Moon's long.}),\ S-3\times(\text{Mars' long.})$$
$$\text{and } S-3\times(\text{Mercury's long.}),$$

where S = Sun's long.+Moon's long.+Mars' long +Mercury's long ,

the problem is to find the longitudes of Sun, Moon, Mars and Mercury separately.

The word *guru* in the Sanskrit text means "value", "magnitude", "measure", and therefore "longitude", in the present context.

16. The sum of the longitudes of all the planets is severally increased or decreased by seven times the individual longitudes of those planets (and the resulting sums or differences are given separately). What are the individual longitudes of those planets ?

Let S_1, S_2, S_3, ..., S_7 be the longitudes of the seven planets and S their sum. Given

$$S \pm 7S_1,\ S \pm 7S_2, \ldots, S \pm 7S_7.$$

the problem is to find S_1, S_2, ..., S_7.

17. If

10(Moon's long.) + 33(Mercury's long.) ± Unknown planet's long. = Saturn's long.,

what are the revolutions of the unknown planet ?[1]

18. If

(i) 23(Mars' long.) − 3(Jupiter's long.) − Unknown planet's long. = Sun's long.,

or (ii) 23(Mars' long.) − 3(Jupiter's long.) + Unknown planet's long. = Saturn's long.,

what are the revolutions of the unknown planet ?[2]

19-21. One who finds, in a variety of ways, the *śuddhi* for the beginning of the year and therefrom the *Ahargaṇa*; from the *Ahargaṇa*, the Sun's longitude; from the *Avamaśeṣa*, the Moon's longitude; and from the *Ahargaṇa*, the longitudes of the planets as also the lord of the civil year and the lords of day and month; knows how to find the illustrious lords of the 30 degrees of the zodiacal signs, the lord of the hour, and the commencement of the day at his local place; and also knows how to find the longitudes of the planets from the lengths of orbits of each planet and how to calculate and apply the longitude correction, is the foremost amongst the astronomers on the Earth girdled by the oceans.

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1. Similar examples occur in *SiŚe*, ii. 74, 75; *SiŚi*, II, xiii. 7.
2. Similar examples occur in *BrSpSi*, xiii, 5, 6.

Section 10

Comments on the *Siddhānta* of Brahmagupta

INTRODUCTION

1. As Jiṣṇu's son, discarding the teachings of the divine *Śāstra*, has said something else on the authority of his own intellect, so I, basing my remarks on his *Siddhānta*, shall (now) point out some of his defects.

QUARTER *YUGAS*

2. As a planet does not make complete revolutions during the quarter *yugas* defined by Jiṣṇu's son (but it does during the quarter *yugas* defined by Āryabhaṭa), so the quarter *yugas* of Śrīmad Āryabhaṭa (and not those of Jiṣṇu's son) are the true ones.

YUGA, *MANU* AND *KALPA*

3. If his (i. e., Jiṣṇu's son's) *yuga* is the same as defined in the *Smṛtis*, how is it that the Moon (according to him) is not beyond the Sun (as stated in the *Smṛtis*) ? If that is unacceptable here because that statement of the *Smṛtis* is false, then, alas!, the *yuga*-hypothesis of the *Smṛtis*, too, is false.

This is in fact a rejoinder to Brahmagupta for the following comments made by him against the teachings of Āryabhaṭa :

"Of the four quarter *yugas*, Kṛtayuga etc., which have been defined to be of equal duration by Āryabhaṭa, not one is equivalent to that defined in the *Smṛtis*."[1]

"Since the (durations of) *manu*, *yuga* and *kalpa*, and the periods of time elapsed since the beginning of *kalpa* or Kṛtayuga are not equivalent to those stated in the *Smṛtis*, it means that Āryabhaṭa has no knowledge of mean motion."[2]

4. If the *kalpa* itself is called *yuga* by you, (O Jiṣṇu's son !), how is it that your *yuga* is not so large (as a *kalpa*)? If the *yuga* defined

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1. *BrSpSi*, i. 9,
2. *BrSPSi*, xi. 10.

by you could be determined by you alone, and the same could not be determined by the sages, it proves that the *yuga* determined by you is not real (but forged).

This comment is made on the following statement of Brahmagupta :

"Since the conjunction of the planets, their *mandoccas*, *śīghroccas*, and ascending nodes occurs in relation to time, asterisms and place, at the interval of a *kalpa*, so the *kalpa* is (really) the correct *yuga* of the planets."[1]

The comment is useless.

5. Since (the lengths of) *manu*, *kalpa*, and quarter *yugas*, stated by Jiṣṇu's son are in no way equivalent to those stated by Puliśa, Romaka, Sūrya and Pitāmaha, so they are not the true ones.

The *Siddhāntas* of Puliśa, Romaka, Sūrya and Pitāmaha are not available in their original forms, so it is not possible to verify the correctness of Vaṭeśvara's comment. However, it may be mentioned that the durations of *manu*, *kalpa* and quarter *yugas* given in the modern *Siddhāntas* ascribed to Romaka, Sūrya and Brahmā are the same as found in the *Siddhānta* of Brahmagupta,

6. If a *manu* has one twilight, the idea of two twilights is false; if it has two twilights, then it cannot have one twilight. So the (single and double) twilights (imagined by Jiṣṇu's son) are the fabrication of his own intellect; they are not those stated by Manu or Puliśa.

There are 14 *manus* in a *kalpa* and the number of twilights preceding and following the 14 *manus* is 15. Hence the comment of Vaṭeśvara. The comment is useless.

7. The term *caraṇa* ("quarter") exists in the world in the sense of "one-fourth part" and nowhere, alas!, in the sense of "one-tenth part" (as supposed by Jiṣṇu's son). The terms *yuga* and *kalpa*, too, are not spoken of as synonymous. So the true meaning of these terms (*caraṇa* and *yuga*) has also been falsified (by Jiṣṇu's son).

This comment is based on the following grounds :

(1) Kaliyuga is a quarter *yuga* and so its duration should be one-fourth of a *yuga* as taken by Āryabhaṭa I, but according to Brahmagupta its duration is one-tenth of a *yuga*.

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1. *BrSpSi*, i. 14.

(2) Brahmagupta calls a *kalpa* as the *yuga* of the planets, their *uccas* and ascending nodes.

The comment is useless.

8. The statement of the lotus-born that the world is subject to creation and destruction is false. For, it is because the *Vedas* are eternal that people have faith in the words of the *Vedas*.

The use of "lotus-born" for Brahmagupta is a taunt. The comment is justified, because Brahmagupta has indeed said :

"The creation of planets and asterisms takes place in the beginning of Brahmā's day, and their destruction occurs at the end of Brahmā's day."[1]

LORDS OF HOURS ETC.

9. The lords of the hours, days, months and years have been stated by Brahmā to succeed in the order of increasing velocity beginning with Saturn and not beginning with the Sun (as done by Jiṣṇu's son). Even the order of the planets is not known to him.

Vaṭeśvara takes Brahmagupta to task for starting his *kalpa* with a Sunday and not with a Saturday as has been done by Vaṭeśvara.

BEGINNING AND END OF *KALPA*

10. If a *kalpa* (of Jiṣṇu's son) begins on a Sunday, how is it that it does not end on a Saturday ? His *kalpa*, being thus contradictory to his own words, is a fabrication of his own mind.

This comment is valid. For, the number of civil days in a *kalpa* according to Brahmagupta = 1577916450000 $\equiv$ 2 (mod 7), so that if a *kalpa* begins on a Sunday it ends on Monday and not on Saturday. And when one *kalpa* does not end on a Saturday, the next one cannot begin on Sunday, as it should according to Brahmagupta. Thus the *kalpa* of Brahmagupta is contradictory to his own words.

The above comment does not apply to Āryabhaṭa I, because the number of days in a *kalpa* of Āryabhaṭa I = 1008 × 1577917500 $\equiv$ 0 (mod 7). The *kalpa* of Vaṭeśvara is also free from the above defect. For, the number of days in his *kalpa* = 1008 × 1577917560 $\equiv$ 0 (mod 7).

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1. ग्रहनक्षत्रोत्पत्तिर्ब्रह्मदिनादौ दिनक्षये प्रलयः । *BrSpSi* (ed. R. S. Sharma), i. 28 (a-b).

11. Since the assumption of Sunday (in the beginning of *kalpa*) (by Jiṣṇu's son) does not come out to be true from the days elapsed since the beginning of *kalpa*, so that assumption is baseless.

This is simply a rejoinder to Brahmagupta for the following comment made by him against Āryabhaṭa I :

"Since the initial day on which the *kalpa* started according to (Āryabhaṭa's) sunrise system of astronomy is Thursday and not Sunday (as it ought to be), the very basis has become discordant."[1]

ELEMENTS OF *PAÑCĀṄGA*

12. Since due to the ignorance of (the correct positions of) the Sun and the Moon, *tithi*, *karaṇa*, *nakṣatra* and *yoga* (computed by Jiṣṇu's son) disagree with observation at the times of eclipses, so he is ignorant even of the five elements of the *Pañcāṅga*.

SIDDHĀNTA OF BRAHMAGUPTA

13. Since the *yuga* etc. stated by Jiṣṇu's son do not in the least agree with those stated by Brahmā, therefore the title "*Brahmokta Siddhānta*" (*Siddhānta* taught by Brahmā) given by him to his *Siddhānta* is false.

BEGINNING OF KALIYUGA

14. Although Jiṣṇu's son has said that three quarter *yugas* had passed in the beginning of Kaliyuga (since the commencement of the current *yuga*),[2] but actually the first quarter of (the third quarter *yuga*) Dvāpara only had then passed. It means that the elapsed and unelapsed parts of the current *yuga* (stated by him) are not correct.

According to Vaṭeśvara :

3 quarter *yugas* = 3×1080000 years

= 3240000 years ;

1. *BrSpSi*, xi. 11.
2. See *BrSpSi*, i. 26.

and, according to Brahmagupta :

Kṛtayuga + Tretā + one-fourth of Dvāpara = 1728000 + 1296000 + 216000 years = 3240000 years.

Hence the above comment.

Brahmagupta had criticised Āryabhaṭa on similar grounds :

"Āryabhata has said that three quarter *yugas* had elapsed in the beginning of Kaliyuga (since the commencement of the current *yuga*), but since the beginning of his one quarter *yuga* and the end of another quarter *yuga* fall in the middle of Kṛtayuga his assertion is not correct."[1]

Vateśvara's comment is in retaliation.

MEAN PLANETS

15. "The mean longitudes of the planets correspond to mean sunrise for the meridian of Laṅkā."[2] This assertion of Jiṣṇu's son is not true except at the equator.

This comment is justified.

COMMENCEMENT OF DAY (*VĀRAPRAVṚTTI*)

16. "To the east of the meridian of Ujjayinī, the commencement of the day occurs after sunrise (and to the west of the meridian of Ujjayinī, before sunrise)." This statement of Jiṣṇu's son is not true, because the beginning of the day depends (also) on the Sun's ascensional difference.[3]

This comment is valid, because Brahmagupta does not mention the Sun's ascensional difference. What he says is :

"The commencement of the day occurs later than sunrise or earlier than sunrise by an amount equal to the *ghaṭīs* of longitude of the local place, according as the local place is to the east or to the west of the meridian of Ujjayinī."[4]

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1. *BrSpSi*, xi. 4.
2. *BrSpSi*, i. 35.
3. See *supra*, sec. 8, vs. 13.
4. *BrSpSi*, i. 36.

AYANACALANA

17-18. The instantaneous motion of the Sun's *ayana* has been contemplated by stating a particular revolution-number for it. If this motion is disregarded, the positions of the Moon and the other planets become wrong, and all calculations based on them become incorrect. Therefore, Brahmagupta, who, due to ignorance of the *Śāstra*, says that the Sun rises at the east point of the horizon when it is at the first point of Meṣa (i.e., *nirayaṇa* Aries) has indeed lost his mind. For, the sages have (definitely) said that the correction for the Sun's *ayana* has to be applied there.

Brahmagupta, in fact, says :

"That *Siddhānta* (only) should be regarded as accurate according to which the Sun, at an equinox, is exactly in the east."[1]

Brahmagupta, however, has criticized the theory of *ayana-calana*. Writes he :

"The *nāḍīs* of the duration of day and of night are respectively the greatest and the least when the Sun is at the last point of the sign Gemini, and the occurrence of the seasons conforms to the motion of the Sun. So there does not exist any *ayana-yuga* depending on the motion of the *ayanas*, and consequently both the *ayanas* are fixed."[2]

MEAN MOTION

19. The lengths of *yuga*, *manu*, and *kalpa*, as also the periods elapsed since the beginning of the current *kalpa* and since the beginning of the current Kṛtayuga, (as stated by Jiṣṇu's son), are not equivalent to those stated by Brahmā. It means that Jiṣṇu's son has no knowledge of mean motion.

This comment is in reply to a similar comment on Āryabhaṭa by Brahmagupta himself :

"The lengths of *manu*, *yuga* and *kalpa* as also the periods elapsed since the beginning of the current *kalpa* and since the beginning of the

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1. *BrSpSi*, xxiv. 4(c-d).
2. *BrSpSi*, xi. 54.

current Kṛtayuga, as stated by Āryabhaṭa, are not equivalent to those stated in the *Smṛtis.* It means that Āryabhaṭa has no knowledge of mean motion." [1]

REVOLUTION-NUMBERS OF PLANETS

20. The longitude of a planet obtained from its forged revolution-number cannot be the same as that obtained from its real revolution-number. The longitudes of a planet calculated from (different) forged revolution-numbers will also differ from one another.

21-22. (Revolution-numbers can be easily forged from a real revolution-number.) The revolution-number for Mars, (for example), may be forged by taking the first four figures as 8522, 0635, 7552 or 9292 (and keeping the other figures as they are). In this way one might forge thousands of other revolution-numbers for Mars, and also for other planets and their apogees. But none of these can be said to be real.

What Vaṭeśvara wants to say is that the revolution-numbers of the planets stated by Brahmagupta are not real but forged ones. It is noteworthy that Vaṭeśvara cites 2,29,68,28,522 as an example of a forged revolution-number of Mars and this exactly is the revolution-number of Mars adopted by Brahmagupta.

Not only the revolution-numbers of the planets but also the other parameters stated by Brahmagupta are not real but forged ones in the eyes of Vaṭeśvara. See *infra,* vs. 27.

EARTH'S CIRCUMFERENCE

23. "The circumference of the Earth is 5000 (*yojanas* in length)."[2] This (statement of Brahmagupta) is gross. This is why the number of *yojanas* derived from the latitude-difference of Sthāṇvīśvara and Ujjayinī (by using this value) is not true.

BrSpSi, xi. 10.

See *BrSpSi*, i. 37. It is to be noted that "*bhūparidhiḥ khakhakhaśarāḥ*" occurring in the Sanskrit text is literally quoted from Brahmagupta.

The rationale of Brahmagupta's value of the Earth's circumference is as follows :

∵ linear daily motion of a planet (according to Brahmagupta)

$$= \frac{18712069200000000}{1577916450000}$$

(see *BrSpSi*, i. 22; xxi. 11)

$$= 11858.7 \textit{ yojanas}$$

$$\therefore \text{ Earth's diameter } = 2 \times \frac{11858{\cdot}7}{15}$$

$$= 1581 \textit{ yojanas.}$$

Hence, taking $\pi = \sqrt{10}$ (which is regarded as the accurate value of π by Brahmagupta),

Earth's circumference $= 1581\sqrt{10} = 5000$ *yojanas.*

If, however, we take $\pi = 3.1416$ (which is regarded as the accurate value of π by Vaṭeśvara), then

Earth's circumference $= 1581 \times 3{\cdot}1416 = 4967$ *yojanas.*

Hence Vaṭeśvara's comment.

24. Due to the ignorance of (the correct value of) the Earth's circumference, the longitude is of no use; due to the ignorance of that, the true *tithyanta* is not known; and that being destroyed, calculations pertaining to the eclipses are destroyed.

LONGITUDE

25. Jiṣṇu's son has obtained the square of the segment of the local circle of latitude (intercepted between the local and prime meridians) from the *yojanas* between the local place and a place on the prime meridian. In doing so he has exhibited extreme grossness of calculation.

Here Vaṭeśvara criticises the following rule of Brahmagupta :[1]

1. See *BrSpSi*, i. 37.

If A be the local place, C a place on the prime meridian, and B the place where the prime meridian intersects the local circle of latitude, then

$$AB^2 = AC^2 - BC^2 .$$

The criticism is justified. Vaṭeśvara has already pointed out the defects of this rule. See *supra*, sec. 8, vs. 7(a-b).

SUN'S *SAṄKRĀNTI*

26. The times of the Sun's entrance into the zodiacal signs (resulting from the astronomical parameters stated by Jiṣṇu's son) are not in agreement with those obtained from the other *Siddhāntas* and *Tantras*, because, due to his ignorance of the (correct) number of civil days in a *yuga* and the (correct) position of the (Sun's) apogee, the Sun's longitude (resulting from his parameters) is not true.

It is noteworthy that Brahmagupta himself has criticised Āryabhaṭa (though wrongly) for giving two different numbers for civil days in a *yuga* and for the inccuracy of the position of the Sun's apogee.[1]

LONGITUDES OE PLANETS

27. On account of forged revolution-numbers, forged civil days and forged positions of (planets') apogees, and due to ignorance of the epicycles, the longitudes of the planets (resulting from his parameters) disagree with observation, and so they are not true.

RADIAN MEASURE

28. Having rejected the value (3438′), prescribed by the *Śāstras*, for the radius of the circle of the asterisms, his adoption of 3270′ for (the measure of) the radius, in order to achieve accuracy in calculation, is a mathematical blunder.

Vaṭeśvara is commenting here on the following passage of Brahmagupta, where he speaks in defence of adopting 3270′ for the radius of the circle while computing his table of Rsines :

1. See *BrSpSi*, xi. 5; *KK*(BC), II, i. 1.

"The radius derived from the minutes of (the circumference of) the circle of asterisms is not a whole number in minutes, so the Rsines based on it are not accurate. It is for this reason that I have adopted another (number viz. 3270′ for the) radius."[1]

It is noteworthy that the phrase "*khamuniradāḥ*" occurring in the Sanskrit text is literally quoted from Brahmagupta.[2]

RSINES AND RVERSED-SINES

29. From the collection of 24 Rsines (for a quadrant) it is evident that one-ninetysixth part of the circle of asterisms has been assumed to be straight. This being so, the assertion of (Jiṣṇu's son) that the Rversed-sine of that part is 7′ is not true.

30. The assertion made by him that all one-ninetysixth parts of a circle are straight like a gnomon is not true. For if two (one-ninetysixth) parts are straight, how can the arrow (or Rversed-sine) exist there.

31. So this is not the (correct) basis for finding the Rsines. The 24 Rsines should, in fact, end with sharply decreasing Rversed-sines. Their ending with the Rversed-sine of 7′ suits Jiṣṇu's son only.

32. The result based on the right-angled triangle and consistent with the 24 Rsines (of Jiṣṇu's son), which Jiṣṇu's son has stated in connection with the Rsine of the Sun's zenith distance while finding the Rsines of colatitude and latitude is not correct.

Vaṭeśvara is referring in vs. 32 to the following formula (stated in *BrSpSi*, iii. 8) :

$$(\text{radius})^2 = (\text{Rsin } z)^2 + (\text{Rsin } a)^2, \qquad (1)$$

where a denotes the Sun's altitude and z the Sun's zenith distance.

Vaṭeśvara interprets this formula as

$$(3438)^2 = (\text{Rsin } z)^2 + (\text{Rsin } a)^2,$$

where Rsin z and Rsin a conform to the 24 Rsines stated by Brahmagupta for which $R = 3270'$.

Vaṭeśvara's comment is useless, because in formula (1) radius $= 3270'$ and not $3438'$.

1. *BrSpSi*, xxi. 16.
2. See *BrSpSi*, ii. 9(c-d).

CORRECTION OF PLANETS

33. Jiṣṇu's son, who is ignorant of spherics, has applied corrections not envisaged by the ancient teachers. This is why his calculations do not tally with observation.

EPICYCLES

34. If we admit that Mars' *śīghra* epicycle needs a correction, then what is that counterfeit *Āgama* on the authority of which a similar correction is not applied to the epicycles of Moon, Venus, and other planets? It simply means that the epicycles (stated by Jiṣṇu's son) are not correct.

Brahmagupta has prescribed a correction to the *śīghra* epicycle of Mars,[1] but not to the epicycles of other planets. Hence this comment.

SHADOW OF GNOMON

35. With intellect blinded by pride and arrogance, Jiṣṇu's son has dealt with the shadow of the gnomon by indications only (whereas this topic needed a detailed treatment), but in (the fury of) intellectual fever he has (unnecessarily) prattled "thirty six determinations pertaining to shadow".

36. The times of rising of (the signs of) the ecliptic on the eastern horizon was dealt with by the ancients : this has been seen (and copied) by Jiṣṇu's son. That is way he is ignorant of its motion elsewhere.

37. The future shadow of the gnomon calculated according to Jiṣṇu's son, too, differs from that obtained by actual observation and differs widely by *aṅgulas*. Everything of his, therefore, is inaccuarate.

Comment made in the first half of stanza 35 is justified as Brahmagupta has not dealt with the topic of the gnomonic shadow in detail in his *Siddhānta*.

The second half of stanza 35 is a comment on the unnecessary enumeration of 36 determinations pertaining to shadow. It is noteworthy that the phrase "*chāyānayanāni ṣaṭtriṁśat*" occurring in the Sanskrit text is literally quoted from Brahmagupta.[2]

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1. See *BrSpSi*, ii. 37 (c-d)-39.
2. See *BrSpSi*, iii. 37.

KOṆAŚAṄKU AND *SAMAŚAṄKU*

38. The *Koṇaśaṅku* (i.e., the Rsine of the Sun's corner altitude) and *Samaśaṅku* (i.e., the Rsine of the Sun's prime vertical altitude) are non-existent in high latitude both in the forenoon and in the afternoon (when the Sun is to the south of the equator) ; but according to him (i.e., Jiṣṇu's son) they exist at any desired place. This shows that he has no knowledge of the Sun's altitude even.

The comment is useless.

ECLIPSES

39. The Earth's shadow, which has been determined (by Jiṣṇu's son) from the diameters, in *yojanas*, unapproved by *Āgama*, of the other bodies (viz. Earth and Sun), goes to the Moon at its own distance (and causes a lunar eclipse). This means that he knows not a whit.

This comment is in criticism of the following statements of Brahmagupta :

"It is Rāhu who, by virtue of the boon bestowed on him by Brahmā, enters into the Earth's shadow and at the end of the fifteenth *tithi* of the light fortnight covers that part of the Moon's disc which penetrates into the Earth's shadow." (*BrSpSi*, xxi. 44)

"It is Rāhu whose diameter is equal to that of the Earth's shadow and who lies on the Moon's orbit that covers the Moon at the time of a lunar eclipse." (*BrSpSi*, xxi. 46 (a-c)).

40. Jiṣṇu's son knows neither spherics (*Gola*), nor *lambana*-configuration, nor the zodiac, nor even the intricacies of the solar eclipse. He is ignorant of both *Gaṇita* and *Gola*.

41. Eclipse is caused by Rāhu, who with one-half equal to the shadow covers the Moon and with (the other) one-half equal to the Moon covers the Sun : this is the assertion of one who has discarded the teachings of all the *Śāstras* (viz. the son of Jiṣṇu).

The actual words of Brahmagupta commented upon are :

"Rāhu, with his diameter equal to that of the Earth's shadow situated in the orbit of the Moon, eclipses the Moon in a lunar eclipse; and the

same Rāhu, with his body equal to the Moon, eclipses the Sun in a solar eclipse.

"That portion of Black Rāhu's diameter which is in excess of the Earth's shadow or the Moon is destroyed on his coming in front of the Sun. This is the reason for Rāhu being equal to the Earth's shadow and the Moon in diameter (during the eclipses of the Moon and the Sun).

"Hence it is neither the Earth's shadow that eclipses the Moon nor the Moon that eclipses the Sun, but it is Rāhu who with his body equal to them, situated there, eclipses the Moon and the Sun."[1]

Brahmagupta has been designated in the above passage as "one who has discarded the teachings of all the *Śāstras*" (*asta-samasta-śāstrārthaḥ*), because he discarded the teachings of Varāhamihira, Śrīṣeṇa, Āryabhaṭa, Viṣṇucandra, and others who opposed the view that Rāhu was the cause of eclipses. Brahmagupta has declared their views as unpopular and against the *Vedas*, *Smṛtis* and *Saṁhitās*. Writes he :

"When Rāhu eclipses the Moon from the eastern side, why does he not eclipse the Sun in the same way (from the eastern side) ? Why is not the duration of a solar eclipse as large as that of a lunar eclipse ? Are the Sun and Rāhu different for different places that in a solar eclipse the measure of eclipse differs from place to place ? This is how Varāhamihira, Śrīṣeṇa, Āryabhaṭa, Viṣṇucandra, and others have argued against the popular view and against the *Vedas*, *Smṛtis* and *Saṁhitās*."[2]

ZENITH DISTANCE OF CENTRAL ECLIPTIC POINT

42. (The assertion of Jiṣṇu's son) that the sum or difference of the degrees of (local) latitude and the declination of the central ecliptic point gives the degrees of the zenith distance of the central ecliptic point[3] is not correct, because the central ecliptic point lies on the vertical circle passing through the central ecliptic point (and not on the meridian).

The comment is valid. Vaṭeśvara has given a better rule for finding the zenith distance of the central ecliptic point. See *infra*, ch. v, sec. 1, vss. 4(c-d)-5(a-b).

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1. *BrSpSi*, xxi. 46-48.
2. *BrSpSi*, xxi. 37-39.
3. See *BrSpSi*, v. 22 (a.b).

VISIBILITY CORRECTION

43. That Jiṣṇu's son, in his *Siddhānta* following sunrise day-reckoning, has applied the visibility correction meant for sunrise and sunset to a planet's longitude for any desired time, is a mathematical blunder.

Brahmagupta has not actually done so. Vaṭeśvara seems to have inferred it from some of the statements of Brahmagupta. The comment is unjustified.

MOON'S PHASE

44. The illuminated portion (*śukla*) has been exhibited in the Moon by him (i. e., by Jiṣṇu's son) by using the Sun's *bhuja*, and not by using the *bhuja* of the rising or setting point of the ecliptic. This shows that Jiṣṇu's son does not know how to exhibit the illuminated portion of the Moon (by using the *bhuja* of the rising or setting point of the ecliptic).

Vaṭeśvara is taking Brahmagupta to task for giving only one method for exhibiting the illuminated portion in the Moon and not two as done by him, one for the Sun's position on the horizon and the other for the Sun's position elsewhere.

CONCLUSION

45. As it is not possible to mention the (numerous) errors committed by Jiṣṇu's son, so these are to serve as illustrations; the intelligent may add others.

46. As Jiṣṇu's son knows not even one out of mathematics (*Gaṇita*), reckoning with time (*Kāla* or *Kālakriyā*) and spherics (*Gola*), so I have not mentioned the errors pertaining to them separately.

This is exactly what Brahmagupta said for Āryabhaṭa :

"As it is not possible to mention the numerous errors committed by Āryabhaṭa, so these are to serve as illustrations; the intelligent may add others."[1]

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1. *BrSpSi*, xxi. 44.

"As Āryabhaṭa knows not even one out of mathematics, reckoning with time and spherics, so I have not mentioned the errors pertaining to them separately."[1]

47. (Jiṣṇu's son knows) neither reckoning with time (i.e., theoretical astronomy), nor the celestial sphere, nor the motion of the celestial sphere, nor even what is visible to the eye (i.e., eclipses etc.). Everything associated with the celestial sphere is subject to motion; ignorance of that has placed him in such a (miserable) plight.

1. *BrSpSi,* xxi. 43.

Chapter II
TRUE MOTION

Section 1. Correction of Sun and Moon

INTRODUCTION

1. Since, on account of *Ucca* and *Nīca*, a planet is not observed at its mean position in its orbit, so I shall (now) explain the method of finding out the true position (of the Sun and Moon) by the method of *Ucca* and *Nīca* (i. e., by the epicyclic theory).

RSINES AT INTERVALS OF 56′15″

2-27(a). The following are the minutes of the Rsines : 56, 112, 168, 224, 280, 336, 392, 448, 504, 559, 615, 670, 725, 780, 835, 889, 943, 997, 1051, 1105, 1158, 1210, 1263, 1315, 1367, 1418, 1469, 1520, 1570, 1620, 1669, 1718, 1767, 1815, 1862, 1909, 1956, 2002, 2047, 2092, 2137, 2180, 2224, 2266, 2308, 2350, 2390, 2430, 2470, 2509, 2547, 2584, 2621, 2657, 2692, 2727, 2761, 2794, 2826, 2858, 2889, 2919, 2948, 2977, 3004, 3031, 3057, 3083, 3107, 3131, 3154, 3176, 3197, 3217, 3236, 3255, 3272, 3289, 3305, 3320, 3334, 3347, 3360, 3371, 3382, 3391, 3400, 3408, 3415, 3421, 3426, 3430, 3433, 3435, 3437, 3437.

Of these Rsines, the seconds are : 15, 29, 41, 50, 56, 57, 53, 43, 25, 59, 25, 40, 45, 38, 18, 45, 58, 55, 37, 1, 08, 56, 25, 34, 21, 47, 49, 28, 43, 32, 55, 52, 21, 22, 53, 54, 25, 24, 52, 46, 06, 53, 04, 39, 39, 01, 45, 51, 18, 05, 12, 38, 22, 25, 44, 21, 13, 21, 45, 23, 15, 20, 39, 10, 53, 49, 55, 13, 41, 19, 06, 03, 09, 24, 47, 18, 57, 43, 36, 36, 43, 56, 15, 41, 12, 49, 32, 20, 13, 11, 14, 23, 36, 54, 17, 44.

RVERSED-SINES AT INTERVALS OF 56′15″

27-49 (a-b). The following are the minutes of the Rversed-sines : 00, 01 04, 07, 11, 16, 22, 29, 37, 45, 55, 66, 77, 89, 103, 117, 132,

148, 164, 182, 200, 220, 240, 261, 283, 306, 330, 354, 379, 405, 432, 460, 489, 518, 548, 579, 610, 643, 676, 710, 745, 780, 816, 853, 890, 928, 967, 1006, 1046, 1087, 1129, 1171, 1213, 1256, 1300, 1344, 1389, 1435, 1481, 1527, 1574, 1622, 1670, 1718, 1767, 1817, 1867, 1917, 1967, 2018, 2070, 2122, 2174, 2226, 2279, 2332, 2386, 2439, 2493, 2547, 2602, 2657, 2711, 2767, 2822, 2877, 2933, 2989, 3044, 3100, 3156, 3212, 3269, 3325, 3381, and 3437.

(Of these Rversed-sines) the seconds are : 27, 50, 8, 21, 30, 33, 31, 24, 12, 55, 32, 3, 29, 48, 1, 8, 8, 1, 47, 26, 57, 20, 35, 41, 38, 25, 3, 31, 49, 55, 51, 34, 5, 24, 29, 21, 59, 23, 31, 23, 0, 19, 22, 6, 32, 39, 26, 53, 59, 43, 5, 5, 40, 51, 38, 58, 52, 20, 19, 50, 51, 22, 23, 52, 49, 12, 1, 16, 55, 57, 23, 10, 19, 48, 36, 43, 7, 49, 46, 59, 26, 6, 59, 4, 19, 45, 19, 1, 51, 47, 48, 54, 3, 15, 29, 44.

The above Rsines and Rversed-sines may be stated in the tabular form as follows :

Table 13. Table of Rsines and Rversed-sines

Serial No.	Arc	Rsine	Rversed-sine
1.	56′ 15″	56′ 15″	0′ 27″
2.	112′ 30″	112′ 29″	1′ 50″
3.	168′ 45″	168′ 41″	4′ 08″
4.	225′ 00″	224′ 50″	7′ 21″
5.	281′ 15″	280′ 56″	11′ 30″
6.	337′ 30″	336′ 57″	16′ 33″
7.	393′ 45″	392′ 53″	22′ 31″
8.	450′ 00″	448′ 43″	29′ 24″
9.	506′ 15″	504′ 25″	37′ 12″
10.	562′ 30″	559′ 59″	45′ 55″
11.	618′ 45″	615′-25″	55′ 32″

Serial No.	Arc	Rsine	Rversed-sine
12.	675′ 00″	670′ 40″	66′ 03″
13.	731′ 15″	725′ 45″	77′ 29″
14.	787′ 30″	780′ 38″	89′ 48″
15.	843′ 45″	835′ 18″	103′ 01″
16.	900′ 00″	889′ 45″	117′ 08″
17.	956′ 15″	943′ 58″	132′ 08″
18.	1012′ 30″	997′ 55″	148′ 01″
19.	1068′ 45″	1051′ 37″	164′ 47″
20.	1125′ 00″	1105′ 01″	182′ 26″
21.	1181′ 15″	1158′ 08″	200′ 57″
22.	1237′ 30″	1210′ 56″	220′ 20″
23.	1293′ 45″	1263′ 25″	240′ 35″
24.	1350′ 00″	1315′ 34″	261′ 41″
25.	1406′ 15″	1367′ 21″	283′ 38″
26.	1462′ 30″	1418′ 47″	306′ 25″
27.	1518′ 45″	1469′ 49″	330′ 03″
28.	1575′ 00″	1520′ 28″	354′ 31″
29.	1631′ 15″	1570′ 43″	379′ 49″
30.	1687′ 30″	1620′ 32″	405′ 55″
31.	1743′ 45″	1669′ 55″	432′ 51″
32.	1800′ 00″	1718′ 52″	460′ 34″
33.	1856′ 15″	1767′ 21″	489′ 05″
34.	1912′ 30″	1815′ 22″	518′ 24″
35.	1968′ 45″	1862′ 53″	548′ 29″
36.	2025′ 00″	1909′ 54″	579′ 21″
37.	2081′ 15″	1956′ 25″	610′ 59″
38.	2137′ 30″	2002′ 24″	643′ 23″
39.	2193′ 45″	2047′ 52″	676′ 31″
40.	2250′ 00″	2092′ 46″	710′ 23

Serial No.	Arc	Rsine	Rversed-sine
41.	2306′ 15″	2137′ 06″	745′ 00″
42.	2362′ 30″	2180′ 53″	780′ 19″
43.	2418′ 45″	2224′ 04″	816′ 22″
44.	2475′ 00″	2266′ 39″	853′ 06″
45.	2531′ 15″	2308′ 39″	890′ 32″
46.	2587′ 30″	2350′ 01″	928′ 39″
47.	2643′ 45″	2390′ 45″	967′ 26″
48.	2700′ 00″	2430′ 51″	1006′ 53″
49.	2756′ 15″	2470′ 18″	1046′ 59″
50.	2812′ 30″	2509′ 05″	1087′ 43″
51.	2868′ 45″	2547′ 12″	1129′ 05″
52.	2925′ 00″	2584′ 38″	1171′ 05″
53.	2981′ 15″	2621′ 22″	1213′ 40″
54.	3037′ 30″	2657′ 25″	1256′ 51″
55.	3093′ 45″	2692′ 44″	1300′ 38″
56.	3150′ 00″	2727′ 21″	1344′ 58″
57.	3206′ 15″	2761′ 13″	1389′ 52″
58.	3262′ 30″	2794′ 21″	1435′ 20″
59.	3318′ 45″	2826′ 45″	1481′ 19″
60.	3375′ 00″	2858′ 23″	1527′ 50″
61.	3431′ 15″	2889′ 15″	1574′ 51″
62.	3487′ 30″	2919′ 20″	1622′ 22″
63.	3543′ 45″	2948′ 39″	1670′ 23″
64.	3600′ 00″	2977′ 10″	1718′ 52″
65.	3656′ 15″	3004′ 53″	1767′ 49″
66.	3712′ 30″	3031′ 49″	1817′ 12″
67.	3768′ 45″	3057′ 55″	1867′ 01″
68.	3825′ 00″	3083′ 13″	1917′ 16″
69.	3881′ 15″	3107′ 41″	1967′ 55″

Serial No.	Arc	Rsine	Rversed-sine
70.	3937′ 30″	3131′ 19″	2018′ 57″
71.	3993′ 45″	3154′ 06″	2070′ 23″
72.	4050′ 00″	3176′ 03″	2122′ 10″
73.	4106′ 15″	3197′ 09″	2174′ 19″
74.	4162′ 30″	3217′ 24″	2226′ 48″
75.	4218′ 45″	3236′ 47″	2279′ 36″
76.	4275′ 00″	3255′ 18″	2332′ 43″
77.	4331′ 15″	3272′ 57″	2386′ 07″
78.	4387′ 30″	3289′ 43″	2439′ 49″
79.	4443′ 45″	3305′ 36″	2493′ 46″
80.	4500′ 00″	3320′ 36″	2547′ 59″
81.	4556′ 15″	3334′ 43″	2602′ 26″
82.	4612′ 30″	3347′ 56″	2657′ 06″
83.	4668′ 45″	3360′ 15″	2711′ 59″
84.	4725′ 00″	3371′ 41″	2767′ 04″
85.	4781′ 15″	3382′ 12″	2822′ 19″
86.	4837′ 30″	3391′ 49″	2877′ 45″
87.	4893′ 45″	3400′ 32″	2933′ 19″
88.	4950′ 00″	3408′ 20″	2989′ 01″
89.	5006′ 15″	3415′ 13″	3044′ 51″
90.	5062′ 30″	3421′ 11″	3100′ 47″
91.	5118′ 45″	3426′ 14″	3156′ 48″
92.	5175′ 00″	3430′ 23″	3212′ 54″
93.	5231′ 15″	3433′ 36″	3269′ 03″
94.	5287′ 30″	3435′ 54″	3325′ 15″
95.	5343′ 45″	3437′ 17″	3381′ 29″
96.	5400′ 00″	3437′ 44″	3437′ 44″

RADIUS, SQUARE OF RADIUS AND RSIN 24°

49(c-d)-50. 3437′ 44″ is the radius ; and 1,18,18,047′ 35″ is the square of the radius ; and 1398′13″ is the value of Rsin 24°.

Assuming $\pi = 3.1416$, as stated by Āryabhaṭa I, we have

$$\text{Radius} = \frac{21600}{2\pi} = \frac{21600}{6.2832} = 3437'\ 44'', \text{ correct to seconds;}$$

$$\text{and } (\text{Radius})^2 = \left(\frac{21600}{2\pi}\right)^2 = \left(\frac{21600}{6.2832}\right)^2 = 11818047'\ 35'', \text{ correct to seconds.}$$

The value of Rsin 24° may be easily obtained from the above table of Rsines by simple interpolation. See *infra*, p. 170.

51. Thus have been stated, in serial order, the ninety-six Rsines (and Rversed-sines) as obtained through mathematical computation, the equality of the first Rsine and the elemental arc being taken as the basis of this computation.

MANDA AND *ŚĪGHRA* EPICYCLES

52-53. 14, 31½, 72, 22, 33, 11, and 46, are the *manda* epicycles of the Sun etc., in terms of the so called degrees;[1] and 233, 138, 65, 260, and 32, are the *śīghra* epicycles of Mars etc., in terms of the so called degrees.[2]

Table 14. *Manda* and *śīghra* epicycles

Planet	*Manda* epicycle	*Śīghra* epicycle
Sun	14°	
Moon	31½°	
Mars	72°	233°
Mercury	22°	138°
Jupiter	33°	65°
Venus	11°	260°
Saturn	46°	32°

1. Also see *SūSi*, ii. 34-35; *ŚiDVr*, iii. 1 (b); *MSi*, iii. 14(a-b), 21; *SiŚe*, iii. 19, 20, 35, 37; *SiŚi*, I, ii. 22.
2. Also see *SūSi*, ii. 36-37; *ŚiDVr*, iii. 1(c-d); *MSi*, iii. 22(c-d)-23(a-b); *SiŚe*, iii, 37(d)-38; *SiŚi*, I, ii. 23-25.

MANDA AND *SĪGHRA* ANOMALIES

54. The longitude of a planet diminished by the longitude of the planet's apogee (*mandatuṅga*) is defined as the planet's *manda* anomaly (*mandakendra*); and the longitude of the planet's *śīghrocca* diminished by the longitude of the planet is defined as the planet's *śīghra* anomaly (*calakendra* or *śīghrakendra*).[1] The anomalistic quadrants are comprised of three signs each.[2]

manda anomaly = longitude of planet—longitude of planet's apogee

śīghra anomaly =longitude of planet's *śīghrocca*—longitude of planet.

The *mandocca* of a planet is the planet's apogee; and the *śīghrocca* of a planet is the Sun or the planet itself, whichever of the two moves faster.

RSINES OF *BHUJA* AND *KOṬI* OF ANOMALY

55. In the odd (anomalistic) quadrant, the Rsines of the arcs traversed and to be traversed by the planet are defined as *bhuja* and *agra* (*koṭi*), (more correctly, *bhujajyā* and *koṭijyā*), (respectively) ; in the even (anomalistic) quadrant, the *bhuja* and *agra* are the Rsines of the arcs to be traversed and traversed (respectively).[3]

The radius when diminished by the Rversed-sine of the degrees of the *bhuja* or *agra* becomes the Rsine of the other (i.e., of *agra* or *bhuja*) (respectively).[4]

That is, if θ be the *manda* or *śīghra* anomaly of a planet, then

$$\begin{aligned} bhuja\ \theta &= \theta, \text{ if } \theta \leqslant 90^\circ \\ &= 180^\circ - \theta, \text{ if } 90^\circ \leqslant \theta \leqslant 180^\circ \\ &= \theta - 180^\circ, \text{ if } 180^\circ \leqslant \theta \leqslant 270^\circ \\ &= 360^\circ - \theta, \text{ if } 270^\circ \leqslant \theta \leqslant 360^\circ; \end{aligned}$$

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1. *Cf. BrSpSi*, ii. 12(a-b); *ŚiDVr*, ii. 10(a-b); *SiŚe*, iii. 12(a-c); *SiŚi*, I, ii. 18(a-b).
2. *Cf. MSi*, iii. 9(d); *SiŚe*, iii. 12 (d); *SiŚi*, I, ii. 19(a).
3. *Cf. BrSpSi*, ii. 12(c-d); *ŚiDVr*. ii. 10(c-d); *MSi*, iii. 10(a-b); *SiŚe*, iii. 13(a-b); *SiŚi*, I, ii. 19.
4. *Cf. SiŚe*, iii. 14(c-d); *SiŚi*, I, ii. 20(c-d).

$$bhujajy\bar{a}\ \theta = \text{Rsin}\ (bhuja\ \theta) \quad (1)$$

$$\text{and}\ \ ko\d{t}ijy\bar{a}\ \theta = \text{Rsin}\ (90^\circ - bhuja\ \theta)\ \text{or}\ \text{Rcos}\ (bhuja\ \theta) \quad (2)$$

$$\text{Also}\ \ ko\d{t}ijy\bar{a}\ \theta = \text{R} - \text{Rvers}\ (bhuja\ \theta) \quad (3)$$ [1]

$$\text{and}\ \ bhujajy\bar{a}\ \theta = \text{R} - \text{Rvers}\ (90^\circ - bhuja\ \theta) \quad (4)$$

In the text, *bhujajyā* and *koṭijyā* have been loosely called *bhuja* and *agra* respectively

OTHER FORMS OF *BHUJAJYĀ* AND *KOṬIJYĀ*

56. The square root of the difference between the squares of the radius and the Rsine of *bāhu* (*bhuja*) or *agra* is stated to be the Rsine of the other (i.e., of *agra* or *bāhu*).[2] The square root of the product of the difference and the sum of the Rsine of *bāhu* or Rsine of *agra* and the radius is also the Rsine of the same (i.e., of *agra* or *bāhu*).

The square root of the product of the diameter minus Rversed-sine and the Rversed-sine is the Rsine ; the square root of the Rversed-sine multiplied by the diameter and diminished by the square of the Rversed-sine is also the Rsine.

$$ko\d{t}ijy\bar{a}\ \theta = \sqrt{[\text{R}^2 - (bhujajy\bar{a}\ \theta)^2]} \quad (5)$$

$$bhujajy\bar{a}\ \theta = \sqrt{[\text{R}^2 - (ko\d{t}ijy\bar{a}\ \theta)^2]} \quad (6)$$

$$ko\d{t}ijy\bar{a}\ \theta = \sqrt{[(\text{R} - bhujajy\bar{a}\ \theta)(\text{R} + bhujajy\bar{a}\ \theta)]} \quad (7)$$

$$bhujajy\bar{a}\ \theta = \sqrt{[(\text{R} - ko\d{t}ijy\bar{a}\ \theta)(\text{R} + ko\d{t}ijy\bar{a}\ \theta)]} \quad (8)$$

$$\text{and}\ \ \text{Rsin}\ \theta = \sqrt{[(2\text{R} - \text{Rversed-sin}\ \theta).\ \text{Rversed-sin}\ \theta]} \quad (9)$$

$$= \sqrt{[2\text{R}.\ \text{Rversed-sin}\ \theta - (\text{Rversed-sin}\ \theta)^2]}. \quad (10)$$

57. The square of the Rsine of *bhuja* or *koṭi* divided by its own Rversed-sine and the result diminished by the radius gives the other (i.e., Rsine of *koṭi* or *bhuja*). The Rsine of three signs minus the degrees of *koṭi* or *bhuja* is also the other (i.e., Rsine of *bhuja* or *koṭi*).

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1. Several results based on this formula are mentioned in *MSi*, iv. 15.
2. *Cf.* *SiŚe*, iii. 14(a-b); *SiŚi*, I, ii. 21(a-b).

That is, if θ denote the *bhuja*, then

$$\text{Rcos}\ \theta = \frac{(\text{Rsin}\ \theta)^2}{\text{Rvers}\ \theta} - \text{R} \tag{11}$$

$$\text{Rsin}\ \theta = \frac{(\text{Rcos}\ \theta)^2}{\text{Rvers}(90^\circ - \theta)} - \text{R} \tag{12}$$

$$\text{and Rsin}\ \theta = \text{Rsin}(90^\circ - \textit{koṭi}) \text{ or Rsin}\ [90^\circ - (90^\circ - \theta)] \tag{13}$$

$$\text{Rcos}\ \theta = \text{Rsin}\ (90^\circ - \textit{bhuja}) \text{ or Rsin}\ (90^\circ - \theta). \tag{14}$$

COMPUTATION OF THE RSINE

First Method

58. Divide the given (arc reduced to) minutes by the elemental arc (i.e., by $56\frac{1}{4}'$) : the quotient gives the serial number of the tabular Rsine (passed over). Then multiply the remainder by the current Rsine-difference and divide by the elemental arc (i.e., by $56\frac{1}{4}$). The result of this division added to the Rsine (passed over) gives the *koṭijyā* or *bhujajyā* (as the case may be).[1]

The term *dhanuṣ* in the Sanskrit text means "elemental arc". In Vaṭeśvara's table of Rsines it is equal to $56\frac{1}{4}$ minutes.

Example. Calculate Rsin ($1^{\text{sign}}\ 28^\circ$).

$1^{\text{sign}}28^\circ = 3480'$.

Dividing 3480 by $56\frac{1}{4}$ we get 61 as quotient and $48\frac{3}{4}$ as remainder.

The Rsine traversed, i.e., 61^{st} Rsine $= 2889'\ 15''$

and 62^{nd} Rsine $= 2919'\ 20''$.

$\therefore$ current Rsine-difference $= 30'\ 5'' = 30.08'$ approx.

Multiplying $48\frac{3}{4}$ by $30.08'$ we get $1466.4'$ and dividing this by $56\frac{1}{4}$ we get $26'\ 4''$ as quotient.

$\therefore$ required Rsine, i. e., Rsin ($1^{\text{sign}}28^\circ$) $= 2889'\ 15'' + 26'\ 4''$

$= 2915'\ 19''$.

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1. Similar rules occur in *BrSpSi*, ii. 10; *MSi*, iii. 10(c-d)-11(a-b); *SiŚe*, iii. 15; *SiŚi*, I, ii 10(c-d)-11(a-b).

Second Method

59. Or, multiply the (given arc converted into) signs etc. by 32. The resulting signs give the serial number of the Rsine (passed over). Multiply the degrees etc. (reduced to degrees) by the current Rsine-difference and divide by 30; and add whatever is obtained to the Rsine (passed over). This is (also) the method of finding the Rsine (of a given arc).

Let the given arc be s signs and d degrees. Multiplying by 32, we get

$$(s \text{ signs } d \text{ degrees}) \times 32$$

$$= S \text{ signs and } D \text{ degrees, say.}$$

Then S gives the serial number of the Rsine passed over. D degrees constitute 32 times the residual arc. Actually the residual arc is $\frac{D}{32}$ degrees or $\frac{D \times 60}{32}$ minutes. Therefore the corresponding Rsine-correction

$$= \frac{\frac{D \times 60}{32} (\text{current Rsine-difference})}{56\frac{1}{4}}$$

$$= \frac{D \times (\text{current Rsine-difference})}{30}.$$

Rationale. Since 96 Rsines correspond to 3 signs, therefore to 1 sign correspond 32 Rsines. Hence the above rule.

Example. Calculate Rsin ($1^{\text{sign}}28°$).

Multiplying $1^s28°$ by 32, we get $61^s26°$. 61 denotes the serial number of the Rsine passed over.

Multiplying 26 by the current Rsine-difference, i.e., 30′5″, we get 782′10″ and dividing this by 30 we get 26′4″.

Hence Rsin ($1^s28°$) = 2889′15″ + 26′4″ = 2915′19″.

Third Method

60. Or, divide the given (arc reduced to) degrees by 15 and add the resulting quotient to the degrees (of the arc) : this gives the serial number of the Rsine passed over. Then multiply the remainder (of the division)

by the current Rsine-difference and divide by 15 ; and add the resulting minutes etc. to the Rsine (passed over) : the result is the Rsine (of the given arc).

Let the given arc be d°. Let

$$\frac{d}{15} = q + \frac{r}{15}.$$

Then the serial number of the Rsine passed over is $d+q$, and the Rsine-correction

$$= \frac{r \times (\text{current Rsine-difference})}{15}.$$

$$\therefore \text{ Rsin } d^\circ = (d+q)^{\text{th}} \text{ Rsine} + \frac{r \times (\text{current Rsine-difference})}{15}.$$

Rationale. Since 96 Rsines correspond to 90 degrees, therefore to d degrees correspond

$$\frac{96 \times d}{90} \text{ or } (d+\frac{d}{15}), \text{ or } (d+q+\frac{r}{15}) \text{ Rsines.}$$

Hence the rule.

Example. Calculate Rsin ($1^s 28^\circ$).

$1^s 28^\circ = 58^\circ$. Dividing 58 by 15, the quotient is 3 and the remainder 13. Therefore

$$\text{Rsine passed over} = 58 + 3 = 61^{\text{th}}.$$

$$\text{and Rsine-correction} = \frac{13 \times (\text{current Rsine-difference})}{15}$$

$$= \frac{13 \times 30'5''}{15} = 26'4''.$$

$$\text{Hence Rsin } (1^s 28^\circ) = 61^{\text{th}} \text{ Rsine} + 26'4'' = 2889'15'' + 26'4''$$
$$= 2915'19''.$$

Fourth Method

61. Or, multiply the given (arc reduced to) degrees by 16 and divide by 15 : the quotient gives the serial number of the Rsine (passed over). Then multiply the remainder (of the division) by the current Rsine-differ-

ence and divide by the same divisor; and add the result (of the division) to the Rsine passed over. The Rsine (of a given arc) may be obtained in this way also.

This rule is equivalent to the previous one.

Example. Calculate Rsin (24°).

Multiplying 24 by 16 and dividing by 15, we get 25 as quotient and 9 as remainder.

$$25^{\text{th}} \text{ Rsine} = 1367'21''$$

$$\text{and current Rsine-difference} = 26^{\text{th}} \text{ Rsine} - 25^{\text{th}} \text{ Rsine}$$

$$= 1418'47'' - 1367'21'' = 51'26''.$$

Multiplying 51'26" by 9 and dividing by 15, we get 30'51.6".

$$\therefore \text{ Rsine } (24^0) = 1367'21'' + 30'51.6'' = 1398'13''.$$

Fifth Method

62. Multiply the (given arc reduced to) minutes by 4 and divide by 225 : the quotient gives the serial number of the Rsine (passed over). Then multiply the remainder (of the division) by the current Rsine-difference and divide by 225. The result (of this division) added to the Rsine (passed over) gives the (desired) Rsine.

This rule is equivalent to that stated in vs. 58.

COMPUTATION OF THE RSINE BY SECOND ORDER INTERPOLATION

First Method

63. Divide the product of half the difference between the traversed and untraversed Rsine-differences (*bhuktābhuktajyāntaradala*) and the residual arc (*vikala*) by one's own elemental arc (*cāpa*), and then subtract that from or add that to the traversed Rsine-difference (*bhukta-jyā*), according as the Rsines are taken in the serial or reverse order : the result is the multiplier (*guṇaka*), (denoting the instantaneous Rsine-difference). That multiplied by the *labdha* (i.e., the result obtained by dividing the residual arc by the elemental arc) gives the *phala* (required result, i.e., the Rsine-difference corresponding to the residual arc).

The interval (h) at which the tabular Rsines are calculated is called the elemental arc (*cāpa* or *dhanuṣ*). In Vaṭeśvara's table of Rsines, it is equal to $56\frac{1}{4}$ mins or 56′15″.

Let $nh+\lambda$, where $\lambda < h$, be the anomaly of a planet. Then Rsin nh is called the traversed Rsine and Rsin $(n+1)h$, the untraversed Rsine.

$$D_{n-1} = \text{Rsin } nh - \text{Rsin } (n-1)h$$

is called the traversed Rsine-difference (*bhukta-jyāntara, gata-jyāntara, atīta-jyāntara,* or simply *bhukta-jyā, gata-jyā,* or *atītajyā*); and

$$D_n = \text{Rsin } (n+1)h - \text{Rsin } nh$$

is called untraversed or current Rsine-difference (*bhogya-jyāntara* or *agata-jyāntara,* or *simply bhogya-jyā* or *agata-jyā*).

The difference between the traversed and untraversed Rsine-differences, *viz.* $D_n \sim D_{n-1}$, is called *bhuktābhuktajyāntara,* or simply *jyāntara, antara* or *vivara.* λ is called the residual arc (*vikala* or *vikalā*), and Rsin $(nh+\lambda)$— Rsin nh, i.e., the Rsine-difference comesponding to the residual arc, is called the residual Rsine-difference (*vikalajyā*).

The above rule tells us how to find the value of the residual Rsine-difference, i.e., Rsin $(nh+\lambda)$ — Rsin nh. The formula stated is :

$$\text{Rsin } (nh+\lambda) - \text{Rsin } nh = \frac{\lambda}{h} \times \text{multiplier},$$

$$\text{where multiplier} = D_{n-1} \mp \frac{\lambda}{h} \cdot \frac{D_n \sim D_{n-1}}{2},$$

— or + being taken according as $D_n \lesseqgtr D_{n-1}$,

Rationale. (Based on Bhāskara II's rationale)

The Rsine-difference for the traversed elemental arc is

$$D_{n-1} = \text{Rsin } nh - \text{Rsin}(n-1)h$$

and the Rsine-difference for the untraversed or current elemental arc is

$$D_n = \text{Rsin } (n+1)h - \text{Rsin } nh.$$

Therefore, the rate of decrease or increase of the Rsine-difference

$$= D_n \sim D_{n-1}, \text{ for an arc of length } 2h$$

$$= \tfrac{1}{2}(D_n \sim D_{n-1}), \text{ for an arc of length } h.$$

Now the Rsine-difference at the beginning of the untraversed elemental arc is D_{n-1} and the decrease or increase of the Rsine-difference for an arc of length h is $\frac{D_n \sim D_{n-1}}{2}$. Therefore the instantaneous Rsine-difference

$$= D_{n-1} \mp \frac{\lambda}{h} \cdot \frac{D_n \sim D_{n-1}}{2}, \qquad (1)$$

$-$ or $+$ sign being taken according as $D_n \lesseqgtr D_{n-1}$.[1]

Hence

$$\text{Rsin}(nh+\lambda) = \text{Rsin } nh + \frac{\lambda}{h} \text{(instantaneous Rsine-difference)}$$

$$= \text{Rsin } nh + \frac{\lambda}{h}\left(D_{n-1} \mp \frac{\lambda}{h} \frac{D_n \sim D_{n-1}}{2}\right). \qquad (2)$$

Alternative Rationale. We have proved below (see p. 176) that, in case $D_n < D_{n-1}$,

$$\text{Rsin}(nh+\lambda) = \text{Rsin } nh + \frac{\lambda}{h}\left(\frac{D_{n-1}+D_n}{2} - \frac{\lambda}{h} \cdot \frac{D_{n-1}-D_n}{2}\right)$$

If we assume that $\frac{D_{n-1}+D_n}{2} = D_{n-1}$, approx., then

$$\text{Rsin}(nh+\lambda) = \text{Rsin } nh + \frac{\lambda}{h}\left(D_{n-1} - \frac{\lambda}{h} \frac{D_{n-1}-D_n}{2}\right)$$

In modern notation, we may write formula (2) as follows :

$$\text{Rsin }(nh+\lambda) = \text{Rsin } nh + \frac{\lambda}{h} \cdot D_{n-1} + \frac{\lambda^2}{h^2} \cdot \frac{D^2_{n-1}}{2},$$

where $D_{n-1} = \text{Rsin } nh - \text{Rsin }(n-1)h$, $D^2_{n-1} = D_n - D_{n-1}$;

Or, in general form, as

$$f(nh+\lambda) = f(nh) + \frac{\lambda}{h} f_1(n-1)h + \frac{\lambda^2}{h^2} \frac{f_2(n-1)h}{2}$$

or, as

$$f(x) = f(x_0) + \frac{(x-x_0)}{h} \Delta f(x_0 - h) + \left(\frac{x-x_0}{h}\right)^2 \frac{\Delta^2 f(x_0-h)}{2}.$$

1. This is fallacious, because on putting $\lambda = h$, we get $\frac{D_{n-1}+D_n}{2}$ as the Rsine-difference at the end of the untraversed elemental arc and not D_n as it ought to be.

Another form of the First Method

64. **Multiply one-half of what is obtained on dividing the residual arc (*vikala*) by the elemental arc (*cāpa*) by the difference between the (traversed and untraversed) Rsine-differences (*jyāntara*), and subtract that from or add that to the traversed Rsine-difference (*bhuktaguṇa*). That difference or sum divided by the elemental arc (*dhanuṣ*) and multiplied by the residual arc (*vikalā*) gives the residual Rsine-difference (i.e., the Rsine-difference corresponding to the residual arc, *vikalajyā*).**

$$\text{Rsin}(nh+\lambda)-\text{Rsin}\, nh=\frac{\lambda}{h}\left[D_{n-1}\mp\frac{\lambda}{2h}(D_n\sim D_{n-1})\right],$$

— or + sign being taken according as $D_n \lesseqgtr D_{n-1}$.

Second Method. Form 1

65. **Multiply half the difference between the traversed and untraversed Rsine-differences (*agatātītajyāntaradala*) by the residual arc (*vikala*) and divide by the elemental arc (*dhanuṣ* or *cāpa*). Add that to half the difference between the (traversed and untraversed) Rsine-differences (*jyāntaradala*). Subtract that from or add that to the traversed Rsine-difference (*bhukta-guṇa*). Then is obtained the (instantaneous Rsine-difference (*bhojya-guṇa*).**

66. **Add 1 to the *labdha* (i.e., to the result obtained on dividing the residual arc by the elemental arc), reduce it to half, and then multiply that by the product of the *labdha* and the *vivara* (*jyāntara*, i.e., the difference between the traversed and untraversed Rsine-differences). Subtract that from or add that to the product of the *labdha* (*dhanuṣāpta*) and the traversed Rsine-difference (*bhukta-jīvā*). Then is obtained the residual Rsine-difference (*vikalajyā*).**

That is,

$$\text{instantaneous Rsine difference}=D_{n-1}\mp\left(\frac{D_n\sim D_{n-1}}{2}+\frac{\lambda}{h}\cdot\frac{D_n\sim D_{n-1}}{2}\right)\quad(1)$$

$$\text{and Rsin}\,(nh+\lambda)-\text{Rsin}\, nh=\frac{\lambda}{h}D_{n-1}\mp\frac{\frac{\lambda}{h}\left(\frac{\lambda}{h}+1\right)}{2}(D_n\sim D_{n-1}),\quad(2)$$

— or + sign being taken according as $D_n \lesseqgtr D_{n-1}$.

One can easily see that formula (2) is equivalent to Brahmagupta's formula,[1] viz.

$$\text{Rsin}(nh+\lambda)-\text{Rsin}\ nh=\frac{\lambda}{h}\left[\frac{D_n+D_{n-1}}{2}\mp\frac{\lambda}{h}\frac{D_n\sim D_{n-1}}{2}\right].$$

Rationale (as gives by Bhāskara II)

The Rsine-difference for the traversed elemental arc is

$$D_{n-1}=\text{Rsin}\ nh-\text{Rsin}(n-1)h$$

and the Rsine-difference for the untraversed or current elemental arc is

$$D_n=\text{Rsin}\ (n+1)h-\text{Rsin}\ nh.$$

The mean of the two is

$$\tfrac{1}{2}(D_n+D_{n-1}).$$

This, says Bhāskara II, must evidently be at the middle, i.e., at the junction of the traversed and the untraversed elemental arcs.

Now the Rsine-difference at the beginning of the current elemental arc is $\frac{1}{2}(D_n+D_{n-1})$ and at the end D_n, and likewise the increase in the Rsine-difference from the beginning of that arc to its and

$$=D_n-\tfrac{1}{2}(D_n+D_{n-1})$$

$$=\mp\tfrac{1}{2}(D_n\sim D_{n-1}),$$

$-$ or $+$ sign being taken according as $D_n \lesseqgtr D_{n-1}$.

Therefore the instantaneous Rsine-difference[2]

$$=\frac{D_n+D_{n-1}}{2}\mp\frac{\lambda}{h}\cdot\frac{D_n\sim D_{n-1}}{2},$$

—or +sign being taken acccrding as $D_n \lesseqgtr D_{n-1}$.

Hence

$$\text{Rsin}\ (nh+\lambda)-\text{Rsin}\ nh=\frac{\lambda}{h}\left[\frac{D_n+D_{n-1}}{2}\mp\frac{\lambda}{h}\frac{D_n\sim D_{n-1}}{2}\right]$$

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1. *Cf.* *KK*(BC), II, i. 4.
2. It is interesting to note that this formula gives D_{n-1} when $\lambda=-h$ and D_n when $\lambda=h$.

$$= \frac{\lambda}{h}\left[D_{n-1} \mp \frac{D_n \sim D_{n-1}}{2} \mp \frac{\lambda}{h}.\frac{D_n \sim D_{n-1}}{2}\right]$$

$$= \frac{\lambda}{h}\left[D_{n-1} \mp \left(\frac{\lambda}{h} + 1\right)\left(\frac{D_n \sim D_{n-1}}{2}\right)\right]$$

$$= \frac{\lambda}{h} D_{n-1} \mp \frac{\frac{\lambda}{h}\left(\frac{\lambda}{h} + 1\right)}{2}(D_n \sim D_{n-1}),$$

$-$ or $+$ sign being taken according as $D_n \lesseqgtr D_{n-1}$.

Alternative rationale.

$$D_{n-1} = \text{Rsin } nh - \text{Rsin }(n-1)h$$

$$= \text{Rsin } nh - \frac{\text{Rsin } nh.\ \text{Rcos } h - \text{Rcos } nh.\ \text{Rsin } h}{\text{R}}$$

$$= \text{Rsin } nh - \text{Rsin } nh.\left(1 - \frac{h^2}{2}\right) + h.\ \text{Rcos } nh,$$

expanding sin h and cos h in powers of h and retaining terms up to the second power of h.

$$= h.\ \text{Rcos } nh + \frac{h^2}{2}.\ \text{Rsin } nh.$$

Similarly,

$$D_n = \text{Rsin }(n+1)\ h - \text{Rsin } nh$$

$$= \frac{\text{Rsin } nh.\ \text{Rcos } h + \text{Rcos } nh.\ \text{Rsin } h}{\text{R}} - \text{Rsin } nh$$

$$= \text{Rsin } nh.\left(1 - \frac{h^2}{2}\right) + h.\ \text{Rcos } nh - \text{Rsin } nh$$

$$= h.\ \text{Rcos } nh - \frac{h^2}{2}.\ \text{Rsin } nh.$$

$$\therefore \frac{D_{n-1} + D_n}{2} = h.\ \text{Rcos } nh, \text{ and } \frac{D_{n-1} - D_n}{2} = \frac{h^2}{2}.\ \text{Rsin } nh.$$

$$\therefore \text{ Rsin } (nh+\lambda) = \frac{\text{Rsin } nh.\text{ Rcos } \lambda + \text{Rcos } nh.\text{ Rsin } \lambda}{R}, \lambda < h,$$

$$= \text{Rsin } nh.\left(1-\frac{\lambda^2}{2}\right)+\lambda.\text{ Rcos } nh$$

$$= \text{Rsin } nh + \frac{\lambda}{h}\left[h.\text{ Rcos } nh - \frac{\lambda}{h}\cdot\frac{h^2}{2}\text{ Rsin } nh\right]$$

$$= \text{Rsin } nh + \frac{\lambda}{h}\left[\frac{D_{n-1}+D_n}{2} - \frac{\lambda}{h}\cdot\frac{D_{n-1}-D_n}{2}\right]$$

$$= \text{Rsin } nh + \frac{\lambda}{h} D_{n-1} - \frac{\frac{\lambda}{h}\left(\frac{\lambda}{h}+1\right)}{2}(D_{n-1}-D_n),$$

where evidently $D_n < D_{n-1}$.

Modern notation. Formula (2) above is essentially the same as Newton-Gauss backward interpolation formula, viz.

$$f(x) = f(x_o)+\frac{x-x_o}{h}\Delta f(x_o-h)+\frac{x-x_o}{h}\cdot\frac{x-x_o+h}{h}\cdot\frac{\Delta^2 f(x_o-h)}{2}.$$

If we replace f by Rsine, x by $nh+\lambda$, x_o by nh, $x-x_o$ by λ, we get formula (2) above.

Form 2

67. Multiply the sum of the residual arc (*vikala*) and the elemental arc (*cāpa*) by the difference between the traversed and untraversed Rsine-differences (*vivara* or *jyāntara*), then divide by twice the elemental arc, and then subtract what is thus obtained from or add that to the traversed Rsine-difference (*gata-guṇa* or *bhukta-jyā*). The multiplier (of this) is the residual arc (*vikala*) and the divisor is the elemental arc (*dhanuṣ*). What is thus obtained is the value of the residual Rsine-difference (*vikalā-guṇa*).

$$\text{Rsin } (nh+\lambda) - \text{Rsin } nh = \frac{\lambda}{h}\left[D_{n-1} \mp \frac{(\lambda+h)\,(D_n \sim D_{n-1})}{2h}\right],$$

$-$ or $+$ sign being taken according as $D_n \lesseqgtr D_{n-1}$.

Form 3

68. **Divide the square of the residual arc (*vikalakṛti*) by the elemental arc (*cāpa*), then add the residual arc (*vikala*), and then multiply by the difference between the traversed and untraversed Rsine-differences (*vivara*) divided by the elemental arc (*cāpa*). Reduce it to half, and subtract that from or add that to the traversed Rsine-difference (*gataguṇa*) as multiplied by the residual arc (*vikala*) and divided by the elemental arc (*dhanuṣ*) : the result is the (residual) Rsine-difference.**

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \frac{\lambda}{h} D_{n-1} \mp \frac{\left(\frac{\lambda^2}{h} + \lambda\right)(D_n \sim D_{n-1})}{2h} .$$

Form 4

69-71. **Multiply and increase the residual arc divided by the elemental arc (*vikala*/*cāpa*) by half the difference between the traversed and untraversed Rsine-differences (*vivarārdha*) ; subtract that from or add that to the traversed Rsine-difference (*bhukta-guṇa*) ; diminish or increase that by the product of the difference between the traversed and untraversed Rsine-differences (*bhuktetarajīvāntara*) and one-half ; multiply whatever is obtained by half of itself and also by 8 : the result is to be known as *dṛḍha* Now diminish or increase the traversed Rsine-difference (*gatā jīvā*) by the product of (i) half the difference between the traversed and untraversed Rsine-differences (*vivarārdha*) and (ii) the residual arc divided by the elemental arc plus one (*vikala*/*cāpa*+1) ; multiply that by 2 and increase or decrease by the difference between the traversed and untraversed Rsine-differences (*antara* or *vivara*). One-eighth of the difference between the square of that and the *dṛḍha*, when multiplied by the residual arc divided by the elemental arc and divided by the difference between the traversed and untraversed Rsine-differences (*vivara*) gives the residual Rsine-difference (*vikalajīvā*).**

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \frac{1}{8}\Big[[2 \left\{ D_{n-1} \mp \frac{D_n \sim D_{n-1}}{2} \left(\frac{\lambda}{h} + 1 \right) \right\}$$

$$\pm (D_n \sim D_{n-1})]^2 \sim dṛḍka \Big] \times \frac{\lambda}{h} \times \frac{1}{D_n \sim D_{n-1}} ,$$

$$\text{where } dṛḍha = 8 \left[\frac{1}{2} \left\{ D_{n-1} \mp \frac{D_n \sim D_{n-1}}{2} \left(\frac{\lambda}{h} + 1 \right) \mp \frac{D_n \sim D_{n-1}}{2} \right\}^2 \right]$$

Form 5

72. Find the sum of one-half of the residual arc (*vikala*) and the elemental arc (*cāpa*) ; by that multiply the difference between the traversed and untraversed Rsine-differences (*jyāntara* or *vivara*) ; by the "quotient" obtained on dividing that by one's own elemental arc (*svacāpa*) multiply the residual arc divided by the elemental arc (i.e., *vikala*/*cāpa*) ; and add whatever is obtained to the "quotient": then is obtained the so called *rāśi*.

73. Now obtain the sum or difference of the traversed Rsine-difference (*bhukta-jyā*) and the difference between the traversed and untraversed Rsine-differences (*vivara*) ; by that multiply the residual arc (*vikala*) and divide by one's own elemental arc (*svacāpa*); increase or diminish that by the difference between the traversed and untraversed Rsine-differences (*vivara*) ; the difference or sum of that and the *rāśi* is the residual Rsine-difference (*vikalajīvam*).

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \frac{\lambda}{h}\,(D_{n-1} \pm V) \pm V \mp \textit{rāśi},$$

where $\textit{rāśī} = \left(\frac{\lambda}{h} + 1\right) Q,\quad Q = \left(\frac{\lambda}{2} + h\right)\frac{V}{h},$

λ = residual arc, h = elemental arc, and $V = \textit{vivara} = D_n \sim D_{n-1}$.

Form 6

74-75. Divide the traversed Rsine-difference (*bhukta-jyā*) by the difference between the traversed and untraversed Rsine-differences (*antara* or *vivara*). Set down the result in three places (one below the other). The last result (standing in the lowest place) does not take part in (our) calculation. Those standing in the first and second places are to be diminished or increased by 1/2. The square of that in the middle, taken as it is without further subtraction or addition, is the so called *sudṛḍha*. Divide the residual arc (*vikala*) by the elemental arc (*dhanuṣ* or *cāpa*) and subtract whatever is obtained from or add that to the result standing in the first place (i.e., in the topmost place). Now obtain the product of (i) the difference of the square of that and the *sudṛḍha* and (ii) the difference between the traversed and untraversed Rsine-differences (*vivara*). One-half of this product is the residual Rsine-difference (*vikalajyā*).

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \tfrac{1}{2}\left[\,\textit{sudṛḍha} \sim \left(\frac{D_{n-1}}{V} \mp \tfrac{1}{2} \mp \frac{\lambda}{h}\right)^2\right]V,$$

where $\textit{sudṛḍha} = \left(\frac{D_{n-1}}{V} \mp \frac{1}{2}\right)^2$, λ = residual arc,

h = elemental arc, and $V = D_n \sim D_{n-1}$.

Form 7

76-77. **Divide the traversed Rsine-difference** (*gatamaurvī, gatajyā* **or** *bhuktajyā*) **by the difference between the traversed and untraversed Rsine-differences** (*vivara*); **square the "result"** (*phala*) **obtained; diminish or increase that square by the "result"** (*phala*); **and then increase that by 1/4 : this is the** *dṛḍha*. **Now diminish or increase the "result"** (*phala*) **by the residual arc divided by the elemental arc** (*vikala/cāpa*), **and diminish or increase that by 1/2 and find the square of that. Now obtain the difference between this (square) and the** *dṛḍha* **and multiply that by half the difference between the traversed and untraversed Rsine-differences (i. e., by half the** *vivara*)**: the result is the residual Rsine-difference** (*vikalajyā*).

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \left[\,\textit{dṛḍha} \sim \left(\frac{D_{n-1}}{V} \mp \frac{\lambda}{h} \mp \tfrac{1}{2}\right)^2\right]\frac{V}{2},$$

where $\textit{dṛḍha} = \left(\frac{D_{n-1}}{V}\right)^2 \mp \frac{D_{n-1}}{V} + \frac{1}{4}$, or $\left(\frac{D_{n-1}}{V} \mp \frac{1}{2}\right)^2$,

λ = residual arc, h = elemental arc, and $V = D_n \sim D_{n-1}$.

The formulae stated in vss. 67, 68, 69-71, 72-73, 74-75 and 76-77, on simplification, reduce to the formula stated in vss. 65-66. The formulae stated in vss. 74-75 and 76-77 are practically the same.

Form 8

78. **Multiply the difference between the traversed and untraversed Rsine-differences** (*vivara*) **by one-half of the residual arc** (*vikala*) **plus one; then subtract that from or add that to the traversed Rsine-difference; and then multiply that by the residual arc : the result is the residual Rsine-difference** (*vikalajīvā*). **Here the residual arc corresponds to unit elemental arc.**

79. Or, after making the subtraction or addition divide by the unit fraction of the residual arc : the result is the residual Rsine-difference (corresponding to unit elemental arc).

That is, when the elemental arc is taken to be unity,

$$\text{Rsin}\,(nh+\lambda) - \text{Rsin}\,nh = \left[D_{n-1} \mp \frac{\lambda+1}{2} V \right] \lambda$$

$$= \frac{D_{n-1} \mp \frac{\lambda+1}{2} V}{1/\lambda}.$$

This follows from the formula of vs. 67 by putting $h=1$.

Form 9

80. The traversed Rsine-difference (*bhuktaguṇa*) (decreased or) increased by the result obtained on multiplying half the sum of the elemental arc and the residual arc by the difference between the traversed and untraversed Rsine-differences and then dividing by the elemental arc, when divided by the elemental arc upon the residual arc, gives the residual Rsine-difference.

$$\text{Residual Rsine difference} = \frac{D_{n-1} \mp \frac{[(h+\lambda)/2]\; V}{h}}{h/\lambda}.$$

This is equivalent to the formula of vs. 67.

JYĀNTARA OR *VIVARA*

81. The difference between the traversed Rsine-difference (*gatamaurvī* or *bhuktajyā*) and the residual Rsine-difference as multiplied by one upon the residual arc, when divided by half the sum of the residual arc and one, gives the difference between the traversed and untraversed Rsine-differences (*maurvikāntara*).

$$V = \frac{\frac{1}{\lambda}\,(\text{residual Rsine-difference}) \sim D_{n-1}}{\frac{1}{2}\,(\lambda+1)}$$

This follows from the formula of vs. 78.

GATAJYĀ

82. Add or subtract the result obtained by multiplying the sum of one-half of the elemental arc (*cāpadala*) and one-half of the residual arc (*vikaladala*) by the difference between the traversed and untraversed Rsine-differences (*vivara*) and dividing by the elemental arc (*cāpa*), from the product of the residual Rsine-difference (*vikalajyā*) and the elemental arc (*cāpa*) divided by the residual arc (*vikala*). Then is obtained the traversed Rsine-difference (*gatajyā* or *bhuktajyā*).

$$D_{n-1} = \frac{\text{residual Rsine-difference} \times h}{\lambda} \pm \frac{\left(\frac{h}{2}+\frac{\lambda}{2}\right)V}{h}.$$

This follows from the formula of vs. 67.

COMPUTATION OF THE ARC BY SIMPLE INTERPOLATION

First Method

83. (From the given Rsine subtract the greatest tabular Rsine that can be subtracted). By the serial number of the tabular Rsine that has been subtracted multiply the elemental arc (56′ 15″); to that add whatever is obtained by dividing the product of the remainder of subtraction (*vikala*) and the elemental arc (*śarāsana*) by the current Rsine-difference (*jyāntara*). Then is obtained the arc corresponding to the given Rsine.[1]

This rule is just the converse of the rule stated in verse 58 above.

Second Method

84. Having subtracted from the given Rsine the (greatest) tabular Rsine (that can be subtracted from it), multiply 225 by the remainder (of subtraction) and divide (the resulting product) by the product of the current Rsine-difference and 4, and add whatever is (thus) obtained to the product of $56\frac{1}{4}'$ and the serial number of the tabular Rsine subtracted: the result is the arc corresponding to the given Rsine.

1. Similar rules occur in *BrSpSi*, ii. 11; *MSi*, iii. 12; *SiŚe*, iii. 16; *SiŚi*, I, ii. 11(c-d)-12(a-b).

The remainder obtained by subtracting the greatest tabular Rsine from the given Rsine is the same thing as the residual Rsine-difference. In what follows we shall call it residual Rsine-difference.

COMPUTATION OF THE ARC BY SECOND ORDER INTERPOLATION

First Method

85-86. The sum or difference of twice the traversed Rsine-difference (*bhuktajyā*) and the difference between the traversed and untraversed Rsine-differences (*antara* or *vivara*) is the *first*; the sum of the traversed Rsine-difference (*bhuktajyā*) and the residual Rsine-difference (*avaśeṣa*) multiplied by 2, is the *second*; these are rectified on being divided by the difference between the traversed and untraversed Rsine-differences (*jyāntara*). Now take the square-root of the difference or sum of the square of half the *first* (*ādyardha-dvigata*) and the *second*, and find the sum or difference of that (square-root) and half the *first*. Diminish that by 1 and multiply by the elemental arc (*cāpa*). Then is obtained the residual arc (*vikala-cāpa*).

Residual arc, i.e., $\lambda = h\left[\sqrt{\left\{\left(\frac{first}{2}\right)^2 \mp \text{second}\right\}} \pm \frac{first}{2} - 1\right]$,

where *first* $= (2D_{n-1} \pm V)/V$

and *second* $= 2(D_{n-1} + \text{residual Rsine-difference})/V$.

Rationale. From rule 65-66 above, we have

$$\text{Residual Rsine-difference} = \frac{\lambda}{h}\left\{D_{n-1} \mp \left(\frac{\lambda}{h}+1\right)\frac{V}{2}\right\}.$$

Let $\frac{\lambda}{h} + 1 = x$. Then we have

$$\text{Residual Rsine-difference} = (x-1)(D_{n-1} \mp xV/2)$$

or $\qquad x^2 \mp (first)\, x \pm second = 0,$

giving $\qquad \lambda = h(x-1)$

$$= h\left[\sqrt{\left\{\left(\frac{first}{2}\right)^2 \mp second\right\}} \pm \frac{first}{2} - 1\right]$$

Second Method

87-88. The product of half the sum of the traversed and untraversed Rsine-differences and the elemental arc is the *first*; the product of the square of the elemental arc and the residual Rsine-difference is the *second*; these are rectified when divided by half the difference between the traversed and untraversed Rsine-differences. Now take the square root of the difference or sum of the square of half the *first* and the *second* and then obtain the difference between that (square root) and half the *first*. This is the residual arc.

$$\text{Residual arc, i.e., } \lambda = \sqrt{\left[\left(\frac{first}{2}\right)^2 \mp second\right]} - \frac{first}{2},$$

$$\text{where } first = h\,\frac{D_{n-1}+D_n}{2} \div \frac{V}{2} = h\,\frac{D_{n-1}+D_n}{V}$$

$$second = \frac{h^2 r}{V/2} = \frac{2h^2 r}{V},$$

$$V = D_n \sim D_{n-1}$$

and r = residual Rsine-difference.

Rationale. From Brahmagupta's formula,[1] we have

$$\text{Residual Rsine-difference} = \frac{\lambda}{h}\left(\frac{D_{n-1}+D_n}{2} \mp \frac{\lambda}{h}\cdot\frac{D_n \sim D_{n-1}}{2}\right)$$

This may be written as

$$\lambda^2 \pm \frac{Sh\lambda}{V} \mp \frac{2h^2\,(\text{residual Rsine-difference})}{V} = 0$$

where $S = D_{n-1}+D_n$.

Solving this for λ we get the desired result.

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1. This is equivalent to formula (2) of vss. 65-66. For,

$$D_{n-1} \mp \frac{D_n \sim D_{n-1}}{2} = \frac{D_{n-1}+D_n}{2},$$

$-$or $+$sign being taken according as $D_n \lesseqgtr D_{n-1}$.

Third Method

89-90. Divide the traversed Rsine-difference by the difference between the traversed and untraversed Rsine-differences : the result is the subtractive or additive "*phala*". Multiply that by itself and to the square of the "*phala*" (thus obtained) add 1/4 and then add or subtract the "*phala*" and also twice the residual Rsine-difference as divided by the difference between the traversed and untraversed Rsine-differences. Take the square root of that ; and find the difference between that and the "*phala*" and diminish or increase that by 1/2 and lastly multiply that by the desired elemental arc. Then is obtained the residual arc, according as the Rsines are taken in the serial or reverse order.

$$\text{Residual arc, i.e., } \lambda = h\left[\sqrt{\left\{(phala)^2 + \frac{1}{4} \pm phala \pm \frac{2r}{V}\right\}} - phala \mp \tfrac{1}{2}\right],$$

where $phala = \dfrac{D_{n-1}}{V}$

and r = residual Rsine-difference.

This result follows on solving the equation

$$\text{residual Rsine-difference} = \frac{\lambda}{h}\left[D_{n-1} \mp \left(\frac{\lambda}{h} + 1\right)\frac{V}{2}\right]$$

for λ/h.

Fourth Method

91-92. Diminish or increase the traversed Rsine-difference by half the difference between the traversed and untraversed Rsine-differences and divide that by the difference between the traversed and untraversed Rsine-differences : this is the *first*. Diminish or increase the square of that by the residual Rsine-difference as divided by half the difference between the traversed and untraversed Rsine-differences ; extract the square root of that and find the difference between that and the *first*. Or, find the square root of the difference between that square and the result obtained by dividing the residual Rsine-difference by half the difference between the traversed and untraversed Rsine-differences, and add that to or subtract that from the *first*. This result or the difference (obtained above) multiplied by the elemental arc gives the residual arc.

Residual arc, i.e., $\lambda = h\left[first \sim \sqrt{\left((first)^2 \mp \frac{r}{V/2}\right)}\right]$

or, $h\left[first \pm \sqrt{\left((first)^2 \sim \frac{r}{V/2}\right)}\right]$,

where $first = \frac{D_{n-1} \mp V/2}{V}$, and r = residual Rsine-difference.

This result is equivalent to the previous one and may be derived from the equation

$$\text{residual Rsine-difference} = \frac{\lambda}{h}\left[D_{n-1} \mp \left(\frac{\lambda}{h} + 1\right)\frac{V}{2}\right]$$

on solving it for λ/h, as before.

Note. In the rest of this chapter, the word "planet" (*graha*) has been generally used to denote the Sun or Moon.

TRUE LONGITUDES OF SUN AND MOON

Bhujaphala and *bhujāntara* corrections

93-94. (Severally) multiply the *bhujajyā* and the *koṭijyā* by the (*manda*) epicycle and divide by 360 : then are obtained the *bhujaphala* and the *koṭiphala*.[1]

The arcs corresponding to the *bhujaphala* of the Sun and the Moon should be subtracted from or added to the mean longitudes of the Sun and the Moon (respectively), according as the planet's own (*manda*) anomaly lies in the half-orbit beginning with the sign Aries or in that beginning with the sign Libra. The Sun and the Moon then become corrected (for the *bhujaphala*).[2]

Multiply the mean daily motions of the planets (to be corrected) by the minutes of the Sun's *bhujaphala* and divide by 21600 plus the Sun's mean daily motion : the results (known as *bhujāntara*) should be applied (to the planets corrected for the *bhujaphala*) (as correction, positive or negative) like (the *bhujaphala* of) the Sun.[3]

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1. *Cf. BrSpSi*, ii. 14(a); *MSi*, iii. 14(c-d); *SiŚe*, iii. 23 (a-b); *SiŚi*, I, ii. 26(a-b).
2. *Cf. ŚiDVr*, ii. 14; *MSi*, iii. 14(d)-15(a-b); *SiŚe*, iii. 25-26(a-b).
3. For other rules giving *bhujāntara* correction, see : *ŚiDVr*, ii. 16; *MSi*, iii. 16(c-d).

The *bhujaphala* correction is the equation of the centre. If θ be a planet's mean anomaly, then

$$\text{Rsin}\ (bhujaphala) = \frac{\text{Rsin}\ \theta \times manda\ \text{epicycle}}{360}.$$

Śrīpati[1] and Bhāskara II[2] have stated the *bhujaphala* correction in the following form also :

$$\text{Rsin}\ (bhujaphala) = \frac{\text{Rsin}\ \theta \times \text{radius of } manda \text{ epicycle.}}{\text{R}}.$$

Both the above formulae are equivalent.

The *bhujāntara* or *bhujāvivara* correction is "the correction due to the Sun's equation of the centre." By this correction "allowance is made for that part of the equation of time, or of the difference between mean and apparent solar time, which is due to the difference between the Sun's mean and true places."

The usual formula for the *bhujāntara* correction is :

bhujāntara correction = (planet's mean daily motion × Sun's *bhujāphala* in minutes)/21600.

Vaṭeśvara has replaced the divisor 21600 by

21600 + Sun's mean daily motion in minutes.

Vaṭeśvara is correct in this respect because in one civil day the Sun's diurnal motion is

(21600 + Sun's mean daily motion) minutes, and not 21600 mins.

Bhāskara II's formula for the *bhujāntara* correction is :[3]

bhujāntara correction

= [(Sun's *bhujaphala*) × (right ascension of the sign occupied by the Sun)/1800] × (planet's mean daily motion)/21600.

1. See *SiŚe*, iii. 54.
2. See *SiŚi*, I, ii. 26(c).
3. See *SiŚi*, I, ii. 61.

The *udayāntara* correction (i e., the correction due to the obliquity of the ecliptic) has been omitted by Vaṭeśvara. But it has been given by Śrīpati[1] and Bhāskara II[2].

Cara correction

95. The (mean) daily motions of the planets (severally) multiplied by (the *asus* of) the Sun's ascensional difference should be divided by 21600 : by the results obtained the longitudes of the respective planets should be diminished or increased according as the computation is made for sunrise or sunset provided the Sun is in the half-orbit beginning with the sign Aries (i.e., in the northern hemisphere). When the Sun is in the half-orbit beginning with the sign Libra (i.e., in the southern hemisphere), the longitudes of the respective planets should be increased or diminished (in the two cases respectively).[3]

That is, the *cara* correction, or the correction for the Sun's ascensional difference,

$$= \frac{\text{planet's mean daily motion} \times \text{Sun's ascensional difference}}{21600}.$$

Like the early Hindu astronomers, Vaṭeśvara has prescribed four corrections to the Sun and Moon, *viz.* (1) *deśāntara* correction (which has been stated in ch. I, sec. 8), (2) *bhujaphala* correction, (3) *bhujāntara* correction, and (4) *cara* correction.

The mean longitude derived from the *Ahargaṇa* corresponds to mean sunrise at Laṅkā. When the *deśāntara* correction is applied to it, we get mean longitude for mean sunrise at the local equatorial place (i.e., at the place where the local meridian intersects the equator). When the *bhujaphala* correction is applied to it, we get true longitude at mean sunrise at the local equatorial place. When the *bhujāntara* correction is applied to it, we get true longitude at true sunrise at the local equatorial place. When the *cara* correction is applied to it, we get true longitude at true sunrise at the local place.

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1. See *SiŚe*, xi. 1.
2. See *SiŚi*, I, ii. 62-63, 65.
3. *Cf. BrSpSi*, ii. 59; *ŚiDVr*, ii. 19; *MSi*, iii. 18-20; *SiŚi*, I, ii. 53.

TRUE MOTION OF SUN AND MOON

Definition

96. The difference between the true longitudes of the Sun for yesterday and today gives the (Sun's) true daily motion for the day elapsed ; and the difference between the true longitudes of the Sun for tomorrow and today gives the (Sun's) true daily motion for the day to elapse.[1]

Similarly is obtained the true daily motion of the Moon or of the desired planet.[2]

Jīvā-bhukti for Sun

97-98. The (mean) daily motion of a planet diminished by the daily motion of the planet's *mandocca* is (defined as) the daily motion of the planet's *manda-kendra* (or *manda* anomaly). Multiply that by the current Rsine-difference and divide by the first Rsine (i.e., by $56\frac{1}{4}$). Multiply that by the planet's own *manda* epicycle and divide by 360 : (the result is the correction for the planet's meau daily motion, called *mandagatiphala*). Subtract that from the planet's mean daily motion, if the planet's *mandakendra* is in the half-orbit beginning with the anomalistic sign Capricorn. If the planet's *mandakendra* is in the half-orbit beginning with the anomalistic sign Cancer, add that to the planet's mean daily motion. Then too is obtained the true daily motion of the planet (Sun or Moon).[3]

The motion for one's own time occurs at the extremity of the planet's hypotenuse.

That is, in the case of the Sun,

true daily motion = mean daily motion $\pm$

$$\pm \frac{\textit{mandakendragati} \times \text{current Rsine-diff.}}{\text{first Rsine}} \times \frac{\textit{manda}\text{ epicycle}}{360},$$

+ or − sign being taken according as the planet is in the half orbit beginning with the anomalistic sign Cancer or Capricorn.

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1. *Cf. BrSpSi*, ii. 29(c-d); *SiŚe*, iii. 41(c-d); *SiŚi*, I, ii. 36(c-d).
2. See *infra*, sec. 2, vs. 11.
3. *Cf BrSpSi*, ii. 41-42(a-b); *KK*, I, i. 20; *ŚiDVr*, iii, 11; *SiŚe*, iii. 40-41(a-b).

Rationale. Let θ and θ' be the mean anomalies of the Sun, for sunrise today and sunrise tomorrow. Then

true longitude for sunrise today

$$= \text{mean longitude for sunrise today} \pm \frac{\text{Rsin }\theta \times \textit{manda} \text{ epicycle}}{360}, \tag{1}$$

true longitude for sunrise tomorrow

$$= \text{mean longitude for sunrise tomorrow} \pm \frac{\text{Rsin }\theta' \times \textit{manda} \text{ epicycle}}{360} \tag{2}$$

Subtracting (1) from (2), we get

$$\text{true daily motion} = \text{mean daily motion} \pm \frac{(\text{Rsin }\theta' - \text{Rsin }\theta) \times (\textit{manda} \text{ epicycle})}{360}$$

$$= \text{mean daily motion} \pm \frac{(\theta'-\theta).\ \text{current Rsine-difference}}{\text{first Rsine}} \cdot \frac{\textit{manda} \text{ epicycle}}{360}, \text{ approx}$$

Hence the formula stated in the text.

Āryabhaṭa II gives the following formula :[1]

$$\text{Sun's true daily motion} = \text{Sun's mean daily motion} \pm \frac{\text{Sun's mean daily motion} \times \textit{koṭiphala},}{\text{R}},$$

+ or — sign being taken according as the planet is in the half-orbit beginning with the anomalistic sign Cancer or Capricorn.

Bhāskara II has also given this formula.[2] He has rightly called this motion as instantaneous daily motion.

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1. See *MSi*, iii. 15(c-d)-16(a-b).
2. See *SiŚi*, I, ii, 37.

karṇa-bhukti

99. **Or, multiply the daily motion of the planet's own (*manda-*) *kendra* by the radius and divide by the *mandakarṇa*. The daily motion of the (planet's) apogee (*mandocca*) increased by the resulting quantity gives the true daily motion of the planet (Sun or Moon).**

That is :

$$\text{true daily motion} = \text{daily motion of planet's apogee} + \frac{(\theta' - \theta) \times R}{\text{planet's } \textit{mandakarṇa}},$$

θ and θ' being the planet's *mandakendra* for sunrise today and for sunrise tomorrow, respectively.

The rationale of this formula is as follows :

Rationale. Planet's true longitude for sunrise today

$$\fallingdotseq \text{longitude of planet's apogee for sunrise today} + \text{arc}\left(\frac{R\sin\theta \times R}{\textit{mandakarṇa} \text{ for sunrise today}}\right). \quad (1)$$

Planet's true longitude for sunrise tomorrow

$$= \text{longitude of planet's apogee for sunrise tomorrow} + \text{arc}\left(\frac{R\sin\theta' \times R}{\textit{mandakarṇa} \text{ for sunrise tomorrow}}\right). \quad (2)$$

Subtracting (1) from (2), we get

planet's true daily motion for today

$$= \text{daily motion of planet's apogee} + \text{arc}\left(\frac{R\sin\theta' \times R}{\textit{mandakarṇa} \text{ for sunrise tomorrow}}\right) - \text{arc}\left(\frac{R\sin\theta \times R}{\textit{mandakarṇa} \text{ for sunrise today}}\right) \quad (3)$$

$$= \text{daily motion of planet's apogee} + \frac{(\theta' - \theta) \times R}{\textit{mandakarṇa} \text{ for today}}, \quad \text{approx.,} \quad (4)$$

$(\theta' - \theta)$ being the daily motion of the planet's *mandakendra*.

In reducing formula (3) to the form (4), Vaṭeśvara has evidently made the rough approximation that

$$\frac{R}{mandakarṇa} = 1, \text{ approx.}$$

Bhāskara I[1] has prescribed the following formula (in the case of the Sun and Moon) :

$$\text{true daily motion} = \frac{\text{mean daily motion} \times R}{mandakarṇa}.$$

Particular formulae for *jīvābhukti*

100. The current Rsine-difference for the Sun multiplied by 7 and divided by 172 and that for the Moon multiplied by 49 and divided by 40, too, give their motion-corrections, in terms of minutes.[2]

From vss. 97-98 above, we have

planet's motion-correction

$$= \frac{\text{current Rsine-diff.} \times mandakendragati \times manda \text{ epicycle}}{\text{first Rsine} \times 360}.$$

Making substitutions, we have

$$\text{Sun's motion-correction} = \frac{\text{current Rsine-diff.} \times 59' \, 8'' \times 14}{\frac{225}{4} \times 360},$$

neglecting motion of Sun's apogee

$$= \frac{\text{current Rsine-diff.} \times 7}{172} \text{ mins., approx.}$$

Moon's motion-correction

$$= \frac{\text{current Rsine-diff.} \times (13° \, 10' \, 34'' - 6' \, 40'') \times 31{\cdot}50}{\frac{225}{4} \times 360}$$

$$= \frac{\text{current Rsine-diff.} \times 49}{40} \text{ mins., approx.}$$

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1. See *MBh*, iv. 13; *LBh*, ii. 8.
2. Similar rules are stated in *KK*, I, i. 19; *ŚiDVṛ*, ii. 15.

One can easily see that the above formulae will give gross values only, particularly in the case of the Moon. To get better result in the case of the Moon, the author prescribes the rule given below.

Jīvābhukti for Moon

101-104. From the (mean) daily motion of the planet's own anomaly first subtract the traversed and untraversed portions of the elemental arcs (at the two extremities). Then divide the remaining arc by the elemental arc and take the tabular Rsine-differences equal to the quotient of the division in the reverse or serial order, (starting from the current Rsine-difference), according as it is an odd quadrant or an even quadrant. The two (traversed and untraversed) portions of the elemental arcs should then be divided by the elemental arc and multiplied by the Rsine-difference of the traversed and untraversed elemental arcs (respectively). The sum of the two results thus obtained and the Rsine-differences of the intervening elemental arcs should be multiplied by the planet's *manda* epicycle and divided by 360. The result reduced to arc should be added to or subtracted from the planet's mean daily motion. This gives the planet's true daily motion from sunrise yesterday to sunrise today.

In case the end of a quadrant happens to fall inside the arc representing the motion of the planet's anomaly, the Rsine-differences corresponding to the arc lying after the end of the quadrant should be taken in the direct or inverse order, according as the planet is in an odd or even quadrant. In such a case, the corrections due to the arcs lying in those two different quadrants should be calculated separately, and should be applied to the planet's mean longitude differently, positively and negatively, as the case may be : this gives the true daily motion of the planet for the day intervening between sunrise yesterday and sunrise today. Thus, by the method of Rsine-differences taken in the direct or inverse order, one may obtain the true daily motion of the planet (i.e., Moon).

The above method is applicable when the daily motion of the anomaly is large enough, as in the case of the Moon. It is evidently based on the formula stated in vss. 97-98.

This method occurs also in the *Mahā-Bhāskarīya*[1] and the *Laghu-Bhāskarīya*[2] of Bhāskara I. It was criticized by Lalla[3] for the reason that it gave true daily motion for the day elapsed and was unfit for use in the calculations for the current day. Bhāskara II has commended Lalla for the criticism.[4]

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1. iv. 15-17.
2. ii. 11-13.
3. See *ŚiDVr*, iii. 16.
4. See Bhāskara II's commentary on *ŚIDVr*, iii. 16.

Section 2

Correction of Planets under the epicyclic theory

CORRECTION OF MARS, JUPITER AND SATURN

1. From the (mean) longitude of the planet, calculate the *manda-phala* (i.e., equation of the centre) and apply half of it to the mean longitude of the planet in the manner stated before (*vide supra*, ch. 2, sec. 1, vs. 93). From that subtracted from the longitude of the *śīghrocca*, calculate the *śīghraphala* and apply half of it to the corrected mean longitude of the planet as a positive or negative correction according as the (*śīghra-*)*kendra* is in the half-orbit beginning with the sign Aries or in that beginning with the sign Libra. From the longitude of the planet (thus obtained) calculate the *mandaphala* (afresh), and apply the whole of it to the mean longitude of the planet. (Then is obtained the true-mean longitude of the planet.) From that subtracted from the longitude of the *śīghrocca*, calculate, as before, the *śīghraphala* (again), and apply the whole of it to the true-mean longitude of the planet. Then is obtained the true longitude of the planet.[1]

Āryabhaṭa I and his followers have prescribed this method for the three superior planets, Mars, Jupiter and Saturn. The method applicable to the two inferior planets, Mercury and Venus, is stated in the next stanza.

Brahmagupta, Śrīpati and Bhāskara II, have prescribed this method for Mars only but they have prescribed iteration of the process also. See *BrSpSi*, ii. 39(d)-40; *SiŚe*, iii. 36; and *SiŚi*, I, ii. 35(c-d).

Āryabhaṭa II has prescribed this method for all the planets. See *MSi*, iii. 28.

CORRECTION OF MERCURY AND VENUS

2. From the longitude of the planet's own *śīghrocca* as diminished by the longitude of the planet (Mercury or Venus), calculate the *śīghra-phala* and apply the whole of it to the mean longitude in the case of

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1. This method is the same as given in *Ā*, iii. 23; *MBh*, iv. 40-43; *LBh*, ii. 33-36; *ŚiDVr*, iii. 4-7; *TS*, ii. 61-68(a-b).

Mercury and Venus. Then from the corrected mean longitude (of the planet) diminished by the longitude of its own *mandocca*, calculate the *mandaphala* and apply the whole of it to the (corrected) mean longitude.

This method does not agree with that given by Āryabhaṭa I and his followers. The method given by them is :

"Subtract the mean longitude of the planet from the longitude of the planet's *śīghrocca*, and therefrom calculate the *śīghraphala*. Add half of it to or subtract that from the longitude of the planet's *mandocca* according as the *śīghrakendra* of the planet is in the half-orbit beginning with the sign Libra or in that beginning with the sign Aries. Treat this sum or difference as the correct longitude of the planet's *mandocca*. Therefrom calculate the *mandaphala* and apply the whole of it to the planet's mean longitude. This will give the planet's true-mean longitude. Then calculate the planet's *śīghraphala* and apply the whole of it to the planet's true-mean longitude. This will give the true longitude of the planet."[1]

ŚĪGHRAKARṆA AND *MANDAKARṆA* OF THE PLANETS

First Method

3-4. The difference or sum of the *koṭiphala* and the radius, according as the (*śīghra-*) *kendra* is in the half-orbit beginning with the sign Cancer or in that beginning with the sign Capricorn, is here defined as the (true) *koṭi* (true upright).[2] The square-root of the sum of the squares of that and the *bāhuphala* (*bhujāphala*) is the (*śīghra-*)*karṇa*.[3] This is the divisor of the radius multiplied by the (*śīghra-*) *bhujāphala*. The arc corresponding to the quotient is here defined as the *śīghraphala*.[4]

Similarly, one should find out the planet's *mandakarṇa*. This is the multiplier of the *bāhuphala* and the *koṭiphala* ; the radius is the divisor. With the help of these (new *bāhuphala* and *koṭiphala*), the *mandakarṇa* should be calculated again ; and this process should be repeated again and again. In this way, the value (of the *mandakarṇa*) becomes fixed.

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1. See *Ā*, iii. 24; *MBh*, iv. 44; *LBh*, ii. 37(a-b)-39; *ŚiDVr*, iii. 8.
2. *Cf. BrSpSi*, ii. 14; xxi. 27; *SiŚe*, iii. 23(c-d).
3. *Cf. MSi*, iii. 25; *SiŚi*, I, ii. 28(a-b).
4. *Cf. BrSpSi*, xxi. 28; *MSi*, iii. 26; *SiŚe*, iii. 24; *SiŚi*, I, ii. 39.

$$karṇa = \sqrt{[(R \pm koṭiphala)^2 + (bāhuphala)^2]}, \quad (1)$$

+ or − sign being taken according as the *kendra* (anomaly) is in the half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer.

The *śīghrakarṇa* is obtained by applying the formula once; the *manda-karṇa* is obtained by the process of iteration. Also see *infra*, sec. 5, vs. 35(a-b). For details of the process of iteration, see *MBh*, iv. 9-12, and my notes on it. Also see *ŚiDVṛ*, iii. 17.

Second Method

5. **Or, the *karṇa* (hypotenuse) is equal to the square-root of the sum of the difference between the squares of the true *koṭi* and the *koṭi-phala*, and the square of the *antyaphalajyā*. The use of *bhujāphala* has not been made here.**

$$karṇa = \sqrt{[(\text{true } koṭi)^2 - (koṭiphala)^2 + (antyaphalajyā)^2]}. \quad (2)$$

This is equivalent to formula (1). For,

$$(antyaphalajyā)^2 = (bhujāphala)^2 + (koṭiphala)^2.$$

The term *antyaphalajyā* literally means "the Rsine of the maximum correction" and is equal to the radius of the epicycle. (See *infra*, sec. 3, vs. 2).

Third Method

6. **The product of their (i.e., of true *koṭi* and *koṭiphala*) sum and difference, increased by the square of the *antyaphalajyā*, is also the square of the *karṇa*. The square-root of that is the *karṇa*. This (also) does not involve the use of *bāhuphala*.**

$$karṇa = \sqrt{[(\text{true } koṭi + koṭiphala)(\text{true } koṭi - koṭiphala) + (antyaphalajyā)^2]}. \quad (3)$$

This formula is equivalent to formula (2).

Fourth Method

7. **By true *koṭi* minus *bhujaphala* multiply their sum, or take the difference (of their squares). By that increase twice the square of the *bhujaphala* or decrease twice the square of the true *koṭi*. Their square-roots, too, are declared as the values of the *karṇa*.**

$$karṇa = \sqrt{[2\,(bhujaphala)^2 + (\text{true } koṭi + bhujaphala)\,(\text{true } koṭi - bhujaphala)]} \quad (4)$$

$$= \sqrt{[2\,(\text{true } koṭi)^2 - (\text{true } koṭi + bhujaphala)\,(\text{true } koṭi - bhujaphala)]} \quad (5)$$

$$= \sqrt{[2\,(bhujaphala)^2 + [(\text{true } koṭi)^2 - (bhujaphala)^2]\,]} \quad (6)$$

$$= \sqrt{[2\,(\text{true } koṭi)^2 - [(\text{true } koṭi)^2 - (bhujaphala)^2]\,]}. \quad (7)$$

Fifth Method

8. (Severally) multiply the *bhujaphala* and the true *koṭi* by their difference (i.e., by true *koṭi* minus *bhujaphala*); then add the former to and subtract the latter from their squares (i.e., the squares of *bhujaphala* and true *koṭi* respectively) : thus is obtained their product (i.e., the product of *bhujaphala* and true *koṭi*).

Multiply this product by 2, then add to it the square of their difference (i.e., the difference of true *koṭi* and *bhujaphala*), and then take the square-root. This square-root, too, is called the *karṇa*.

$$karṇa = \sqrt{[\,2 \times bhujaphala \times \text{true } koṭi + (\text{true } koṭi - bhujaphala)^2]}, \quad (8)$$

where

$$bhujaphala \times \text{true } koṭi$$

$$= (bhujaphala)^2 + (bhujaphala)\,(\text{true } koṭi - bhujaphala)$$

$$= (\text{true } koṭi)^2 - (\text{true } koṭi)\,(\text{true } koṭi - bhujaphala).$$

Sixth Method

9. Multiply the radius by twice the *koṭiphala*. When the *kendra* is in the half-orbit beginning with the sign Capricorn, add it to the sum of the squares of the radius and the *antyaphalajyā* ; and when the *kendra* is in the half-orbit beginning with the sign Cancer, subtract it (from the sum of the squares of the radius and the *antyaphalajyā*). The square-root of this (sum or difference) is the *karṇa*.[1]

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1. *Cf. SiŚi*, I, ii. 28(c-d)-29.

The difference between the square of the *karṇa* and the sum of the squares of the *antyaphalajyā* (*paraphala*) and the radius, divided by the diameter, is the *koṭiphala*.

$$karṇa = \sqrt{[R^2 + (antyaphalajyā)^2 \pm 2R \times koṭiphala]}, \qquad (9)$$

+ or — sign being taken according as the *kendra* is in the half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer; and

$$koṭiphala = \frac{(karṇa)^2 \sim [(antyaphalajyā)^2 + R^2]}{2R}.$$

Seventh Method

10. (In one place, severally) multiply the *bhujaphala* and the true *koṭi* by their sum ; and (in the other place, severally) multiply the true *koṭi* and the *bhujaphala* by their own difference. Now find the sum and difference of these (two) products, respectively. The square-roots of these (sum and difference), too, are declared as the valnes of the *karṇa*.

$$karṇa = \sqrt{[bhujaphala\ (bhujaphala + \text{true } koṭi) + (\text{true } koṭi)\ (\text{true } koṭi - bhujaphala)]} \qquad (10)$$

$$= \sqrt{[(\text{true } koṭi)\ (bhujaphala + \text{true } koṭi) - bhujaphala\ (\text{true } koṭi - bhujaphala)]}. \qquad (11)$$

TRUE DAILY MOTION OF THE PLANETS

First Method. Definition

11(a-c). In this way calculate the true longitudes of one of the planets for (sunrise) tomorrow and for (sunrise) today. The difference between these two gives the true daily motion (of that planet) for today.[1] If the latter longitude, i.e., the longitude for (sunrise) today, is greater (than the other), the motion is called retrograde.

Second Method

11(c-d)-15. Or, calculate, as before,[2] the planet's *manda-gatiphala*, and apply half of it (to the planet's mean daily motion). Subtract (whatever is obtained) from the daily motion of the planet's *śīghrocca*,

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1. *Cf. BrSpSi*, ii. 29(c-d); *SiŚe*, iii. 41(c-d).
2. See *supra*, ch. 2, sec. 1, vss. 97-98.

in another place. What remains (after subtraction) is known as the (*śīghra*)*kendragati*. Multiply that by the current Rsine-difference corresponding to the arc of the planet's own *śīghrabhujaphala* and by 61[1] and divide by the measure of the (planet's) *śīghrakarṇa*. Subtract whatever is now obtained from the planet's *śīghrakendragati* : then is obtained the *śīghragatiphala*, which is positive or negative. Having applied half of it to the *mandasphuṭagati* reversely, calculate afresh from it, as before, the *mandagatiphala* and apply the whole of it to the mean daily motion of the planet in the manner stated before. From the *śīghroccagati* diminished by that, calculate the *śīghragatiphala* and apply the whole of it (to the *mandaspaṣṭagati*) : then is obtained the *spaṣṭagati* (or the true daily motion of the planet).

In case the *śīghragatiphala*, when it is to be subtracted, cannot be subtracted (from the *mandaspaṣṭagati*), then the *mandaspaṣṭagati* itself should be subtracted from the *śīghragatiphala*. In this case, the remainder gives the retrograde motion.[2]

This method is similar to that of finding the true longitude of the superior planets. See *supra*, vs. 1. In fact it has been derived from that rule on the basis of the rule stated in the first part of vs. 11 above.

The above method has been given for the superior planets by Bhāskara I in his *Mahā-Bhāskarīya*.[3] He has also given its counterpart applicable to the inferior planets.[4] But this counterpart is missing from the manuscripts of the *Vaṭeśvara-siddhānta* that are available to us.

The method stated in the text may be briefly explained as follows :

1. Apply half the *mandagatiphala* to the mean daily motion of the planet.
2. Then apply half the *śīghragatiphala* to the corrected mean daily motion of the planet
3. Now calculate the *mandagatiphala* afresh and apply the whole of it to the mean daily motion of the planet. Then is obtained the so called true-mean daily motion of the planet.

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1. $3438/56\frac{1}{4} = 61$ approx.
2. *Cf. SiŚe*, iii. 43(c-d).
3. See *MBh*, iv. 58-61.
4. See *MBh*, iv. 62-63.

4. Then calculate the *śīghragatiphala* afresh and apply the whole of it to the true-mean daily motion of the planet. Then is obtained the true daily motion of the planet.

The *mandagatiphala* and the *śīghragatiphala* are obtained by the application of the following formulae :

$$\textit{mandagatiphala} = \frac{\textit{mandakendragati} \times \text{current Rsine-diff.}}{\text{first Rsine}} \times \frac{\textit{manda}\text{ epicycle}}{360}.$$

(See *supra*, sec. 1, vss. 97-98)

$$\textit{śīghragatiphala} = \textit{śīghrakendragati} - \textit{spaṣṭaśīghrakendragati},$$

where
$$\textit{spaṣṭaśīghrakendragati} = \frac{\textit{śīghrakendragati} \times \text{current Rsine-diff.}}{56\frac{1}{4}} \times \frac{3438}{\textit{śīghra-karṇa}}$$

$$= \frac{\textit{śīghrakendragati} \times \text{current Rsine-diff.} \times 61}{\textit{śīghrakarṇa}}.$$

See *infra*, sec. 3, vs. 18.

MANDAPHALA AND *ŚĪGHRAPHALA* CORRECTIONS

First Method

16. Or, when the longitude of the planet's *mandocca* diminished by the mean longitude of the planet is defined as the *mandakendra*, and the (mean) longitude of the planet diminished by the longitude of the planet's *śīghrocca* is defined as the *śīghrakendra*, then the *mandaphala* (derived therefrom) should be added to or subtracted from the mean longitude of the planet, according as the planet's (*manda-*) *kendra* is in the half-orbit beginning with the sign Aries or in that beginning with the sign Libra. The *śīghraphala*, on the other hand, should be applied reversely (i.e., it should be added to or subtracted from the true-mean longitude of the planet, according as the *śīghrakendra* is in the half-orbit beginning with the sign Libra or in that beginning with the sign Aries).

Second Method

17. When the planet is in an odd anomalistic quadrant, calculate the *bhujāphala* from the Rsine of the traversed part of that quadrant ;

and when the planet is in an even anomalistic quadrant calculate the *bhujāphala* from the Rsine of three signs and also from the Rversed-sine of the traversed part of that quadrant. If the planet is in the first anomalistic quadrant, the *bhujāphala* (thus calculated) should be subtracted from the (mean) longitude of the planet; if the planet is in the second anomalistic quadrant, the two *bhujāphalas* (for that quadrant) should be respectively subtracted from and added to the (mean) longitude of the planet; if the planet is in the third anomalistic quadrant, the (corresponding) *bhujāphala* should be added to the (mean) longitude of the planet; and if the planet is in the fourth anomalistic quadrant, the two *bhujāphalas* (for that quadrant) should be respectively added to and subtracted from the (mean) longitude of the planet.

18. Or, in the case of the two even quadrants, the *bhujāphala* calculated from the difference of the Rsine of three signs and the Rversed-sine of the traversed part of that quadrant should be subtracted from or added to the (mean) longitude of the planet (according as the quadrant is the second or the fourth). Or else, the difference of the two *bhujāphalas* (one calculated from the Rsine of three signs and the other calculated from the Rversed-sine of the traversed part of that quadrant) should be applied in that way.

(This is the law of correction in the case of *mandaphala*.) In the case of *śīghraphala*, addition and subtraction should be applied reversely.

The above rules are based on the following formulae. If θ is the *manda* or *śīghra* anomaly of a planet, then

$$\begin{aligned} \text{Rsin}\,\theta &= \text{Rsin}\,\theta,\ \text{if}\ \theta \leqslant 90^\circ \\ &= R - \text{Rvers}\,(\theta - 90^\circ),\ \text{if}\ 90^\circ \leqslant \theta \leqslant 180^\circ \\ &= -\,\text{Rsin}\,(\theta - 180^\circ),\ \text{if}\ 180^\circ \leqslant \theta \leqslant 270^\circ \\ &= -R + \text{Rvers}\,(\theta - 270^\circ),\ \text{if}\ 270^\circ \leqslant \theta \leqslant 360^\circ. \end{aligned}$$

Therefore,

$$\begin{aligned} \textit{bhujajyā} &= \text{Rsin}\,\theta,\ \text{in the first quadrant} \\ &= \text{R} - \text{Rvers}\,(\theta - 90^\circ),\ \text{in the second quadrant} \\ &= \text{Rsin}\,(\theta - 180^\circ),\ \text{in the third quadrant} \\ &= \text{R} - \text{Rvers}\,(\theta - 270^\circ),\ \text{in the fourth quadrant.} \end{aligned}$$

Third Method

19. The residue of the (planet's) own anomalistic revolutions (lit. the residue of the revolutions of the planet's epicycle) multiplied by 4 and divided by the number of civil days (in a *yuga*) gives the anomalistic quadrants passed over by the planet (as the quotient). The remainder multiplied by 3 (and divided by the number of civil days in a *yuga*) gives the signs traversed by the planet. (The new remainder multiplied by 30 and divided by the same divisor gives the degrees traversed by the planet; the new remainder multiplied by 60 and divided by the same remainder gives the minutes traversed by the planet). From the signs etc., thus obtained, one should calculate, as before, the *bhuja* and the *koṭi* (of the anomaly).

20. (From the *bhuja* and *koṭi* of the *manda* and *śīghra* anomalies obtained in this way) one should also calculate the *mandaphala* and the *śīghraphala*. (Severally) multiply them by the number of civil days in a *yuga* and divide by the number of minutes in a revolution (i.e., by 21600). Apply the entire minutes (of the two results) thus obtained to the residue of the revolutions of the planet (obtained from the *Ahargaṇa*): then is obtained the true residue (of the revolutions of the planet).

21. (The residue of the revolutions of the planet), which has the number of civil days in a *yuga* for its denominator, should also be corrected for the correction due to the Sun's *bhujāphala* (i.e., *bhujāntara* correction), the correction due to the *asus* of the Sun's ascensional difference (i.e., *cara* correction), and the correction due to the distance of the local place from the prime meridian (i.e., *deśāntara* correction), in the manner stated before. The mean longitudes of the planets, Mars, etc., may be corrected in this way (also).[1]

The *mandaphala* and *śīghraphala* have been multiplied by the number of civil days in a *yuga* and divided by 21600, because

$$\textit{mandaphala} \text{ in minutes} = \frac{\textit{mandaphala} \text{ in minutes}}{21600} \text{ revolutions}$$

$$= \frac{\dfrac{\textit{mandaphala} \text{ in minutes}}{21600} \times \text{civil days in a } \textit{yuga}}{\text{civil days in a } \textit{yuga}} \text{ revolutions.}$$

Similarly, in the case of *śīghraphala*.

1. Same method occurs in *SiŚe*, iii. 56-57.

The term *kudinabhājitaṁ* in verse 21 is used in the sense of *bhagaṇaśeṣaṁ*. *Kudinabhājitaṁ* literally means "a quantity which has *kudina* (civil days in a *yuga*) in its denominator".

Fourth Method

22-23. The residue of the anomalistic quadrants of the planet (see previous rule) is the so called "*gata*" (="traversed part"); and that subtracted from the number of civil days in a *yuga* is the "*gamya*" (="part to be traversed"). These two (*gata* and *gamya*) should be (severally) multiplied by 96 and divided by the number of civil days in a *yuga* : the quotients obtained would give the serial numbers of the tabular Rsines corresponding to the *gata* and the *gamya*, passed over. The remainders of the divisions should be (severally) multiplied by the current Rsine-difference and divided by the number of civil days in a *yuga*; the quotients added to the corresponding Rsines (passed over) give the *bhujajyā* and the *koṭijyā* or the *koṭijyā* and the *bhujajyā* (of the planet's anomaly), depending upon the quadrant. The corresponding *phalas* (i.e., the corrections *mandaphala*, *śīghraphala*, etc.) should be multiplied by the number of civil days in a *yuga* and divided by 21600: the results (thus obtained) should be applied to the residue of the revolutions of the planet (as in the previous rule).[1]

Let A be the *Ahargaṇa*, R the revolutions of the planet's anomaly, and C the number of civil days in a *yuga*. Let

$$\frac{AR}{C} = r + \frac{r_1}{C}\text{ revs.} = r\text{ revs.} + \frac{4r_1}{C}\text{ quadrants}$$

$$= r\text{ revs.} + (Q + r_2/C)\text{ quadrants.}$$

Then r_2 is the residue of the anomalistic quadrants, and

$$gata = r_2$$

$$gamya = C - r_2.$$

Since there are 96 Rsines in a quadrant, therefore the serial numbers of the tabular Rsines corresponding to the *gata* and the *gamya* passed over, are given by the quotients

1. This rule occurs also in *BrSpSi*, xiv. 20-23; *SiŚe*, iii. 55-57(first rule).

$$\frac{96r_2}{C} \text{ and } \frac{96(C-r_2)}{C}.$$

If $96r_2/C = a + r_3/C$ and $96(C-r_2)/C = b + r_4/C$, then

a^{th} Rsine $+ r_3 \times$ current Rsine-difference$/C$

and b^{th} Rsine $+ r_4 \times$ current Rsine-difference$/C$

give the Rsine and Rcosine or the Rcosine and Rsine of the planet's anomaly, depending on whether the quadrant is odd or even.

CORRECTION OF PLANETS DERIVED FROM RISINGS OF A STAR OR RISINGS OF A PLANET

24. When the longitude of a planet is derived from the number of risings of a star or a planet (in a *yuga*), the longitude correction, the *bhujāntara* correction, and the *cara* correction are also determined (lit. stated) on the same basis. (See below)

(i) The longitude correction

25-26. The (star's or planet's) own *udayabhoga* multiplied by the *deśāntara-yojanas* (for the local place) and divided by (the *yojanas* of) the (local) circumference of the Earth (gives the longitude correction. It) should be added or subtracted as before.

The number of Moon's risings in the east (in a *yuga*) is stated to be 1,52,44,84,224; and the motion of the Moon from one of its risings to the next is 818′17″28‴. (Those for the other planets may be obtained.)

By the term *udayabhoga* is meant "the motion of a planet from one rising of the planet to the next", i.e., "the motion of the planet for one civil day of the planet".

The term *deśāntara-yojanas* means "the distance of the local place from the prime meridian, in terms of *yojanas*".

The Moon's risings in a *yuga* and the Moon's motion from one moon-rise to the next are obtained as follows :

revolution-number of asterisms = 1,58,22,37,560 (1)

revolution-number of Moon = 5,77,53,336 (2)

Subtracting (2) from (1), we get

Moon's risings in a *yuga* = 1,52,44,84,224.

Consequently, Moon's motion from one moonrise to the next

$$= \frac{57753336}{1524484224} \text{ revs.}$$

$$= 818'17''28'''.$$

In general, the number of risings of a planet in a *yuga*
= revolution-number of the asterisms − revolution-number of the planet.

(ii) The *bhujāntara* correction

27-28(a-b). Multiply the motions of the planets (from one rising of a heavenly body to the next) by the minutes of the difference between the true and mean positions of the heavenly body for whose rising the planets have been calculated, and divide (each product) by the number of minutes in a revolution as increased by the *udayabhoga* of that heavenly body. When the true position of the heavenly body is greater than its mean position, the results (obtained) should be added to the mean longitudes of the respective planets; when smaller, they should be subtracted from them. This is the *bhujāntara* correction.

Suppose, for example, that the planets are calculated for the time of sunrise, then

the *bhujāntara* correction for a planet

$$= \frac{(\text{daily motion of the planet}) \times (\text{Sun's true position} - \text{Sun's mean position})}{21600 + \text{Sun's daily motion}}.$$

Similarly, when the planets are calculated for the time of rising of the heavenly body P, then

the *bhujāntara* correction for a planet Q

$$= \frac{(\text{motion of the planet } Q \text{ from one rising of } P \text{ to the next}) \times (\text{true position of } P - \text{mean position of } P)}{21600 + udayabhoga \text{ of } P}.$$

If the heavenly body be a star, there will be no difference between its true and mean positions and so the *bhujāntara* correction will be zero. See *BrSpSi*, xiv. 27(c-d).

(iii) The *cara* correction

28(c-d)-29. The motions (of the planets) corresponding to the (civil) day of that heavenly body for whose rising the planets have been calculated should be multiplied by the ascensional difference of that heavenly body and divided by the number of *asus* in a day and night of that heavenly body : the resulting quotients should be subtracted from or added to the longitudes of the respective planets (calculated for the time of rising of that heavenly body) (according as the heavenly body is in the northern or southern hemisphere).

The resulting longitudes being diminished or increased (as the case may be) by the motions of the respective planets for the time-interval between the rising of that heavenly body and the rising of Aśvinī (ζ Piscium), the longitudes correspond to the time of rising of Aśvinī.

Section 3

Correction of Planets under the eccentric theory

INTRODUCTION

1. This (aforesaid) correction of the planets has been stated by the methods prescribed under the epicyclic theory. I shall now describe the correction by the methods prescribed under the eccentric theory.[1]

ANTYAPHALAJYĀ

2. The radius multiplied by the epicycle and divided by 360 gives the *antyaphalajyā* (= Rsine of the maximum correction) or the radius of the epicycle.[2] The arc corresponding to that gives the maximum correction.

KARṆA OR HYPOTENUSE

First Method

3. The sum or difference of the *koṭijyā* and the *antyaphalajyā* according as the *kendra* (i.e., anomaly) is in the half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer (gives the true *koṭi*). The square root of the sum of the squares of that and the *bāhujyā* gives the *karṇa* (i.e., hypotenuse).[3]

$$karṇa = \sqrt{[(\text{true } koṭi)^2 + (bhujajyā)^2]}, \quad (1)$$

$$\text{where true } koṭi = koṭijyā \pm antyaphalajyā,$$

according as the planet is in the anomalistic half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer.

The true *koṭi* is the perpendicular dropped from the planet (situated on the eccentric) upon the line drawn through the Earth's centre at right angles to the apse line. The *karṇa* is the line joining the planet with the Earth's centre. The *karṇa* is thus the hypotenuse of the right-angled triangle which has *bāhujyā* for its base and true *koṭi* for its upright.

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1. *Cf.* *SiŚe*, iii. 48(a-b).
2. *Cf.* *SiŚe*, iii. 48(c-d)-49 (a).
3. *Cf.* *SiŚe*, iii. 49(b-d); *SiŚe*, I, ii. 27.

Second Method

4. Or, find the difference between the squares of the true *koṭi* and the *koṭijyā*, and then add the square of the radius to that or take their difference (according as the planet's *kendra* is in the half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer) : the square root of that is the *karṇa*. This *karṇa* has been obtained without the use of the *bāhujyā*.

That is, when the planet is in the anomalistic half-orbit beginning with the sign Capricorn,

$$karṇa = \sqrt{[((\text{true } koṭi)^2 - (koṭijyā)^2) + (\text{Radius})^2]};$$

and when the planet is in the anomalistic half-orbit beginning with the sign Cancer,

$$karṇa = \sqrt{[(\text{Radius})^2 - ((koṭijyā)^2 - (\text{true } koṭi)^2)]}. \quad (2)$$

Third Method

5. Or, find the product of their sum or difference, and then take the sum or difference of that and the square of the radius, according as the planet's *kendra* is in the half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer : the square-root of that is again the *karṇa*, not involving the use of the *bāhujyā*.

That is, when the planet is in the anomalistic half-orbit beginning with the sign Capricorn,

$$karṇa = \sqrt{[(\text{true } koṭi + koṭijyā)\ (\text{true } koṭi - koṭijyā) + (\text{Radius})^2]};$$

and when the planet is in the anomalistic half-orbit beginning with the sign Cancer,

$$karṇa = \sqrt{[(\text{Radius})^2 - (koṭijyā + \text{true } koṭi)\ (koṭijyā - \text{true } koṭi)]}. \quad (3)$$

Fourth Method

6-7(a-b). Or, multiply the *antyaphalajyā* by twice the *koṭijyā*. When the *kendra* is in the half-orbit beginning with the sign Capricorn, add that (product) to the sum of the squares of the radius and the *antyaphalajyā*; when the *kendra* is in the half-orbit beginning with the

sign Cancer, subtract that from the sum of the squares of the radius and the *antyaphalajyā*. The square-root of that (sum or difference) is again the *karṇa*, not involving the use of the *bāhujyā*.[1]

$$karṇa = \sqrt{[(\text{Radius})^2 + (antyaphalajyā)^2 \pm 2\, koṭijyā \times antyaphalajyā]}, \quad (4)$$

$+$ or $-$ sign being taken according as the planet is in the anomalistic half-orbit beginning with the sign Capricorn or in that beginning with the sign Cancer.

KOṬIJYĀ AND TRUE *KOṬI*

7(c-d)-8. Find the difference between the sum of the squares of the radius and the *antyaphalajyā* and the square of the *karṇa*. Halve whatever is obtained and divide that by the *antyaphalajyā* : the result is the *koṭijyā*.[2]

The square-root of the product of the sum of the *bāhujyā* (lit. the other *jyā*) and the *karṇa*, and the difference of the same, (is the true *koṭi*).

$$(1)\ koṭijyā = \frac{(karṇa)^2 \sim [(\text{radius})^2 + (antyaphalajyā)^2]}{2\,(antyaphalajyā)};$$

$$(2)\ \text{true } koṭi = \sqrt{[(karṇa + bāhujyā)\,(karṇa - bāhujyā)]}.$$

Fifth Method

9. Or, multiply the sum of the *bhujajyā* and the true *koṭi* by the difference of the true *koṭi* and the *bhujajyā*. Subtract that (product) from and add that (product) to twice the square of the true *koṭi* and twice the square of the *bhujajyā* (respectively or otherwise, as the case may be). The square-roots of the two results (in either case) give the *karṇa*.

That is: when true $koṭi > bhujajyā$, then

$$karṇa = \sqrt{[2\,(\text{true } koṭi)^2 - (bhujajyā + \text{true } koṭi)\,(\text{true } koṭi - bhujajyā)]}$$

$$\text{or} \sqrt{[2\,(bhujajyā)^2 + (bhujojyā + \text{true } koṭi)\,(\text{true } koṭi - bhujajyā)]};$$

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1. *Cf. SiŚe*, iii. 50.
2. Same rule occurs in *SiŚe*, iii. 51.

and when true *koṭi* $<$ *bhujajyā*, then

karṇa $=\sqrt{}$[2 (*bhujajyā*)^2 $-$ (*bhujajyā* $+$ true *koṭi*) (*bhujajyā* $-$ true *koṭi*)],

or $\sqrt{}$[2 (true *koṭi*)^2 $+$ (*bhujajyā* $+$ true *koṭi*) (*bhujajyā* $-$ true *koṭi*)]. (5)

Sixth Method

10(a-c). (In one place, severally) multiply the *bhujajyā* and the true *koṭi* by their sum, and (in the other place, severally) multiply the true *koṭi* and the *bhujajyā* by their difference. Take the sum and difference (or difference and sum, as the case may be) of the two results in the respective order. The square-roots of the two results (thus obtained) give the *karṇa*.

That is, when true *koṭi* $>$ *bhujajyā*, then

karṇa $=\sqrt{}$[*bhujajyā* (*bhujajyā* $+$ true *koṭi*) $+$ true *koṭi* (true *koṭi* $-$ *bhujajyā*)],

or $\sqrt{}$[true *koṭi* (*bhujajyā* $+$ true *koṭi*) $-$ *bhujajyā* (true *koṭi* $-$ *bhujajyā*)];

and when true *koṭi* $<$ *bhujajyā*, then

karṇa $=\sqrt{}$[*bhujajyā* (*bhujajyā* $+$ true *koṭi*) $-$ true *koṭi* (*bhujajyā* $-$ true *koṭi*)],

or $\sqrt{}$[true *koṭi* (*bhujajyā* $+$ true *koṭi*) $+$ *bhujajyā* (*bhujajyā* $-$ true *koṭi*)]. (6)

Seventh Method

10(c-d). Or, the *karṇa* is also equal to the square root of twice the product (of the *bhujajyā* and the true *koṭi*) increased by the square of their difference.

karṇa $=\sqrt{}$[(*bhujajyā* $\sim$ true *koṭi*)^2 $+$ 2 *bhujajyā* $\times$ true *koṭi*].

CORRECTION OF THE PLANETS

First Method

11. Multiply the *bhujajyā* by the radius and divide by the *karṇa*. The arc corresponding to the quotient is the correction. This is to be added to the longitude of the *mandocca* and subtracted from the longitude of the *śīghrocca*. Then is obtained the true (longitude of a) planet.[1]

1. *Cf.* *SiŚe*, iii. 52(a).

What is meant is this : Multiply the planet's *mandakendra-bhujajyā* by the radius and divide that by the planet's *mandakarṇa* (obtained by the process of iteration described above in sec. 2, vss. 3-4) : the arc corresponding to the quotient is the correction which added to the longitude of the planet's *mandocca* gives the true longitude in the case of the Sun and Moon and the true-mean longitude in the case of the planets, Mars etc. Now, multiply the planet's *śīghrakendra-bhujajyā* by the radius and divide by the planet's *śīghrakarṇa* : the arc corresponding to the quotient is the correction which subtracted from the longitude of the planet's *śīghrocca* gives the true longitude of the planet, Mars etc.

The addition or subtraction of the correction should be made in the manner stated in the next stanza.

Method of Correction

12. (When the planet is) in the first quadrant, the above correction should be applied as it is; in the second quadrant, after subtracting it from 6 signs; in the third quadrant, after increasing it by 6 signs; and in the fourth quadrant, after subtracting it from 12 signs.[1]

The Four Quadrants

13. If the *koṭijyā* happens to fall beyond the *antyaphalajyā*, the quadrant is the first or the fourth; and if that (*antyaphalajyā*) happens to fall inside the *koṭijyā*, the quadrant is either of the middle ones (i.e., the second or the third).

14. Or, (in the process of calculating the true *koṭi*) if the *antyaphalajyā* has been subtracted, the quadrants are the middle ones; and if the *antyaphalajyā* has been added, the quadrants are the remaining ones (i.e., the first and fourth).

This is how the quadrants are designated. The application of the (planetary) correction is dependent on them. Other things are as stated above.

Second Method

15-17. To the mean longitude of the planet apply half the difference between the longitude of the mean planet and the longitude of the mean planet corrected for the *mandaphala* : when the longitude of the corrected

1. *Cf. SiŚe*, iii. 52(b-d).

mean planet is less than the longitude of the mean planet, subtraction is to be made; when greater, addition is to be made. (What is obtained is called the longitude of the once-corrected planet.)

Half the difference between that (i.e., the longitude of the once-corrected planet) and the same corrected for the *śīghraphala* should then be applied to that (i.e., to the longitude of the once-corrected planet) as a positive or negative correction in the manner stated. (What is obtained is called the longitude of the twice-corrected planet.)

Now, the entire difference between that (longitude of the twice-corrected planet) and the same corrected for the *mandaphala* (calculated afresh) should be applied to the longitude of the mean planet (as before). (What is obtained is called the longitude of the true-mean planet.) Then the entire difference between that (i.e., the longitude of the true-mean planet) and the same corrected for the *śīghraphala* should be applied to that (longitude of the true-mean planet). Then is obtained the longitude of the true planet (or, what is the same thing, the true longitude of the planet).

To Mercury and Venus, two corrections are applied (viz. *śīghraphala* and *mandaphala*); to the Sun and the Moon only one correction is applied (viz. *mandaphala*).

The process described in verses 15-17(a-b) is meant for the superior planets (Mars, Jupiter and Saturn) only and forms the counterpart of the rule stated above in sec. 2, vs. 1. More detailed description of this process is given by Bhāskara I. See *MBh*, iv. 48-51(a-b).

The procedure envisaged for Mercury and Venus is indeed the counterpart of the process described in sec. 2, vs. 2, above.

TRUE DAILY MOTION OF THE PLANETS

First Method

18. Multiply the (*śīghra* or *manda*) *kendra-bhukti* by its own current Rsine-difference and divide by the first Rsine; multiply that by the radius and divide by the (*śīghra* or *manda*) *karṇa*. By the resulting quantity diminish or increase the daily motion of the *śīghrocca* or *mandocca* (respectively): (then is obtained the *spaṣṭagati* or *manda-spaṣṭagati* respectively).

What is meant is this :

"Multiply the *mandakendragati* by the current Rsine-difference and divide by the first Rsine; multiply that by the radius and divide by the *mandakarṇa*. By the resulting quantity increase the daily motion of the *mandocca* : then is obtained the *mandaspaṣṭagati*. Similarly, multiply the *śīghrakendragati* by the current Rsine-difference and divide by the first Rsine; multiply that by the radius and divide by the *śīghrakarṇa*; by the resulting quantity diminish the *śīghroccagati* : then is obtained the *spaṣṭagati* (i.e., true daily motion)."

This method forms the counterpart of the first method for finding the longitude of a planet described in vs. 11 above.

The rationale of this method is as follows :

Let u, θ, and t be respectively the longitude of the planet's *mandocca*, the planet's *manda* anomaly and the planet's true-mean longitude for sunrise today, and u', θ', and t' respectively the longitude of the planet's *mandocca*, the planet's *manda* anomaly and the planet's true-mean longitude for sunrise tomorrow. Then

$$t-u = \frac{\text{Rsin }\theta \times \text{R}}{H}, \text{ approx.,} \tag{1}$$

$$\text{and } t'-u' = \frac{\text{Rsin }\theta' \times \text{R}}{H}, \text{ approx.,} \tag{2}$$

neglecting the variation of the planet's *mandakarṇa* H.

Subtracting (1) from (2), we get

mandaspaṣṭagati of planet – *mandoccagati*

$$= \frac{(\text{Rsin }\theta' - \text{Rsin }\theta) \times \text{R}}{H}$$

$$= \frac{(\theta' - \theta) \times \text{current Rsine-diff.}}{\text{first Rsine}} \cdot \frac{\text{R}}{H}$$

$$= \frac{\textit{mandakendragati} \times \text{current Rsine-diff.} \times \text{R}}{\text{first Rsine} \times \textit{mandakarṇa}}.$$

Therefore,

$$\textit{mandaspaṣṭagati} = \textit{mandoccagati} + \frac{\textit{mandakendragati} \times \text{current Rsine-diff.} \times \text{R}}{\text{first Rsine} \times \textit{mandakarṇa}}$$

Now, let U, ϕ and T be respectively the longitude of the planet's *śīghrocca*, the planet's *śīghrakendra* and the planet's true longitude for sunrise today, and U', ϕ' and T' respectively the longitude of the planet's *śīghrocca*, the planet's *śīghra-kendra* and the planet's true longitude for sunrise tomorrow. Then

$$U - T = \frac{\text{Rsin}\ \phi \times \text{R}}{H'}\text{, approx.,} \qquad (3)$$

$$U' - T' = \frac{\text{Rsin}\ \phi' \times \text{R}}{H'}\text{, approx.,} \qquad (4)$$

neglecting the variation of the planet's *śīghrakarṇa* H'.

Subtracting (3) from (4), we get

śīghroccagati – *spaṣṭagati* of planet

$$= \frac{(\text{Rsin}\ \phi' - \text{Rsin}\ \phi) \times \text{R}}{H'}$$

$$= \frac{(\phi' - \phi) \times \text{current Rsine-diff}}{\text{first Rsine}} \cdot \frac{\text{R}}{H'}$$

$$= \frac{\textit{śīghrakendragati} \times \text{current Rsine-diff.} \times \text{R}}{\text{first Rsine} \times \textit{śīghrakarṇa}}.$$

Therefore,

$$\textit{spaṣṭagati} = \textit{śīghroccagati} - \frac{\textit{śīghrakendragati} \times \text{current Rsine-diff.} \times \text{R}}{\text{first Rsine-diff.} \times \textit{śīghrakarṇa}}.$$

It may be observed that the above method of Vaṭeśvara conforms totally to the eccentric theory. The methods given by Brahmagupta (*BrSpSi*, ii. 41-44), Lalla (*ŚiDVṛ*, iii. 18-19) and Śrīpati (*SiŚe*, iii. 40-42) conform partly to the epicyclic theory and partly to the eccentric theory.

Second Method

19. The differences of these (i.e., the difference between *manda-spaṣṭagati* and *madhyamagati* and the difference between *spaṣṭagati* and *mandaspaṣṭagati*) should be applied, half or full, positively or negatively, as the case may be, to the (mean) daily motion of the planet as in the case of the longitude of a planet (*vide supra*, vss. 15-17): this will give the true daily motion of the planet.

This method forms the counterpart of the second method for finding the true longitude of a planet described above in vss. 15-17.

ŚĪGHRAPHALA AND *MANDAPHALA*

20. **Multiply the *śīghrabhujajyā* (i.e., the Rsine of the *bhuja* corresponding to the *śīghrakendra*) and *mandabhujajyā* (i.e., the Rsine of the *bhuja* corresponding to the *mandakendra*) by their own *antyaphalajyā* (i.e., by *śīghra-antyaphalajyā* and *manda-antyaphalajyā* respectively) and divide by the *śīghrakarṇa* and the radius respectively: the arcs corresponding to the quotients are the *śīghraphala* and *mandaphala* respectively.**

That is :

śīghra-phala or *śīghra* correction

$$= \text{arc}\left\{\frac{\textit{śīghra-bhujajyā} \times \textit{śīghra-antyaphalajyā}}{\textit{śīghrakarṇa}}\right\}$$

and

$$\textit{mandaphala} = \text{arc}\left\{\frac{\textit{mandabhujajyā} \times \textit{manda-antyaphalajyā}}{\text{radius}}\right\}.$$

Antyaphalajyā means "the radius of the epicycle". Likewise, *manda-antyaphalajyā* means "the radius of the *manda* epicycle" and *śīghra-antyaphalajyā* means "the radius of the *śīghra* epicycle." See *supra*, vs. 2.

MEAN PLANET FROM TRUE PLANET

21. **From the longitude of the *śīghrocca* diminished by the longitude of the true planet calculate the *śīghraphala*. Apply the whole of it to the longitude of the true planet, which stands undestroyed, as a positive or negative correction contrarily to the application of the *śīghraphala* correction, and apply the process of iteration: thus is obtained the longitude of the true-mean planet. From that diminished by the longitude of the *mandocca* calculate the *mandaphala*. Apply the whole of it, too, to the longitude of the true-mean planet, which stands undestroyed, as a negative or positive correction contrarily to the rule stated for the *mandaphala* correction, and apply the process**

of iteration : then is obtained the longitude of the mean planet. The other things (such as the *Ahargaṇa* etc.) are to be derived from it.[1]

The first process of iteration prescribed in the above rule can be dispensed with, because the longitude of the true-mean planet may be obtained directly from the longitude of the true planet by using the formula :

long. of true-mean planet = long of true planet $\pm$ *śīghraphala*,

where

$$\textit{śīghraphala} = \frac{\text{Rsin}\,\phi \times \textit{śīghra}\text{ epicycle}}{360} \quad \text{or} \quad \frac{\text{Rsin}\,\phi \times r}{\text{R}},$$

ϕ being the *bhuja* corresponding to the *śīghra* anomaly of the true planet and r the radius of the *śīghra* epicycle, + or − sign being taken according as the *śīghrakendra* is in the half-orbit beginning with the sign Libra or in the half-orbit beginning with the sign Aries.

However, the second process of iteration cannot be avoided in that way, because the *mandakarṇa* and true *manda* epicycle of the planet are themselves obtained by the process of iteration. See Vaṭeśvara's *Gola*, ii. 14.

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1. *Cf. SiŚi*, I, ii. 45.

Section 4

Correction of Planets without using the Rsine table

INTRODUCTION

1. Correction of the planets with the help of the (tabular) Rsines has been duly described by me. I shall now describe that correction without the use of the (tabular) Rsines.

PIṆḌARĀŚI OR SINE OF *BHUJA*

2. Diminish and multiply the degrees of half a circle (i.e., 180) by the degrees of the *bhuja*. Divide that by 40500 minus that; and then multiply (the quotient) by 4. The result is called the *piṇḍarāśi*.

Or, multiply the degrees of the *bhuja* by 180 and diminish that (product) by the square of the degrees of the *bhuja*. Divide that by 10125 minus one-fourth of that (difference). Then (too) is obtained the *piṇḍarāśi*.[1]

The *piṇḍarāśi* is the sine of the *bhuja*. Let θ be the degrees of the *bhuja*. Then, according to the above rule,

$$\sin\theta = \frac{4\,(180-\theta)\,\theta}{40500-(180-\theta)\,\theta}, \quad \text{or} \quad \frac{180\,\theta-\theta^2}{10125-\dfrac{180\,\theta-\theta^2}{4}}.$$

For the rationale of these formulae, the reader is referred to my notes on *MBh*, vii. 17-19.[2]

MANDAPHALA AND *ŚĪGHRAPHALA*

3. (The *piṇḍarāśi*) when multiplied by the Rsine of the maximum correction (i.e., the radius of the *manda* epicycle) gives the Rsine of

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1. *Cf. BrSpSi*, xiv. 23-24; *MBh*, vii. 17-19; *SiŚe*, iii. 17; *GK*, II, p. 80-82, rules 69 and 70.
2. Or see R. C. Gupta, "Bhāskara I's approximation to sine," *IJHS*, vol. 2, No. 2, 1967, pp. 128-134.

the correction (i.e., *mandaphala*) and when multiplied by the radius, gives the Rsine of the *bhuja*; similarly, when multiplied by the other numbers.

One should calculate the *mandaphala* with the help of the degrees of the *bāhu* in the manner stated above; and also the *śīghraphala* with the help of the degrees of its own *bāhu* and *koṭi*, in the same way.

ARC FROM RSINE

4. Here, take one-fourth of the Rsine plus the radius as the divisor of the Rsine multiplied by 10125; or, divide the given Rsine multiplied by 40500 by the sum of the Rsine and the product of the radius and 4.

5. Subtract the quotient (thus obtained) from the square of 90 (i.e., from 8100); and subtract the square root of that from 90. Whatever is obtained as the remainder is the arc or the correction (as the case may be), derived without taking recourse to the (tabular) Rsines.[1]

That is,

$$\theta = 90 - \sqrt{(90^2 - Q)},$$

where
$$Q = \frac{10125.\ \text{Rsin}\,\theta}{\text{Rsin}\theta/4+\text{R}},\ \text{or}\ \frac{40500.\ \text{Rsin}\,\theta}{\text{Rsin}\,\theta+4\,\text{R}}.$$

This formula may be easily derived from the previous rule.

EIGHT TYPES OF PLANETARY MOTION

6-7. When the true longitude of a planet is less than the mean longitude of the planet, add one-half of the difference between the true and mean longitudes of the planet to the longitude of the planet's *śīghrocca* as diminished by the true longitude of the planet; and when the true longitude of the planet is greater (than the mean longitude of the planet), subtract the same (one-half of the difference between the true and mean longitudes of the planet from the longitude of the planet's *śīghrocca* as diminished by the true longitude of the planet). (The result is the planet's corrected *śīghrakendra*).

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1. *Cf. BrSpSi*, xiv. 25-26; *SiŚe*, iii. 18.

In the (successive) signs of this corrected *śīghrakendra*, the planet is "very fast" (in the first sign); "fast" (in the second sign) and "natural or mean" (in the third sign); in the two halves of the next (i.e., fourth) sign, it is "slow" in the first half and "very slow" in the other half; in the next (i.e., fifth) sign it is "retrograde"; and in the next (i.e., sixth) sign, it is "very retrograde".

8. In the (six) signs obtained by subtracting the corrected *śīghrakendra* from a circle (i.e., 360°), the planet is said to have the same motion. But, when the corrected *śīghrakendra* is subtracted from a circle, the "(very) slow" motion is designated as "reretrograde or direct" motion.[1]

The eight varieties of planetary motion contemplated above are : (1) retrograde, (2) very retrograde, (3) reretrograde or direct, (4) very slow, (5) slow, (6) natural or mean, (7) fast, and (8) very fast.[2]

Brahmagupta criticises Āryabhaṭa I for having not mentioned the abovementioned eight varieties of motion. Writes he :

"The statement that Āryabhaṭa knows the eight varieties of planetary motion is not correct."[3]

ŚĪGHRA ANOMALIES FOR RETROGRADE AND DIRECT MOTIONS

9. Mars becomes retrograde when its *śīghrakendra* is 163° ; Mercury, when its *śīghrakendra* is 145°; Jupiter, when its *śīghrakendra* is 126°; Venus, when its *śīghrakendra* is 165°; and Saturn, when its *śīghrakendra* is 113°.

They become direct when their *śīghrakendras* become 360°—163°, 360°—145°, 360°—126°, 360°—165° and 360°—113° (i.e., 197°, 215°, 234°, 195° and 247°) respectively.[4]

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1. *Cf. BrSpSi*, ii. 50-51; *MBh*, iv. *ŚiDVr*, iii. 15; *SiŚe*, iii. 59, 60.
2. *SūSi*, ii. 12.
3. *BrSpSi*, xi. 9(a-b).
4. *Cf. BrSpSi*, ii. 48-49; *ŚiDVr*, iii. 20; *KPr*, iii. 8; *MSi*, iii. 31; *SiŚe*, iii. 58; *SiŚi*, I ii. 41; *KK*, I, ii. 8-17; *SūSi*, ii. 53-54.

The following table gives the *śīghrakendras* for retrograde motion as stated by the various Hindu astronomers.

Table 15. *Śīghrakendras* for retrograde motion.

Planet	*Śīghrakendra*				
	Āryabhaṭa I (*KK*)	Brahmagupta, Lalla, Śrīpati and Bhāskara II	*SūSi*	Āryabhaṭa II	Vaṭeśvara
Mars	164°	163°	164°	163°	163°
Mercury	146°	145°	144°	145°	145°
Jupiter	130°	125°	130°	125°	126°
Venus	165°	165°	163°	166°	165°
Saturn	116°	113°	115°	113°	113°

The term *akuṭila* used in the Sanskrit text means "different from retrograde, or direct". The term used in the same sense by Brahmagupta is *anuvakra*, which means "that (motion) which occurs when retrograde (motion) ends". Lalla uses the term *vakratyāga*, meaning "the end of retrograde motion", and Śrīpati, *avakracāra*, meaning "contrary to retrograde motion."

PERIODS OF RETROGRADE AND DIRECT MOTION

10. The civil days (of duration) of retrograde motion for the planets, beginning with Mars, are 65, 21, 112, 52 and 132 (respectively).

These subtracted from the days of their synodic period, are the days (of duration) of their direct motion.

SYNODIC PERIODS OF PLANETS

11. 780, 116, 399, 584, and 378 are, in days, the synodic periods of the planets, Mars etc., in their respective order.

Table 16. Synodic periods and periods of retrograde and direct motion of the planets according to Vaṭeśvara.

Planet	Synodic period in days	Period in days of retrograde motion	Period in days of direct motion
Mars	780	65	715
Mercury	116	21	95
Jupiter	399	112	287
Venus	584	52	532
Saturn	378	132	246

ŚĪGHRA ANOMALIES OF PLANETS AT RISING IN THE EAST AND SETTING IN THE WEST

12. (The planets, Mars etc.) rise (heliacally) in the east when their *śīghrakendras* amount to 28, 203, 13, 183 and 17 degrees respectively; they set heliacally in the west when their *śīghrakendras* amount to 360−28, 360−203, 360−13, 360−183 and 360−17 degrees (i.e., 332, 157, 347, 177 and 343 degrees) respectively.[1]

The following table gives the *śīghrakendras* at heliacal rising of the planets in the east according to the various Hindu astronomers.

Table 17. *Śīghra* anomalies of heliacal risings of the planets in the east

Planet	*Śīghrakendra* at heliacal rising in the east: Āryabhaṭa I Lalla and Sumati	Brahmagupta and Bhāskara II	Āryabhaṭa II	Śrīpati	Vaṭeśvara
Mars	28°	28°	28°	28°	28°
Mercury	205°	205°	205°	205°	203°
Jupiter	14°	14°	14°	14°	13°
Venus	183°	183°	182°30′	183°	183°
Saturn	20°	17°	17°30′	17°	17°

1. *Cf. KK*, I, ii. 8-17; *BrSpSi*, ii. 52-53; *ŚiDVr*, iii. 22; *MSi*, iii. 32; *SiŚe*, iii. 61-62; *SiŚi*, I, ii. 42-43.

ŚĪGHRA ANOMALIES OF MERCURY AND VENUS AT RISING IN THE WEST

13. In the same way, Mercury and Venus rise in the opposite direction (i.e., in the west) when their *śīghrakendras* are 49 and 24 degrees respectively.[1]

The (number of) days to elapse or elapsed are obtained from the minutes to elapse or elapsed with the help of the motion of the *śīghrakendra* of the planet.

The following table gives the *śīghrakendras* at heliacal rising of Mercury and Venus in the west according to the various Hindu astronomers.

Table 18. *Śīghra* anomalies at heliacal rising of Mercury and Venus in the west

Planet	*Śīghrakendras* at heliacal rising in the west				
	Āryabhaṭa I (*KK*) and Sumati	Brahmagupta, Śrīpati and Bhāskara II	Lalla	Āryabhaṭa II	Vaṭeśvara
Mercury	51°	50°	51°	49°	49°
Venus	24°	24°	23°	20°	24°

PERIODS OF HELIACAL SETTING AND RISING

14. 120, 16, 30, 8 and 36 are the days during which the planets, Mars etc., remain in heliacal setting in the west.[2] **Mercury and Venus are said to remain in heliacal setting in the east for 32 and 75 days respectively.**[3]

15. After 660, 34, 369, 251, and 342 days (since rising in the east in the case of Mars, Jupiter and Saturn and in the west in the case of Mercury and Venus) the planets, Mars etc., again set in the west.[4]

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1. *Cf. KK*, I, ii. 10, 14; *BrSpSi*, ii. 53; *ŚiDVr*, iii. 23; *MSi*, iii. 33-34; *SiŚe*, iii. 62; *SiŚi*, I, ii. 43.
2. *Cf. ŚiDVr*, iii. 25(a-b); *KPr*, iii. 12(a-b).
3. *Cf. ŚiDVr*, iii. 24(c-d).
4. *Cf. ŚiDVr*, iii. 25 (c-d); *KPr*, iii. 12 (c-d).

The following tables give the days during which the planets remain in heliacal setting and rising in the west and east according to Lalla and Vaṭeśvara.

Table 19. Days of heliacal setting in the west, and heliacal rising in the east

Planet	Days of setting in the west		Days of rising in the east	
	Lalla	Vaṭeśvara	Lalla	Vaṭeśvara
Mars	120	120	660	660
Mercury	16	16	37	34
Jupiter	30	30	372	369
Venus	8	8	251	251
Saturn	36	36	342	342

Table 20. Days of heliacal setting in the east, and heliacal rising in the west

Planet	Days of setting in the east		Days of rising in the west	
	Lalla	Vaṭeśvara	Lalla	Vaṭeśvara
Mercury	32	32	37	34
Venus	71	75	251	251

Below we summarise the motion of the planets (*grahacāra*) in the tabular form.

Table 21. Motion of Mars

Śīghrakendra	Phenomenon	Duration
28°	Rises in the east	660 days
163°	Retrograde motion begins	
197°	Retrograde motion ends	
332°	Sets in the west	120 days
28°	Rises in the east	

Synodic period = 780 days

Period of direct motion = 715 days

Period of retrograde motion = 65 days

Table 22. Motion of Mercury

Śīghrakendra	Phenomenon	Duration
49°	Rises in the west	34 days
145°	Retrograde motion begins	
157°	Sets in the west	16 days
203°	Rises in the east	34 days
215°	Retrograde motion ends	
311°	Sets in the east	32 days
49°	Rises in the west	

Synodic period = 116 days

Period of retrograde motion = 21 days

Period of direct motion = 95 days

Table 23. Motion of Jupiter

Śīghrakendra	Phenomenon	Duration
13°	Rises in the east	
126°	Retrograde motion begins	369 days
234°	Retrograde motion ends	
347°	Sets in the west	30 days
13°	Rises in the east	

Synodic period = 399 days

Period of retrograde motion = 112 days

Period of direct motion = 287 days

Table 24. Motion of Venus

Śīghrakendra	Phenomenon	Duration
24°	Rises in the west	
165°	Retrograde motion begins	251 days
177°	Sets in the west	
183°	Rises in the east	8 days
195°	Retrograde motion ends	251 days
336°	Sets in the east	75 days
24°	Rises in the west	

Synodic period = 584 days

Period of retrograde motion = 52 days

Period of direct motion = 532 days

Table 25. Motion of Saturn

Śīghrakendra	Phenomenon	Duration
17°	Rises in the east	342 days (17° to 247°)
113°	Retrograde motion begins	
247°	Retrograde motion ends	
343°	Sets in the west	36 days (343° to 17°)
17°	Rises in the east	

Synodic period = 378 days

Period of retrograde motion = 132 days

Period of direct motion = 246 days.

Section 5

Correction of Planets by the use of *mandaphala* and *śīghraphala* tables

INTRODUCTION

1. The correction of the planets has been explained by me in various ways with the help of *bhujaphala*, *koṭiphala* and *karṇa*. Now I, endowed with a boon, carefully describe the correction (of the planets) by means of (tables of) the *bhuja* (*phalas*) of the anomaly.

MANDAPHALA AND *ŚĪGHRAPHALA* CORRESPONDING TO THE TABULAR RSINE-DIFFERENCES

2(a-b). The tabular Rsine-differences for the elemental arcs of the (*manda* or *śīghra*) anomaly, multiplied by the corresponding epicycle and divided by 360 give the minutes of the arcs of the (*manda* or *śīghra*) *phala*-differences.

Here the author constructs a table of *mandaphala*-differences and a table of *śīghraphala*-differences, corresponding to the elemental arcs of the (*manda* and *śīghra*) anomalies. It is to be noted that in constructing the table of *śīghraphala*-differences use of hypotenuse-proportion is not made because these *śīghraphala*-differences are supposed to correspond to true *śīghra* anomaly. This point will be clear from the rule stated below in vss. 3-4.

In practical calculation, tables of *mandaphala* and *śīghraphala* have been in use amongst Pañcāṅga-makers in India from very early times. Tables giving the *mandaphala* for the Sun and Moon, at the intervals of 15° of the *mandakendra*, are given in the *Pūrva-Khaṇḍakhādyaka*. Tables for the Sun's *mandaphalajyā*, based on the teachings of the *Uttara-khaṇḍa-khādyaka*, at the intervals of 1° of the *mandakendra*, are found in the *Sūryacandra-sāraṇī* (MS No. 1657 of the Akhila Bhāratīya Sanskrit pariṣad, Lucknow). Sumati, in his *Sumati-mahā-tantra*, gives tables of *mandaphala* and *śīghraphala* for all the planets at the equal intervals of 18° in the case of the Sun and Moon and at unequal intervals in the case of

Mars, etc. These are based on the old *Sūrya-siddhānta* or the *Āryabhaṭa-siddhānta*. The same Sumati, in his *Sumati-karaṇa*, gives tables for the *mandaphalas* of the Sun and Moon at the intervals of 1° of the *manda-kendra*, assuming Sun's maximum *mandaphala* = 2°11′16″ and Moon's maximum *mandaphala* = 5°1′45″. Besides these tables, there exist works, called *Sāraṇīs*, which give tables of the various astronomical elements besides the *mandaphala* and *śīghraphala* for the planets.

Tables of *mandaphala* and *śīghraphala* for the Sun and Moon occur also in the Almagest of Ptolemy, but, it must be noted that, they differ from the Hindu tables, as they are based on slightly different theories of planetary motion. For example, in all Hindu tables the *mandaphala* is maximum when *mandakendra* = 90°, but it is not so in the tables given by Ptolemy.

TRUE LONGITUDE OF A PLANET

2(c-d). The *phala*-differences and the fraction of the current *phala*-difference (i.e., residual *phala-difference*), obtained by proportion, corresponding to *manda* and *śīghra* anomalies should be applied to the planet's (mean) longitude again and again (until the true longitude is fixed).

This method of repeated application of the *mandaphala* and *śīghra-phala* corrections has been prescribed by Brahmagupta for the planets Mercury, Jupiter, Venus and Saturn. See *BrSpSi*, ii. 35.

Śrīpati has also prescribed this method for Mercury, Jupiter, Venus and Saturn, but he has slightly modified the method. He applies the *manda-phala* and *śīghraphala* to the mean longitude of the planet; thus he gets the first approximation to the true longitude of the planet. Then he takes the mean of the mean and true longitudes of the planet and treats it as the mean longitude of the planet, and iterates the process. See *SiŚe*, iii. 39.

Bhāskara II also prescribes the above mentioned method for all the star-planets except Mars. In the case of Mars, the first approximation for the true longitude is obtained by applying half *mandaphala* and half *śīghraphala*; but in finding the next successive approximations these corrections are applied in full. See *SiŚi*, I, ii. 35(c-d).

COMPUTATION AND APPLICATION OF *ŚĪGHRAPHALA*

3. In the larger (eccentric) quadrant one should diminish the minutes of the *śīghrabhuja* (i.e., *bhuja* of the *śīghra* anomaly) by the corresponding (*śīghrabhuja*) *phala* : (then one gets the corrected *śīghrabhuja*). Whatever (corrected *śīghrabhuja*) is thus obtained should be divided by the elemental arc. In the shorter (eccentric) quadrant, one should add the same (*śīghrabhujaphala* to the minutes of the *śīghrabhuja*) and divide (the corrected *śīghrabhuja*) by the elemental arc.

4. That (quotient of division of the *śīghrabhuja* by the elemental arc) is the *phala* (i.e., the number of the *śīghraphala*-differences). The remainder (of the division) should be multiplied by the current *śīghraphala*-difference and divided by the interval of the *śīghraphala*-differences (i.e., by the elemental arc) : this gives the additional *śīghraphala* for the shorter quadrant. In the larger quadrant, one should perform the division and obtain the *phala* (i.e., the number of the *śīghraphala*-differences) and the additional *śīghraphala*, in the same way. Therefrom one should calculate the (entire) *śīghraphala*. This should be added to or subtracted from the longitude of the planet as before.

The first and fourth quadrants of the *śīghra* eccentric are the larger ones; the second and third quadrants of the *śīghra* eccentric are the shorter ones.

The above rule tells how to derive the corrected *śīghrabhuja* from the mean *śīghra-bhuja* in the larger and shorter quadrants, and gives the details of finding the *śīghraphala* from the table of the *śīghraphala*-differences and its application to the longitude of a planet.

MANDAPHALA AND *ŚĪGHRAPHALA*

Alternative Method

5. The product of the interval and the number of the *phala*-differences (i.e., *mandaphala*-differences or *śīghraphala*-differences) respectively increased and diminished by the (*antya*)*phala* (i.e., *manda antyaphala* or *śīghra antyaphala*) gives the measures of the larger and shorter quadrants. From these too, by proportion, one may obtain the *kendraphala* (i.e., *mandaphala* or *śīghraphala*).

That is :

$$\text{larger quadrant} = 90^\circ + \textit{antyaphala}$$

and $$\text{shorter quadrant} = 90^\circ - \textit{antyaphala},$$

because

$$(\text{elemental arc}) \times (\text{no. of } \textit{phala}\text{-differences}) = 90^\circ.$$

The formulae for deducing the *mandaphala* and *śīghraphala* from the larger and shorter quadrants contemplated by the author are probably :

$$\textit{mandaphala} = \frac{\textit{manda-antyaphala} \times \textit{mandakendra-bhuja}}{\text{larger or shorter quadrant}},$$

$$\textit{śīghraphala} = \frac{\textit{śīghra-antyaphala} \times \textit{śīghrakendra-bhuja}}{\text{larger or shorter quadrant}},$$

the larger or shorter quadrant being taken in the denominator according as the planet is in the larger or shorter quadrant.

As stated above the larger quadrants are the first and fourth quadrants of the eccentric, and the shorter quadrants are the second and third quadrants of the eccentric. Brahmagupta says :

"Three signs plus the arc of the *antyaphala* (maximum correction) is the measure of the first quadrant of the eccentric; half a circle diminished by that is the measure of the second and third quadrants each; the fourth quadrant is equal to the first.

"The first quadrant of the eccentric extends up to 3 signs as increased by the arc of the *antyaphala*; the third up to 9 signs as diminished by the arc of the *antyaphala*; the second and fourth up to 6 signs and 12 signs respectively."[1]

Similar statements have been made by Śrīpati[2] and Bhāskara II[3] also.

1. *BrSpSi*, xiv. 15-16.
2. See *SiŚe*, iii. 53.
3. See *SiŚi*, I. ii. 34(a-b).

TRUE MOTION OF *MANDA* ANOMALY FOR A RETROGRADE PLANET

6. Multiply the *kalāntara* (i.e., the difference, in minutes, between the planet's *mandakarṇa* and the radius) by the *(manda-)kendragati* and divide (the product) by the planet's (*maṇḍa*) *karṇa*, as stated before;[1] the result is the *mandakendragatiphala* for the retrograding planet. This should be applied to the planet's (*mandakendra*)*gati* as a positive or negative correction reversely to that stated above.[2]

Let H be the *śīghrakarṇa*. Then

$$\textit{spaṣṭa-mandakendragati} = \frac{\textit{mandakendragati} \times R}{H}$$

$$= \textit{mandakendragati} \mp \frac{\textit{mandakendragati}(H \sim R)}{H}$$

$-$ or $+$ sign being taken according as $R \lessgtr H$: and

spaṣṭa-mandakendragati for a retrograde planet

$$= \textit{mandakendragati} \mp \frac{\textit{mandakendragati}\ (H \sim R)}{H}$$

$-$or $+$ being taken according as $H \lessgtr R$.

ŚĪGHRA ANOMALY AT A STATIONARY POINT

First Method

7. Multiply the arc (of the planet's *spaṣṭaśīghrabhujajyā*) corresponding to the smaller quadrant by the minutes of the *śīghrakendragati* and divide (the product) by the *śīghroccagati* : (the result is the *madhyamaśīghrabhuja*) of the planet whose *vakrakendra* (*bhuja*) is equal to that (*madhyamaśīghrabhuja*), the *anuvakrakendra* is equal to six signs plus that (*madhyamaśīghrabhuja*).

Since

$$\textit{spaṣṭaśīghrakendragati} = \frac{\textit{śīghrakendragati} \times R}{H},$$

---

1. Vide *supra*, chap. II, sec. 3, vs. 11.
2. See *supra*, chap. II, sec. 1. vss. 97-98.

where H denotes the *śīghrakarṇa*, therefore when the planet is stationary (i.e., when the planet begins or ends its retrograde motion)

$$\textit{śīghroccagati} = \textit{spaṣṭaśīghrakendragati}$$

$$= \frac{\textit{śīghrakendragati} \times \mathrm{R}}{H}.$$

$$\therefore \quad \frac{H}{\mathrm{R}} = \frac{\textit{śīghrakendragati}}{\textit{śīghroccagati}}.$$

Therefore,

$$\textit{madhyamaśīghrabhuja} = \frac{\textit{spṣṭaśīghrabhuja} \times H}{\mathrm{R}}$$

$$= \frac{\textit{spaṣṭaśīghrabhuja} \times \textit{śīghrakendragati}}{\textit{śīghroccagati}}.$$

Hence, we have

$$\textit{vakrakendra} = 6 \text{ signs} - \frac{\textit{spaṣṭaśīghrabhuja} \times \textit{śīghrakendragati}}{\textit{śīghroccagati}}$$

$$\textit{anuvakrakendra} = 6 \text{ signs} + \frac{\textit{spaṣṭaśīghrabhuja} \times \textit{śīghrakendragati}}{\textit{śīghroccagati}}.$$

Second Method

8. **The (*śīghra*) *kendragati* multiplied by the radius and divided by the *śīghroccagati* gives the *vakrakarṇa* (i.e., the planet's *śīghrakarṇa* when it begins or ends retrograde motion). From that one should calculate, as before,[1] the (*śīghra*) *bhujajyā* and (*śīghra*) *koṭijyā* (and the *śīghrakendragati*). One should then multiply the *śīghrakendragati* by the current Rsine-difference and divide (the product) by the first Rsine: this again gives the (*vakra*) *karṇa* (when multiplied by the radius and divided by the *śīghroccagati* as before). One should perform this cycle of operations again and again until the (*vakra*) *karṇa* is fixed. The arc of the (*madhyamaśīghra*) *koṭijyā* (derived from that)[2] increased by three signs gives the (*śīghra*) anomaly when the planet becomes retrograde from direct.**

1. See *supra*, chap. II, sec. 3, vss. 7(c-d)-8.
2. See vs. 7 above.

Bhujaphaladhanuṣo bhogyajīvā means "the Rsine-difference corresponding to *bhujaphala-dhanuṣ*," i. e., "the Rsine-difference corresponding to the elemental arc in which the planet is situated".

The author uses the following three formulae in the above rule :

(1) $spaṣṭaśīghrakendragati = \dfrac{śīghrakendragati \times R}{H}$,

where H denotes the *śīghrakarṇa*, which reduces to

$$śīghroccagati = \frac{śīghrakendragati \times R}{H},$$

where the planet is stationary and becomes retrograde from direct.

Thus

$$vakrakarṇa = \frac{śīghrakendragati \times R}{śīghroccagati}.$$

(2) $koṭijyā = \dfrac{H^2 \sim (R^2 - antyaphalajyā^2)}{2.\ antyaphalajyā}$

[*vide supra*, chap. II, sec. 3, vss. 7(c-d)-8]

and $bhujajyā = \sqrt{[R^2 - (koṭijyā)^2]}$,

(3) $spaṣṭagati = śīghroccagati - \dfrac{śīghrakendragati \times \text{current Rsine-diff.} \times R}{\text{first Rsine} \times H}$.

[*vide supra*, chap. II, sec. 3, vs. 18; *BrSpSi.* ii. 43-44; *SiŚe*, ii. 42-43]
which reduces to

$$0 = śīghroccagati - \frac{śīghrakendragati \times \text{current Rsine-diff.} \times R}{\text{first Rsine} \times vakrakarṇa},$$

when the planet is stationary and changes from direct motion to retrograde, so that

$$vakrakarṇa = \frac{śīghrakendragati \times \text{current Rsine-diff.} \times R}{\text{first Rsine} \times śīghroccagati}.$$

GRAPHICAL REPRESENTATION OF RETROGRADE MOTION

(1) Stationary Points

9-10. Where the thread stretched from the initial point of Capricorn or Cancer, on the *śīghra* epicycle, to the centre of the Earth meets the (*śīghra*) epicycle, there lies the centre of the planet when it takes up direct or retrograde motion. According to some (astronomers) this happens at the beginning and end of the shorter (*śīghra*) quadrants (i.e., at the beginning of the second and the end of the third *śīghra* quadrants).

I shall now discuss it systematically and in many ways with the help of *koṭis* and *karṇas*.

(2) *Koṭiphala* for Stationary Points

11. By the sum ("*yoga*") of the squares of the radius and the *antyaphalajyā* divide the radius as multiplied by twice the square of the *antyaphalajyā* : what is obtained is the *kuṭilakoṭi-phala* (i.e., *koṭiphala* for a stationary point).

Let R and r denote the radius and the *antyaphalajyā*. Also let k be the *koṭiphala* for a stationary point. Then

(1) $$k = \frac{2r^2 . R}{R^2 + r^2}.$$

(3) *Bāhuphala* for Stationary Points

12. Having subtracted the square of the square of the *antyaphalajyā*, from the square of the product of the radius and the *antyaphalajyā*, multiply the remainder by the sum ("*yoga*") of the squares of the radius and the *antyaphalajyā*.

13. Then having subtracted that (product) from the square of the product ("*vadha*") of the square of the radius and the *antyaphalajyā*, subtract the square root of the remainder from the "*vadha*" (i.e., the product of the square of the radius and the *antyaphalajyā*) ; and divide that (difference) by the "*yoga*" (i.e., the sum of the squares of the radius and the *antyaphalajyā*) : the result is the *kuṭilabāhuphala* (i.e., *bāhuphala* for a stationary point)

(2) $kuṭilabāhuphala = \dfrac{R^2 . r - \sqrt{[(R^2 . r)^2 - ((R. r)^2 - (r^2)^2)(R^2 + r^2)]}}{R^2 + r^2}$,

where R denotes the radius and r the (*śīghra*) *antyaphalajyā*.

The right hand side of (2) simplifies to

$$\frac{r\,(R^2 - r^2)}{R^2 + r^2}$$

and one may write

$$kuṭilabāhuphala = \frac{r\,(R^2 - r^2)}{R^2 + r^2}.$$

(4) *Upakoṭi, Upabhuja, Upakarṇa,* and *Kuṭilakarṇa.*

14-17. The radius minus the *koṭiphala* is the *upakoṭi* (i.e., adjacent *koṭi*) ; the *antyaphalajyā* minus the *bāhuphala* is the *upabhuja* (i.e., adjacent *bhuja*). The square root of the sum of the squares of the *koṭiphala* and that (*upabhuja*) is the *upakarṇa* (i.e., adjacent *karṇa*). The square root of the sum of the squares of the radius and the *antyaphalajyā*, diminished by that (*upakarṇa*), is the *kuṭilakarṇa*. The square root of the sum of the squares of the *upakoṭi* and the *bāhuphala* is also the *kuṭilakarṇa*. The *bhujaphala* multiplied by the *upakarṇa* and divided by the *upabhuja* also gives the *kuṭilakarṇa*. The product of the *upakarṇa* and the *upakoṭi* divided by the *koṭi*(*phala*) is also the *kuṭilakarṇa*. The square root of the sum of the squares of the radius and the *antyaphalajyā* multiplied by the *bhujaphala* and divided by the *antyaphalajyā* is also the *kuṭilakarṇa*.

(3) $upakoṭi = R - koṭiphala$

(4) $upabhuja = antyaphalajyā - bāhuphala$

(5) $upakarṇa = \sqrt{[(koṭiphala)^2 + (upabhuja)^2]}$

(6) $kuṭilakarṇa = \sqrt{(R^2 + r^2)} - upakarṇa$

(7) $kuṭilakarṇa = \sqrt{[(upakoṭi)^2 + (bāhuphala)^2]}$

(8) $kuṭilakarṇa = \dfrac{upakarṇa \times bhujaphala}{upabhuja}$.

(9) $kuṭilakarṇa = \frac{upakarṇa \times upakoṭi}{koṭiphala}$

(10) $kuṭilakarṇa = \frac{\sqrt{(R^2 + r^2)} \times bhujaphala}{antyaphalajyā}$;

where R denotes the radius and r the *antyaphalajyā*.

The following is the rationale of the above formulae :

Rationale. See Fig. 1. The circle centred at E, the Earth, is the mean orbit of the planet called deferent (*kakṣāvṛtta*) and the circle centred at M, the true-mean planet, is the *śīghra* epicycle, U is the *śīghrocca* (apex of fast motion), A is the first point of Cancer on the epicycle, T is the point where the line AE intersects the epicycle, and MT is parallel to EU. Then, according to our author, T is the position of the planet when it abandons direct motion and takes up retrograde motion.

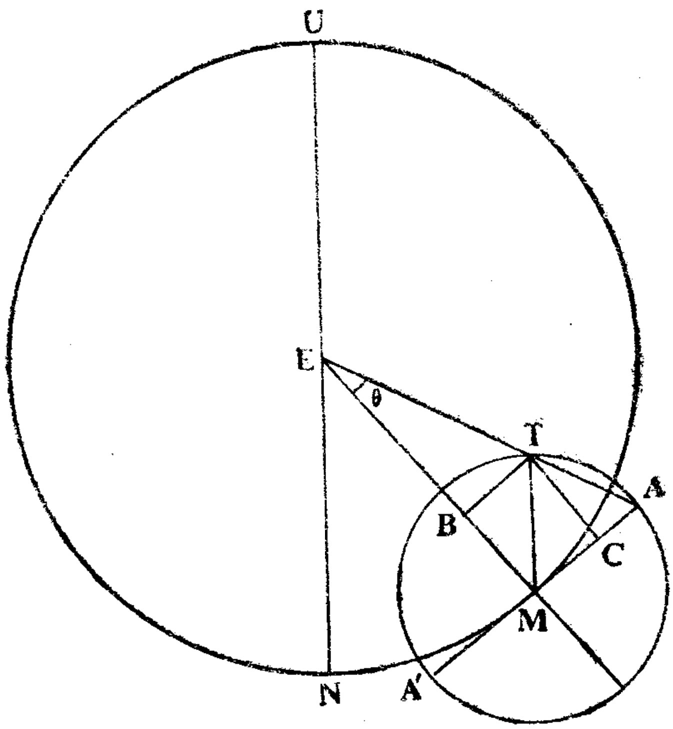


Fig. 1

In Fig. 2, M′ is the new position of M, A′ is the first point of Capricorn, T′ is the point where the line A′E intersects the *śīghra* epicycle, and

M′T′ is parallel to EU. Then, according to the author, T′ is the position of the planet when it abandons retrograde motion and takes up direct motion again.

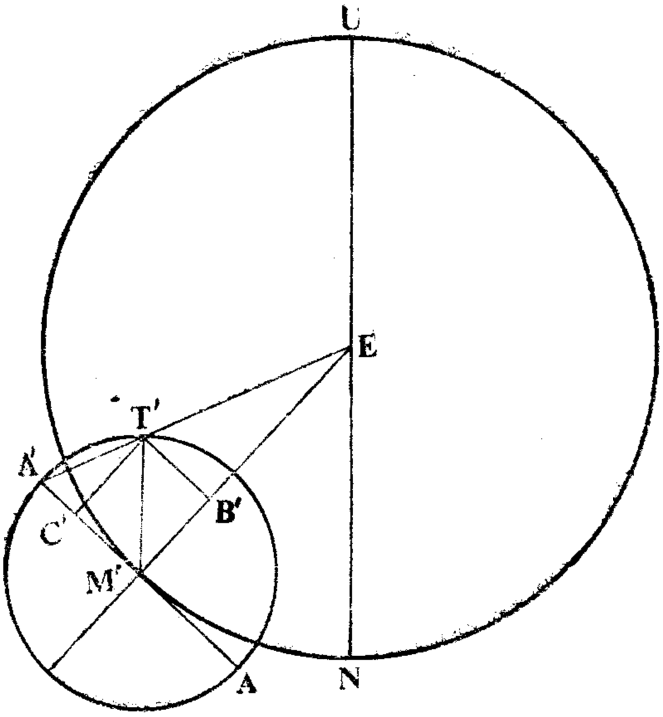


Fig. 2

Again, in Fig 1, let TC be perpendicular to MA and TB perpendicular to EM. Then

ET is the *kuṭilakarṇa* and TA the *upakarṇa*;

CM (=TB) is the *bhujaphala* and AC the *upabhuja* (*phala*);

MB is the *koṭiphala* and BE the *upakoṭi* (*phala*) ; and

TM is the *antyaphalajyā*.

Let R denote the radius of the deferent, r the *antyaphalajyā* (i. e., the radius of the epicycle), and H the *kuṭilakarṇa*. Then, denoting the angle MET by θ, we have

$$\cos\theta = \frac{ME}{AE} = \frac{R}{\sqrt{(R^2+r^2)}}.$$

Now from triangle TME we have

$$TM^2 = ME^2 + TE^2 - 2ME.\ TE.\cos\theta$$

or $$r^2 = R^2 + H^2 - 2R.\ H.\ R/\sqrt{(R^2+r^2)}$$

or $$\sqrt{(R^2+r^2)}.\ H^2 - 2R^2.\ H + (R^2 - r^2)\sqrt{(R^2+r^2)} = 0,$$

giving $H = \frac{R^2 - r^2}{\sqrt{(R^2 + r^2)}}$.

Hence, we have :

(1) *kuṭila-koṭiphala* $= R -$ *upakoṭi*

$$= R - H\cos\theta$$

$$= R - \frac{R^2 - r^2}{\sqrt{(R^2 + r^2)}} \cdot \frac{R}{\sqrt{(R^2 + r^2)}}$$

$$= \frac{2r^2 . R}{R^2 + r^2}.$$

(2) *kuṭilabāhuphala* $= H \sin\theta$

$$= \frac{R^2 - r^2}{\sqrt{(R^2 + r^2)}} \cdot \frac{r}{\sqrt{(R^2 + r^2)}}$$

$$= \frac{r(R^2 - r^2)}{R^2 + r^2}.$$

(3) *upakoṭi* = BE = ME − MB = R − *koṭiphala.*

(4) *upabhuja* = CA = MA − MC = *antyaphalajyā* − *bāhuphala.*

(5) *upakarṇa* = TA = $\sqrt{(TC^2 + CA^2)}$ = $\sqrt{}$[(*koṭiphala*)^2 + (*upabhuja*)^2].

(6) *kuṭilakarṇa* = TE = AE − AT = $\sqrt{(R^2 + r^2)}$ − *upakarṇa.*

(7) *kuṭilakarṇa* = TE = $\sqrt{(BE^2 + TB^2)}$ = $\sqrt{}$[(*upakoṭi*)^2 + (*bāhuphala*)^2].

(8) Since the triangles TEB and ATC are similar, therefore

kuṭilakarṇa TE $= \frac{AT \times TB}{AC} = \frac{upakarṇa \times bhujaphala}{upabhuja}$,

and

(9) *kuṭilakarṇa* TE $= \frac{AT \times BE}{CT} = \frac{upakarṇa \times upakoṭi}{koṭiphala}$.

(10) Since the triangles TEB and AEM are similar, therefore

kuṭilakarṇa TE $= \frac{AE \times TB}{AM} = \frac{\sqrt{(R^2 + r^2)} \times bhujaphala}{antyaphalajyā}$,

and

$$(11)\ \textit{kuṭilakarṇa}\ TB = \frac{AE \times BE}{ME} = \frac{\sqrt{(R^2 + r^2)} \times \textit{upakoṭi}}{R}.$$

Formula (11) has been omitted by our author.

(5) Particular cases of Stationary *Karṇa.*

18. When the *karṇa* is equal to the *bāhujyā*, then the *kendra* (i.e., *śīghra* anomaly) is equal to three signs increased by the *antyaphala* (i.e., maximum *śīghra* correction) ; when the *karṇa* is equal to the radius, then the *kendra* is equal to (three signs) increased by the arc corresponding to half the *antyaphalajyā*.

19. In a shorter quadrant (i.e., second or third quadrant), the *karṇa* when equal to the *koṭijyā*, is equal to the square root of twice the square of the *antyaphalajyā*, as increased by the square of the radius, minus the *antyaphalajyā*.

20. When, in a shorter quadrant, the *karṇa* happens to be equal to the *antyaphalajyā*, then the *koṭijyā* is equal to the Rsine of one sign as multiplied by the radius and divided by the *antyphalajyā*.

In other words :

(1) When *karṇa* = *bāhujyā*, then

$$\textit{śīghra}\ \text{anomaly}\ (\textit{kendra}) = 90^\circ + \textit{antyaphala}.$$

(2) When *karṇa* = radius, then

$$\textit{śīghra}\ \text{anomaly}\ (\textit{kendra}) = 90^\circ + \text{arc}\ (\textit{antyaphalajyā}\ /2).$$

(3) When *karṇa* = *koṭijyā*, then

$$\textit{karṇa}\ \text{or}\ \textit{koṭijyā} = \sqrt{[2\ (\textit{antyaphalajyā})^2 + R^2]} - \textit{antyaphalajyā}.$$

(4) When *karṇa* = *antyaphalajyā*, then

$$\textit{koṭijyā} = \frac{\text{Rsin}\ (30^\circ) \times R}{\textit{antyaphalajyā}} = \frac{R^2}{2\ (\textit{antyaphalajyā})}.$$

The following is the rationale of the above formulae :

Rationale. From Fig. 1, we have

$$\text{ET}^2 = (\text{EM} - \text{MB})^2 + \text{TB}^2$$
$$= \text{EM}^2 + \text{MB}^2 + \text{TB}^2 - 2\text{EM}\times\text{MB}$$
$$= \text{EM}^2 + \text{MT}^2 - 2\text{EM}\times\text{MB},$$

i.e., $(\textit{karṇa})^2 = \text{R}^2 + (\textit{antyaphalajyā})^2 - 2\text{R}\times\textit{koṭiphala}$

$$= \text{R}^2 + (\textit{antyaphalajyā})^2 - 2(\textit{antyaphalajyā})\times\textit{koṭijyā}, \qquad \text{(A)}$$

because $\textit{koṭiphala} = \dfrac{\textit{koṭijyā}\times\textit{antyaphalajyā}}{\text{R}}$.

(1) When *karṇa* = *bāhujyā* = $\sqrt{[\text{R}^2 - (\textit{koṭijyā})^2]}$, then from (A) we have

$$\text{R}^2 - (\textit{koṭijyā})^2 = \text{R}^2 + (\textit{antyaphalajyā})^2 - 2(\textit{antyaphalajyā})\times\textit{koṭijyā}$$

or $(\textit{koṭijyā})^2 - 2(\textit{antyaphalajyā})(\textit{koṭijyā}) + (\textit{antyaphalajyā})^2 = 0$

or $(\textit{koṭijyā} - \textit{antyaphalajyā})^2 = 0$

or *koṭijyā* = *antyaphalajyā*

or Rsin $(90^\circ - \textit{bhuja})$ = Rsin (*antyaphala*).

∴ *bhuja* = 90° − *antyaphala*.

∴ *kendra* = 90° + *antyaphala*.

(2) When *karṇa* = radius, then from (A) we have

$$\text{R}^2 = \text{R}^2 + (\textit{antyaphalajyā})^2 - 2(\textit{antyaphalajyā})(\textit{koṭijyā}).$$

∴ *koṭijyā* = (*antyaphalajyā*)/2

or Rsin $(90^\circ - \textit{bhuja})$ = (*antyaphalajyā*)/2

∴ *bhuja* = 90° − arc (*antyaphalajyā*/2).

∴ *kendra* = 90° + arc (*antyaphalajyā*/2).

(3) When *karṇa* = *koṭijyā*, then (A) gives

$$(\textit{koṭijyā})^2 = \text{R}^2 + (\textit{antyaphalajyā})^2 - 2(\textit{antyaphalajyā})(\textit{koṭijyā})$$

or $(koṭijyā)^2 + 2(antyaphalajyā)(koṭijyā) - [R^2 + (antyaphalajyā)^2] = 0$

$\therefore\ koṭijyā = \sqrt{[2(antyaphalajyā)^2 + R^2]} - antyaphalajyā.$

(4) When *karṇa* = *antyaphalajyā*, then from (A) we have

$(antyaphalajyā)^2 = R^2 + (antyaphalajyā)^2 - 2(antyaphalajyā)(koṭijyā),$

whence

$$koṭijyā = R^2/2(antyaphalajyā)$$

$$= R\sin(30°) \times R\ /\ (antyaphalajyā).$$

(6) Other particular cases of the *Karṇa*

21. When the remainder obtained by diminishing the radius by the *antyaphalajyā* be not less than the *antyaphalajyā*, the *karṇa* will be equal to that (*antyaphalajyā*) or greater than that.

22. When the planet is situated at its *ucca*, the *karṇa* is equal to the sum of the radius and the *antyaphalajyā*. When the *karṇa* is equal to their difference, the anomaly (*kendra*) is exactly equal to 6 signs.

The statement in vs. 22 is obvious and needs no explanation. That in vs. 21 may be explained as follows:

When $R - antyaphalajyā \geqslant antyaphalajyā$, then

$$antyaphalajyā \leqslant R/2.$$

so that $koṭiphala \leqslant R/2$.

Therefore from (A) we have

$$(karṇa)^2 = R^2 + (antyaphalajyā)^2 - 2R \times koṭiphala$$

$$\geqslant R^2 + (antyaphalajyā)^2 - 2R.R/2$$

$$\geqslant (antyaphalajyā)^2.$$

$\therefore\ karṇa \geqslant antyaphalajyā.$

COMPUTATION OF RETROGRADE MOTION

23-24. Obtain the product of the *manda* epicycle and the daily motion of the planet's *mandakendra* and divide that by 360 (lit. degrees in the circle of asterisms). Subtract that from or add that to the

planet's (mean) daily motion (according as the planet is in the half-orbit beginning with the *manda* anomalistic sign Capricorn or in the half-orbit beginning with the *manda* anomalistic sign Cancer). Find the difference between that and the daily motion of the planet's *śīghrocca*, multiply that by the radius and divide by the *śīghrakarṇa* which has been already obtained. Take the difference between that and the daily motion of the planet's *śīghrocca* : whatever is thus obtained is the direct or retrograde (true daily) motion of the planet.

At the commencement of retrograde motion as well as at the commencement of re-retrograde motion, the velocity of the planet is zero.

Let M, M′; t, t′; and T, T′ be the mean longitudes, true-mean longitudes and true longitudes of a planet at sunrise today and at sunrise tomorrow respectively. Then, denoting the *bhujas* of the *manda* anomalies at sunrise today and at sunrise tomorrow by θ and θ', we have

$$t = M \pm \frac{\text{Rsin}\,\theta \times \textit{manda}\text{ epicycle}}{360}, \text{ approx.}$$

$$t' = M' \pm \frac{\text{Rsin}\,\theta' \times \textit{manda}\text{ epicycle}}{360}, \text{ approx.}$$

$\therefore$ *mandaspaṣṭagati* $= t' - t$

$$= (M' - M) \pm \frac{(\text{Rsin}\,\theta' - \text{Rsin}\,\theta) \times \textit{manda}\text{ epicycle}}{360}, \text{ approx.}$$

$$= \text{mean daily motion} \pm \frac{(\theta' - \theta) \times \textit{manda}\text{ epicycle}}{360}, \text{ approx.}$$

$$= \text{mean daily motion} \pm \frac{\textit{mandakendragati} \times \textit{manda}\text{ epicycle}}{360}, \text{ approx.},$$

+ or − sign being taken according as the planet is in the half-orbit beginning with the *manda* anomalistic sign Cancer or Capricorn.

Now, if U and U′ be the longitudes of *śīghroccas* and K and K′ the *śīghrakendras* at sunrise today and at sunrise tomorrow respectively, then

$$T = U - \textit{spaṣṭa śīghrakendra} \text{ at sunrise today}$$

$$= U - K \times R/H$$

$$T' = U' - \textit{spaṣṭa śīghrakendra} \text{ at sunrise tomorrow}$$

$$= U' - K' \times R/H,$$

neglecting the difference of the *śīghrakarṇas* at sunrise today and at sunrise tomorrow.

Therefore

$$spaṣṭagati = (U' - U) - \frac{(K' - K) \times R}{H}$$

$$= śīghroccagati - \frac{śīghrakendragati \times R}{H}$$

$$= śīghroccagati - \frac{(śīghroccagati - mandaspaṣṭagati) \times R}{H}, \quad (1)$$

where H is the planet's *śīghrakarṇa*.

When the expression on the right hand side of (1) is positive the motion is direct, otherwise retrograde.

BHUJAJYĀ AND *KOṬIJYĀ*

Case 1. When their sum is given.

25. To half the sum of the *bhujajyā* and the *koṭijyā* add the square root of the difference of (i) half the difference between the squares of the radius and the sum of the *bhujajyā* and the *koṭijyā* and (ii) the square of half the sum of the *bhujajyā* and the *koṭijyā*. This gives one of the two *jīvās* (*bhujajyā* or *koṭijyā*).

That is, if b and k denote the *bhujajyā* and the *koṭijyā* respectively, then

$$\frac{b+k}{2} + \sqrt{\left[\left(\frac{b+k}{2}\right)^2 - \tfrac{1}{2}[(b+k)^2 - R^2]\right]} = b \text{ or } k,$$

according as $k \lesssim b$.

This is evident, because

$$\sqrt{\left[\left(\frac{b+k}{2}\right)^2 - \tfrac{1}{2}[(b+k)^2 - R^2]\right]} = \sqrt{\left[\frac{(b+k)^2}{4} - \tfrac{1}{2}[b^2 + k^2 + 2bk - R^2]\right]}$$

$$= \sqrt{[(b+k)^2/4 - bk]}, \text{ because } b^2+k^2=R^2$$

$$= \frac{b \sim k}{2}$$

and $\frac{b+k}{2} + \frac{b \sim k}{2} = b$ or k, according as $k \lesseqgtr b$.

Case 2. When their difference is given.

26. Multiply the square of the radius by 2, then diminish that by the square of the difference between the *bhujajyā* and the *koṭijyā*, and then take the square root (of that). Severally increase and diminish that by the difference between the *bhujajyā* and the *koṭijyā*, and divide them by 2. Then are obtained the *bhujajyā* and the *koṭijyā* (separately). From them one may obtain the *karṇa*.

That is, if b and k denote the *bhujajyā* and the *koṭijyā* respectively, then

$$b = \frac{\sqrt{[2R^2 - (b-k)^2]} + (b-k)}{2}$$

and

$$k = \frac{\sqrt{[2R^2 - (b-k)^2]} - (b-k)}{2};$$

or,

$$k = \frac{\sqrt{[2R^2 - (k-b)^2]} + (k-b)}{2}$$

and

$$b = \frac{\sqrt{[2R^2 - (k-b)^2]} - (k-b)}{2}.$$

The above formulae are obvious, because the radical in each case is equal to $b+k$.

Case 3. When their sum and difference are given

27. The sum of twice the halves of the (two given) quantities is the sum of the (*bhuja*) *jyās*; the other (viz. the sum of the *koṭijyās*) is zero (as the *koṭijyās* cancel in the process of addition). The difference (of the half-quantities) is the *koṭijyā*. The half-quantities (themselves) may be obtained from the hypotenuse, *bhujajyā* and *koṭijyā* (i.e., from the sides of a right-angled triangle).

Let *bhujajyā* + *koṭijyā* = a and *bhujajyā* − *koṭijyā* = b. Then

$$2\ bhujajy\bar{a} = a + b, \quad \text{or} \quad bhujajy\bar{a} = \frac{a}{2} + \frac{b}{2}$$

$$\text{and} \quad ko\d{t}ijy\bar{a} = \frac{a}{2} - \frac{b}{2}$$

Also, $\frac{a}{2} = \sqrt{\left[\frac{R^2}{2} - \left(\frac{b}{2}\right)^2\right]}$ and $\frac{b}{2} = \sqrt{\left[\frac{R^2}{2} - \left(\frac{a}{2}\right)^2\right]}$.

ŚĪGHRA ANOMALY AT PLANET'S HELIACAL RISING

28. The *kālaliptās* for visibility or invisibility (i. e., the *asus* of the limits of visibility or invisibility) of a planet multiplied by 30 and divided by the *asus* of rising of the sign occupied by the planet's own *udayalagna*[1] in case the rising takes place in the east, or by the *asus* of rising of the sign occupied by the planet's own *astalagna*[2] in case the rising takes place in the west, (give the degrees of the ecliptic which rise during the planet's *kālaliptās*). These increased by the degrees (of the planet's *śīghraphala*) give the planet's *śīghrakendra* (at the time of its rising in the east), (and the same subtracted from 360° gives the planet's *śīghrakendra* at the time of its setting in the west). (Thus are obtained the *śīghrakendras* at the time of heliacal rising of Mars, Jupiter and Saturn.)

29. In the case of Mercury and Venus, when they are in swift (i. e., direct) motion, the degrees of the planet's true *bhuja* should be obtained with the help of the Rsine of the above result (viz. the Rsine of the degrees of the ecliptic which rise during the *kālaliptās* for the planet) and they should be increased by the degrees of the ecliptic which rise during the *kālaliptās* for the planet, and when they are in retrograde motion, the true *bhuja* of the planet (as increased by 180°) should be diminished by the degrees of the ecliptic which rise during the *kālaliptās* for the planet. Then is obtained the *śīghrakendra* at the time of heliacal rising of Mercury and Venus.

Let l be the measure of the arc of the ecliptic which rises during the *kālaliptās* for the planet. Then :

(1) *śīghrakendra* for the time of heliacal rising of Mars, Jupiter and Saturn $= l +$ planet's *śīghraphala*;

(2) *śīghrakendra* for the time of heliacal rising of Mercury and Venus (when they are in direct motion) $=$ planet's true *bhuja* $+ l$;

(3) *śīghrakendra* for the time of heliacal rising of Mercury and Venus (when they are in retrograde motion) $= 180° +$ planet's true *bhuja* $- l$.

---

1. The *udayalagna* of a planet is that point of the ecliptic which rises when the planet rises.
2. The *astalagna* of a planet is that point of the ecliptic which rises when the planet sets.

Sudhakara Dvivedi gives the following rules :

"Divide the (*śīghra*) *antyaphalajyā* by the radius and multiply by the *kālāṁśajyā*. Reduce the resulting Rsine to arc and to that add the *kālāṁśa* : the sum thus obtained is the *śīghrakendra* of the planet Mars, Jupiter or Saturn at the time of its heliacal rising in the east.

Multiply the radius by the *kālāṁśajyā* and divide by the (*śīghra*) *antyaphalajyā*. The arc corresponding to that increased by the *kālāṁśa* gives the *śīghrakendra* of Mercury or Venus at the time of its heliacal rising in the west. The same arc increased by 180° and diminished by the *kālāṁśa* is the *śīghrakendra* of Mercury or Venus at the time of its heliacal rising in the east.[1]

The *śīghrakendras* for heliacal rising subtracted from 360° give the degrees of the *śīghrakendras* for their setting in the opposite direction."[2]

The rationale of the above rules is as follows :

1. *Rising in the east of Mars, Jupiter and Saturn.* See Fig. 1.

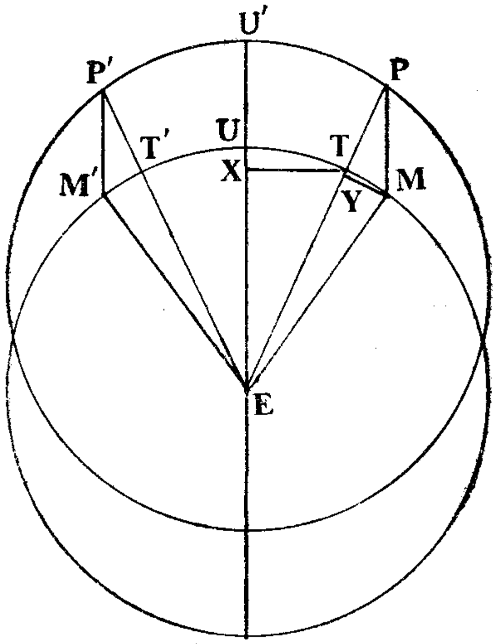


Fig. 1

1. Mercury and Venus rise in the west when they are in direct motion and in the east when they are in retrograde motion.

2. त्रिज्याविभक्तान्त्यफलज्यकेह कालांशजीवागुणिताऽऽप्तचापम् ।
कालांशयुक्तं चलकेन्द्रमैन्द्र्युद्गमे भवेदीज्यकुजार्कजानाम् ॥
कालांशजीवागुणिता त्रिभज्या विभाजिता स्वान्त्यफलज्ययैव ।
कालांशयुक्तं च तदीयचापं परोदये स्याच्चलकेन्द्रमानम् ॥
ज्ञशुक्रयोश्चक्रदलान्वितं तच्चापं तथा काललवोनितं स्यात् ।
चलाख्यकेन्द्रं बुधशुक्रयोर्वै पूर्वोदयेऽथोदयकेन्द्रहीनैः ॥
चक्रांशकैस्तैश्चलकेन्द्रभागैर्ग्रहाः परस्यां दिशि यान्ति चास्तम् ।

Let the circle centred at E be the mean orbit of the planet called concentric (*kakṣāvṛtta*) and the other circle the *śīghra* eccentric (*śīghra-prativṛtta*).

U is the position of the *śīghrocca* (i. e., the mean Sun) on the concentric, M is the mean (true-mean) position of the planet and T the true position at the time of its rising.

TU is equal to the *kālāṁśa* (or more correctly the portion of the ecliptic that rises during the *kālāṁśa*). The arc MU is the required *śīghrakendra*.

In the right-angled triangle EXT and PYM, we have

$$YM/PM = XT/ET, \quad \text{or } YM = XT \times PM/ET,$$

$$\text{or Rsin MT} = \frac{\textit{śīghrāntyaphalajyā} \times \text{Rsin UT}}{R}.$$

Therefore,

$$\text{arc MU} = \text{arc UT} + \text{arc MT} = \text{arc UT} + \text{arc (Rsin MT)}$$

$$= \textit{kālāṁśa} + \text{arc}\left[\frac{\textit{śīghrāntyaphalajyā} \times \textit{kālāṁśajyā}}{R}\right],$$

or arc MU = arc UT + arc MT = l + planet's *śīghraphala*.

Note. The planets Mars, Jupiter and Saturn are slower than the Sun, so they are visible in the morning before sunrise. They remain visible until they set in the west.

When the planet Mars, Jupiter or Saturn sets its true position is T′ and its mean (true-mean) position is M′ such that arc UM′ = arc UM. Hence at the time of setting the *śīghrakendra* of the mean (true-mean) position of the planet = 360° − arc MU.

2. *Rising in the west of Mercury and Venus.* This happens when Mercury and Venus are in direct motion. See Fig. 2.

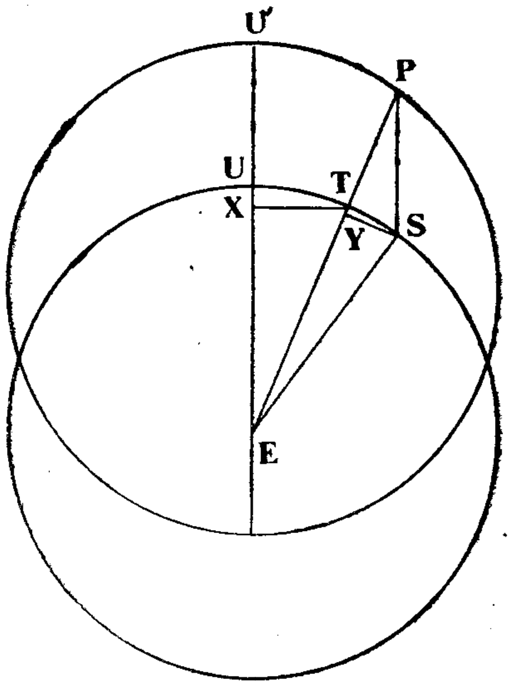


Fig. 2

At the time of rising, U is the *śīghrocca*, S the mean position and T the true (true-mean) position of the planet. Since the point S is also the mean position of the Sun, therefore arc ST is the *kālāṁśa*. We require arc US, the *śīghrakendra* of S.

From the right-angled triangles EXT and PYS, which are similar, we have

TX = SY × ET/PS = *kālāṁśajyā* × R/*śīghrāntyaphalajyā*,

or, Rsin UT = *kālāṁśajyā* × R / *śīghrāntyaphalajyā*.

Therefore,

arc US = arc ST + arc UT

= *kālāṁśa* + arc (*kālāṁśajyā* × R/*śīghrāntyaphalajyā*)

or arc US = arc UT + arc ST = planet's true *bhuja* + *l*.

Note. The planet will set in the east when the *śīghrakendra* of its mean position is equal to 360° — arc US.

3. *Rising in the east of Mercury and Venus.* This happens when they are in retrograde motion. See Fig 3.

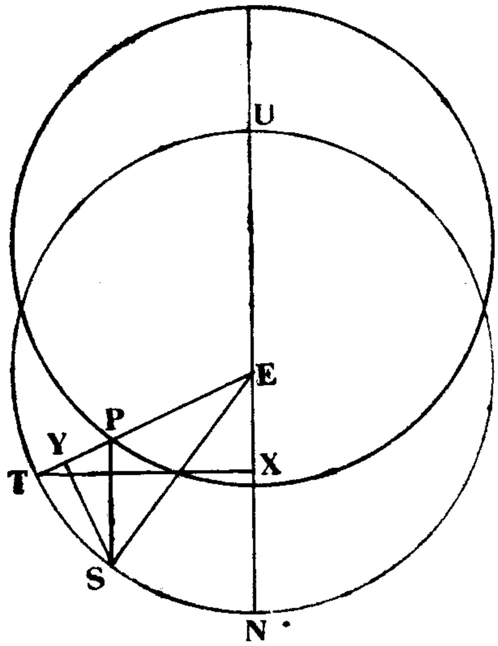


Fig. 3

At the time of rising, S is the mean position of the planet and also the mean position of the Sun. T is the true (true-mean) position of the planet, and TS is the *kālāṁśa*. We require arc UNS, the *śīghrakendra* of the mean position of the planet.

From the right-angled triangles EXT and PYS, which are similar, we have

TX = SY × ET / PS

or Rsin NT = *kālāṁśajyā* × R / *śīghrāntyaphalajyā*

∴ arc NT = arc (*kālāṁśajyā* × R / *śīghrāntyaphalajyā*)

∴ arc UNS = arc UN + arc NS

= 180^0 + arc NT — arc ST

= 180^0 + arc (*kālāṁśajyā* × R / *śīghrāntyaphalajā*) — *kālāṁśa*,

or arc UNS = arc UN + arc NT — arc TS = 180^0 + planet's true *bhuja* — *l*.

Note. For setting in the west, the *śīghrakendra* is

180^0 — arc NS

or 180^0 — arc (*kālāṁśajyā* × R / *śīghrāntyaphalajyā*) + *kālāṁśa*, which is the same as 360^0 — arc UNS.

Putumana Somayāji (c. 1732 A.D.), author of the *Karaṇa-paddhati*, gives expressions for the *śīghrakarṇas* of the planets when they rise or set.[1]

PERIODS OF HELIACAL SETTING AND RETROGRADE MOTION

30. The minutes of arc between the *śīghrakendras* for setting and rising (of a planet) divided by the daily motion of the *śīghrakendra* (of the planet) give the days (during which the planet remains heliacally set). In the same way, from the difference in minutes of arc between the *śīghrakendras* for retrograde and re-retrograde motions (of a planet) one may obtain the days of the planet's retrograde motion.

That is, if k, k' denote the *śīghrakendras*, in minutes, for setting and rising respectively, and K, K' the *śīghrakendras*, in minutes, for the beginnings of retrograde and re-retrograde motions respectively, then the planet remains set for

$$(k'-k) / m \text{ days}$$

and retrograde for

$$(K'-K) / m \text{ days},$$

m being the daily motion of the planet's *śīghrakendra*.

PERIODS OF HELIACAL RISING

31. The number of civil days in a *yuga* divided by the revolution-number of the planet's *śīghrakendra* gives the days of the planet's synodic period. These (days) diminished by the days of planet's setting give the days of rising. In the case of Mercury and Venus, the days of rising (or setting) are the combined days of rising (or setting) in the east and west taken together.

Synodic period of a planet, in terms of days,

$$= \frac{\text{civil days in a } yuga}{\text{revolution-number of planet's } \acute{s}\bar{\imath}ghrakendra}$$

1. *See KP*, vii. 30-32.

$$= \frac{\text{civil days in a } yuga}{\text{rev.-no. of planet's } \acute{s}\bar{\imath}ghrocca - \text{rev.-no. of planet}},$$

days of planet's rising = synodic period in days — days of planet's setting.

In the case of Mercury and Venus :

days of planet's rising = days of planet's rising in the east + days of planet's rising in the west,

and

days of planet's setting = days of planet's setting in the east + days of planet's setting in the west.

Lalla gives a method for finding the days elapsed since or to elapse before a planet sets or rises heliacally or becomes retrograde.[1]

COMPUTATION OF PLANET'S TRUE LONGITUDE MISCELLANEOUS METHODS

Method 1. Brahmagupta's method

32. Apply the entire *mandaphala* to the mean longitude of the planet. Then apply the entire *śīghraphala* obtained from that (i. e., the corrected mean longitude) subtracted from the longitude of the planet's *śīghrocca*.

From that calculate the *mandaphala* afresh and apply (the whole of it) to the mean longitude of the planet, and to this corrected longitude apply the *śīghraphala* as calculated from that (i. e., from the corrected longitude) subtracted from the longitude of the planet's *śīghrocca*. Repeat the process until the longitude is fixed. Then is obtained the true longitude of the planet.

This method is the same as given in vs. 2 above. It has been prescribed by Brahmagupta, Śrīpati and Bhāskara II for Mercury, Jupiter, Venus and Saturn. See *BrSpSi*, ii. 35; *SiŚe*, iii. 39; *SiŚi*, I, ii. 35(c-d).

Method 2. Āryabhaṭa I's method for inferior planets

33. Apply half the *śīghraphala* reversely to the longitude of the planet's *mandocca*. Subtract that from the mean longitude of the planet

1 . See *ŚiDVr*, iii. 26.

and calculate the *mandaphala*; apply the whole of it to the mean longitude of the planet. Subtract that from the longitude of the planet's *śīghrocca* and calculate the *śīghraphala*; and apply the whole of it to the corrected longitude. Thus are obtained the true longitudes of the two (inferior) planets (Mercury and Venus).

This method is applicable to Mercury and Venus and is the same as prescribed by Āryabhaṭa I and his followers. See *Ā*, iii. 24; *LBh*, ii. 37(c-d)-39; *ŚiDVṛ*, iii. 8. Also see *MBh*, iv. 44, where the same method is stated in a slightly different way.

Method 3. Method of *Sūrya-siddhānta*

34. Apply one-half of the *śīghraphala* to the mean longitude of the planet; and to the resulting longitude apply one-half of the *mandaphala*. (From the longitude thus obtained, calculate the *mandaphala* and) apply the whole of the *mandaphala* to the mean longitude of the planet; (from that calculate the *śīghraphala* and) apply the whole of the *śīghraphala* to that. Then is obtained the true longitude of the planet.

This method agrees with that given in *SūSi*, ii. 44; *KK*, I, ii. 18; *SiTV*, *spaṣṭādhikāra*, 247.

COMPUTATION OF *MANDAKARṆA*

35(a-b). Multiply the so called *mandaphala* (i.e., the *mandabhuja-phala*) by the *karṇa* and divide by the radius. (Taking the quotient, thus obtained, as the *bhujaphala*, calculate the *karṇa* again; and taking this as the *karṇa*, repeat the process. Taking the resulting quantity again as the *bhujaphala*, calculate the *karṇa* again; and taking this as the *karṇa*, repeat the same process.) Continue this process of iteration until one *karṇa* becomes equal to the next one. (The *karṇa* which is thus obtained by iteration is the true *mandakarṇa*.)

This rule is the recapitulation of the rule stated in sec. 2, vs. 4(b-d).

Method 4. Unknown author's method

35 (c-d). First calculate the *mandaphala* from the mean longitude of the planet; then the remaining (*śīghraphala*) correction from the corrected *śīghrocca* (i. e., from the *śīghrocca* as diminished by the corrected mean longitude).

To the mean longitude of the planet apply the *mandaphala*: (the result is the true-mean longitude of the planet). Subtract that from the longitude of the planet's *śīghrocca* and therefrom calculate the *śīghraphala* and apply that to the true-mean longitude of the planet. (Then is obtained the true longitude of the planet.)

This is the same method as stated in vs. 32 above, but iteration is not prescribed here.

Regarding this method, astronomer Lalla writes :

"Some (astronomers) say that Mercury and Venus should be corrected for *mandaphala* calculated from the planet's mean longitude diminished by the longitude of the planet's *mandocca* and for *śīghraphala* calculated from the longitude of the planet's *śīghrocca* diminished by that of the planet, each correction being applied once." (*ŚiDVṛ*, iii. 9.)

In the case of Mercury and Venus, Vaṭeśvara too recommends the single application of the two corrections but he has reversed the sequence of application of the two corrections. He prescribes first the application of the *śīghraphala* and then the application of the *mandaphala*. See *supra*, sec. 2, vs. 2.

COMPUTATION OF *ŚĪGHRAKARṆA* WITHOUT USING *KOṬI*

36. **The *śīghrakarṇa* may be obtained, without using the *koṭi* by dividing the product of the *śīghrāntyaphalajyā* and the *śīghrakendrabhujajyā* by the *śīghraphalajyā*;[1] and the radius by dividing the product of the *mandakendrabhujajyā* and the *mandāntyaphalajyā* (by the *mandaphalajyā*).**

That is,

$$\textit{śīghrakarṇa} = \frac{\textit{śīghrakendrabhujajyā} \times \textit{śīghrāntyaphalajyā}}{\textit{śīghraphalajyā}},$$

$$\text{and radius} = \frac{\textit{mandakendrabhujajyā} \times \textit{mandāntyaphalajyā}}{\textit{mandaphalajyā}}.$$

The rationale of these formulae is as follows :

1. Same rule occurs in *KK*, I, viii. 2 (a-b).

In Fig. 1, the circle centred at E, the Earth, is the concentric, the other equal circle is the *śīghra* eccentric, and U the *śīghrocca.* M is the true-mean planet, M′ the true planet, and MB is the perpendicular on EM′.

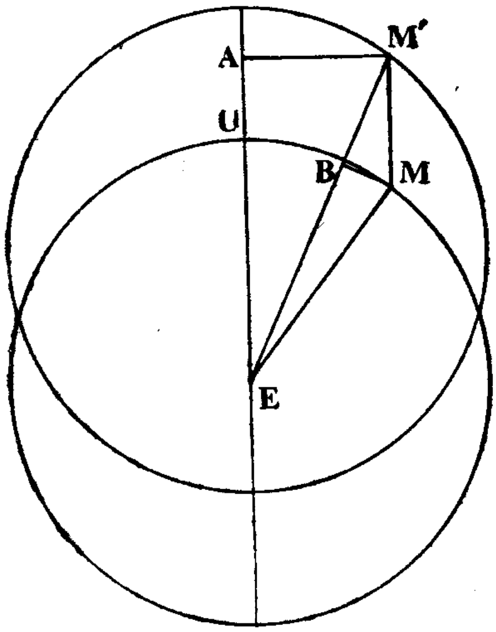


Fig. 1

Comparing the triangles EM′A and M′BM, which are similar, we have

$$EM' = \frac{MM' \times M'A}{BM}, \qquad (1)$$

where

EM′ = *śīghrakarṇa,*

MM′ = *śīghrāntyaphalajyā*

M′A = *śīghrakendrabhujajyā* ,

and BM = *śīghraphalajyā.*

Hence formula (1).

Next consider Fig. 2. Here the circle centred at E is the concentric, the other equal circle is the *manda* eccentric, and U is the *mandocca.* M′B is the perpendicular on EM produced and MA the perpendicular on EU.

Comparing the triangles MM′B and EMA, which are similar, we have

$$EM = \frac{MM' \times MA}{M'B}, \qquad (2)$$

where EM = radius R,

MM′ = *mandāntyaphalajyā*,

MA = *mandakendrabhujajyā*,

and M′B = *mandaphalajyā*.

Hence formula (2).

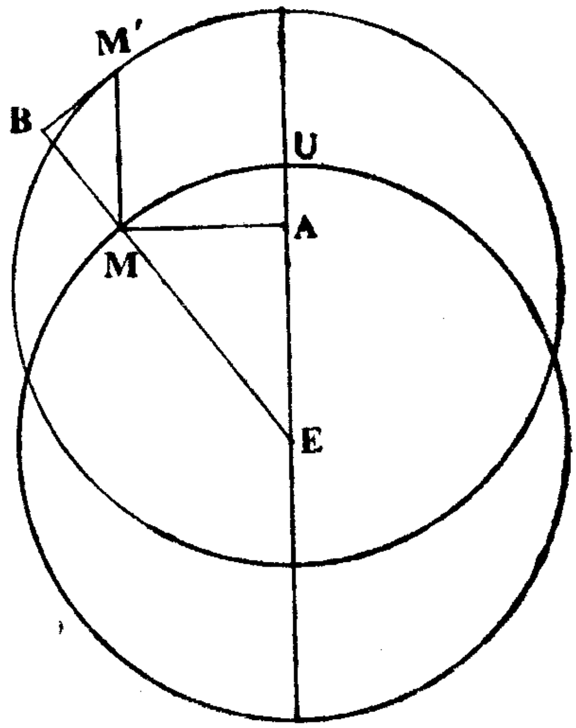


Fig. 2

Section 6 : Elements of the *Pañcāṅga*

CALCULATION OF *TITHI*

1. Subtract the Sun's longitude from the Moon's longitude and divide the resulting difference, reduced to degrees, by 12. The quotient gives the number of *tithis* elapsed (since new moon). (The remainder of the division gives the elapsed part of the current *tithi* and the remainder subtracted from 12 degrees gives the unelapsed part of the current *tithi*.) The elapsed and unelapsed parts of the current *tithi* when multiplied by 60 and divided by the difference, in degrees, between the daily motions of the Moon and the Sun give the *nāḍīs* elapsed since the beginning of the current *tithi* and the *nāḍīs* to elapse before the end of the current *tithi* (respectively).[1]

CALCULATION OF *NAKṢATRA*

2. Reduce the longitude of the planet to degrees. Multiply them by 3 and divide by 40. The resulting quotient gives the number of *nakṣatras* (beginning with Aśvinī) traversed by the planet. The traversed and the untraversed parts of the current *nakṣatra* when multiplied by 20 and divided by the daily motion of the planet (in terms of minutes) give the time, in terms of days etc., elapsed since the planet crossed into the current *nakṣatra* and the time to elapse before the planet crosses into the next *nakṣatra*, (respectively).[2]

There are 27 *nakṣatras* in the whole zodiac of 21600 minutes, so that 1 *nakṣatra* contains 800 minutes. Therefore the number of *nakṣatras* in D degrees

$$= \frac{D}{800/60} = \frac{60\,D}{800} = \frac{3\,D}{40}\,. \qquad (1)$$

The use of the multiplier 20 has been made because the numerator and denominator in (1) have been abraded by 20.

1. *Cf. KK*, I, i. 22 ; *BrSpSi*, ii. 62 : *ŚiDVr*, ii. 22 ; *MSi*, iii. 40 ; *SiŚe*, iii. 71 ; *SiŚi*, I, ii. 66.
2. *Cf. KK*, I, i. 21 ; *BrSpSi*. ii. 61 ; *ŚiDVr*, ii. 23 (a-b) ; *MSi*, iii. 40 ; *SiŚe*, iii. 75 ; *SiŚi*, I, ii. 77.

Exactly the same rule occurs in *Siddhānta-śekhara*, iii. 75.

TRUE LENGTHS OF *NAKṢATRAS*

3. This is gross ; the accurate (determination) is being given now. First I state the *nakṣatras* whose *bhogas* (*bhoga* = extent, length or measure) are described as *adhyardha* (i. e., one and a half times the mean daily motion of the Moon), *sama* (i. e., equal to the mean daily motion of the Moon) and *ardha* (i. e., one-half of the mean daily motion of the Moon), particularly, the true *bhoga* of (*nakṣatra*) Abhijit.

4-5. Rohiṇī, the three Uttarās (i. e., Uttarā Phālgunī, Uttarāṣāḍha, and Uttara Bhādrapada), Viśākhā, and Punarvasu are designated as *Adhyardhabhogī* (i. e., those having their *bhoga* equal to one and a half times the mean daily motion of the Moon, i. e., 1185′ 52″) ; Śatabhiṣak, Āśleṣā, Ārdrā, Svātī, Bharaṇī and Jyeṣṭhā are called *Ardhabhogī* (i. e., those having their *bhoga* equal to half the Moon's mean daily motion, i. e., 395′ 17·5″) ; the remaining (fifteen) *nakṣatras* are called *Samabhogī* (i. e., those having their *bhoga* equal to the Moon's mean daily motion, i. e., 790′ 35″).

The *samabhoga* is equal to the mean daily motion of the Moon (i. e., 790′ 35″) ; that increased by one-half of itself (i. e., 1185′ 52″) is called *adhyardhabhoga* ; and one-half of the Moon's mean daily motion (i. e., 395′ 17·5″) is called *ardhabhoga*.[1]

The abovementioned division of the zodiac into unequal *nakṣatras* is very ancient. According to Brahmagupta,[2] it occurred in the *Siddhāntas* of Pitāmaha, Sūrya, Vaśiṣṭha, Romaka and Puliśa. Śrīpati[3] has ascribed it to the ancient Sages (Maharṣis).

The Jainas who based their astronomy on the *Paitāmaha-siddhānta* and the *Vedāṅga-Jyautiṣa* have also divided the zodiac into unequal *nakṣatras* in the same way. They have used the term *dvyardha-kṣetra* in place of *adhyardha-bhoga*, *sama-kṣetra* in place of *sama-bhoga*, and *apārdha-kṣetra* in place of *ardha-bhoga*. The Jainas divided their zodiac into 54900 parts

1. *Cf. KK*, II, i. 7-10; *BrSpSi*, xiv. 47-49 (a-b); *SiŚe*, iii. 79-80 (a-b); *SiŚi*, I, ii. 71-73.
2. See *BrSpSi*, xiv. 47. Also see *KK*, II, i. 6 (c-d), where it is ascribed to the *Paitāmaha-siddhānta*.
3. See *SiŚe*, iii. 78.

(called *gaganakhaṇḍas*) and measured the *nakṣatras* in terms of these parts, as well as in terms of *muhūrtas*. The number of *muhūrtas* in a sidereal month, according to them, is $819\frac{27}{67}$.

LENGTH OF ABHIJIT

6-7. The sum of the *bhogas* of the (twenty seven) *nakṣatras* subtracted from 360° gives the *bhoga* of Abhijit.[1]

Or, divide the civil days in a *yuga* by the revolution-number of the Moon, and diminish the result by 27 days : the remainder is the *bhoga* of Abhijit, in terms of *ghaṭīs*, etc.

Or, multiply the Moon's revolution-number by 27; subtract (the resulting product) from the number of civil days in a *yuga*; mnltiply (the resulting difference) by the Moon's mean daily motion and divide by the Moon's revolution-number : the result is the *bhoga* of Abhijit, in terms of minutes etc.

Or, 21600 minutes being divided by the Moon's mean daily motion, the remainder is called the *bhoga* of Abhijit.[2]

The *bhoga* of Abhijit is thus equal to 4° 14′ 15″, or 19 *ghaṭīs* 18 *vighaṭīs*, or $9\frac{24}{60}$ *muhūrtas*, approx. According to the Jainas, it is equal to $9\frac{27}{67}$ *muhūrtas*.

CALCULATION OF TRUE *NAKṢATRA*

8. (Reduce the longitude of the desired planet to minutes.) Subtract the *bhogas* of as many *nakṣatras* (Aśvinī etc.) as can be subtracted from (those minutes of) the longitude of the planet. (The remainder is the traversed portion of the *nakṣatra* being traversed by the planet, i. e., of the current *nakṣatra*. The same remainder subtracted from the *bhoga* of the current *nakṣatra* gives the untraversed portion of the current *nakṣatra*.) Divide the traversed and untraversed minutes of the current *nakṣatra* by the mean daily motion of the planet (in terms of minutes) : then are obtained the days etc. elapsed since the planet entered into the

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1. *Cf. KK*, II, i. 11 (a-b); *SiŚe*, iii. 80 (c-d); *SiŚi*, I, ii. 74 (a-b).
2. *Cf. BrSpSi*, xiv. 50-52; *SiŚe*, iii. 80 (c-d)-81.

current *nakṣatra* and those to elapse before the planet moves out of the current *nakṣatra*, (respectively).[1]

SITUATION OF *NAKṢATRA* ABHIJIT

9. Abhijit lies in the last quarter of Uttarāṣāḍha and in the initial 4 *ghaṭīs* (=1/15 part) of Śravaṇa.[2] **One taking his birth in that *nakṣatra* dies before long.**

Jātasya iṣṭaṁ kṛtaṁ bhavati literally means "if the *iṣṭākāla* of the newly born child has been made", so that the sense here is "if the birth of a child has taken place".

CONJUNCTION OF MOON WITH THE *NAKṢATRAS*

10. The six *nakṣatras* beginning with Revatī (viz. Revatī, Aśvinī, Bharaṇī, Kṛttikā, Rohiṇī, and Mṛgaśirā), the twelve *nakṣatras* beginning with Ārdrā (viz. Ārdrā, Punarvasu, Puṣya, Āśleṣā, Maghā, Pūrvā Phālgunī, Uttarā Phālgunī, Hasta, Citrā, Svātī, Viṣākhā and Anurādhā) and the nine *nakṣatras* beginning with Jyeṣṭhā (viz. Jyeṣṭhā, Mūla, Pūrvāṣāḍha, Uttarāṣāḍha, Śravaṇa, Dhaniṣṭhā, Satabhiṣak, Pūrva Bhādrapada and Uttara Bhādrapada) have their conjunction with the Moon in the initial half, central half, and the last half of the *nakṣatra*, (respectively).[3]

CALCULATION OF *KARAṆA*

11. Subtract the longitude of the Sun from the longitude of the Moon and reduce the difference to degrees. Divide them by 6 and subtract 1 from the quotient and then divide that by 7: the remainder (of this second division) gives the number of *karaṇas* (that have elapsed since Baba). (The remainder of the first division, which is in terms of degrees, denotes the elapsed part of the current *karaṇa*; and the same remainder subtracted from 6° gives the unelapsed part of the current *karaṇa*.) Reduce the elapsed and unelapsed parts of the current *karaṇa* to minutes and divide them by the degrees of the difference between the daily motions of the

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1. Same rule occurs in *SiŚe*, iii. 82; *SiŚi*, I, ii. 74 (c-d)-75.
2. The same occurs in *ŚiDVṛ*, xi. 10 (c-d). A similar rule is quoted by Makkibhaṭṭa in his comm. on *SiŚe*, iii. 82. Also see *Jyotiścandrārka*, i. 136; *Vidyāmādhavīya*, ii. 64 (a-b)
3. Also see *BṛSaṁ*, iv. 7.

Sun and the Moon : then are obtained the *nāḍīs* elapsed since the beginning of the current *karaṇa* and the *nāḍīs* to elapse before the end of the current *karaṇa*, (respectively).[1]

IMMOVABLE *KARAṆAS*

12. (There are four immovable *karaṇas* known as Śakuni, Catuṣpada, Nāga and Kiṁstughna.) Śakuni falls in the second half of Kṛṣṇa Caturdaśī (i.e., in the second half of the fourteenth *tithi* in the dark half of a lunar month); Catuṣpada, in the first half of Amāvāsyā (*kuhū*); Nāga, in the second half (of the same); and Kiṁstughna, in the first half of Śukla Pratipad (i. e., in the first half of the first *tithi* in the light half of a lunar month).[2]

CALCULATION OF *YOGA*

13. Add the longitudes of the Sun and the Moon and reduce the sum to minutes. Divide them by 800 : the quotient gives the number of *yogas* elapsed (since Viṣkambha). (The remainder of the division gives the elapsed part of the current *yoga* and the same subtracted from 800 minutes gives the unelapsed part of the current *yoga*.) Multiply the elapsed and unelapsed parts of the current *yoga* by 60 and divide (the products) by the sum of the daily motions of the Sun and Moon (in terms of minutes) : the results are the *nāḍīs* elapsed since the beginning of the current *yoga* and the *nāḍīs* to elapse before the end of the current *yoga*, (respectively).[3]

VYATIPĀTA AND *VAIDHṚTA*

14-15(a-b). The *pāta* called *Vyatipāta*, which is malignant like the poison produced by the combination of equal quantities of clarified butter and honey, occurs when the sum of the longitudes of the Sun and the Moon is 180°, the *ayanas* (of the Sun and the Moon) are different and the declinations (of the Sun and the Moon) are equal. Similarly, the (*pāta* called) *Vaidhṛta* occurs when the sum of the longitudes of the Sun and the Moon is equal to 360°, the declinations (of the Sun and the

1. For other rules see *KK*, I, i. 24; *BrSpSi*, ii. 66; *ŚiDVṛ*, ii. 24; *SiŚe*, iii. 77; *SiŚi*, I, ii. 66.
2. *Cf. KK*, I, i. 23; *BrSpSi*, ii. 65; *ŚiDVṛ*, ii. 25; *SiŚe*, iii. 83.
3. *Cf. BrSpSi*, ii. 63; *ŚiDVṛ*, ii. 23 (c-d); *MSi*, iii. 40; *SiŚe*, iii. 76.

Moon) are equal and the *ayanas* (of the Sun and the Moon) are the same.[1]

The longitudes to be used are obviously tropical, i.e., those corrected for precession of the equinoxes.

DAYS ELAPSED OR TO ELAPSE

15(c-d)-16(a-b). From the minutes of the defect or excess divided by (the minutes of) the sum of the daily motions (of the Sun and the Moon) are obtained the days and *nāḍīs* (to elapse before or elapsed since the occurrence of *Vyatipāta* or *Vaidhṛta*). The longitudes of the Sun, Moon and the Moon's ascending node should be increased or decreased by their own motions for those days and *nāḍīs* (according as the *pāta* is to occur or has occurred). (Thus are obtained the longitudes of the Sun, Moon and the Moon's ascending node at the stipulated time of occurrence of the *pāta*).[2]

POSSIBILITY OF OCCURRENCE OF *PĀTA*

16(c-d)-17. The Moon being at the *ayanasandhi* and the Moon's latitude and declination being of unlike directions, if the (Moon's) declia-tion minus the (Sun's) greatest declination is less than the Moon's latitu-de, there is absence of *pāta*; in the contrary case, it takes place.

The Moon's *ayanasandhi* lies 35° to the east of the Sun's *ayanasan-dhi*, upwards.

The points where the ecliptic intersects the equator are the Sun's *golasandhi* points and the points of the ecliptic which lie at the distances of 90° from these points are the Moon's *ayanasandhi* points.

Similarly, the points where the Moon's orbit crosses the equator are the Moon's *golasandhi* points and the points of the Moon's orbit which lie at the distances of 90° from these points are the Moon's *ayanasandhi* points.

Bhāskara II writes:[3]

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1. *Cf. KK*, I, i. 25; *BrSpSi*, xiv. 33-34; *ŚiDVr*, xii. 1; *SiŚe*, viii. 1-2.
2. *Cf. KK*, I, i. 25; *BrSpSi*, xiv. 35; *ŚiDVr*, xiii, 2; *SiŚe*, viii. 4.
3. See *SiŚi*, I, xii. 3-5, commentary.

"The *golasandhi* of the Sun indeed lies at the intersection of the ecliptic and the equator; that of the Moon, at the intersection of the Moon's orbit and the equator, because the Moon moves in its own orbit. When it is situated there, then and then only it rises in the east. What is meant is this: When the Moon is situated there, the Moon's declination as corrected for its latitude is zero. When it is three signs ahead or three signs behind, its true declination is a maximum. When the Moon is situated there, it attains the maximum point of its northerly or southerly course and turns back ; hence these are the *ayanasandhis* of the Moon."

The instruction in the latter half of stanza 17 shows that at the time of composition of this work the Moon's *ayanasandhi* was 35° to the east of the Sun's *ayanasandhi.*

Stanzas 16(c-d) and 17 (a-b) are important as the instruction contained in them compares with that given by Bhāskara II in his *Siddhānta-śiromaṇi* (I, xii. 7). According to Bhāskara II, when the Moon is at its *ayanasandhi*, then so long as

Moon's true declination < Sun's declination

equality of declination of the Sun and Moon cannot happen. Vaṭeśvara has taken the Moon at its *ayanasandhi* and the Sun at its own *ayanasandhi* and his condition is

Moon's declination − Sun's greatest declination < Moon's latitude

i.e., Moon's true declination < Sun's greatest declination

= Sun's declination.

SPECIAL INSTRUCTION FOR DECLINATION

18. When the declinations (of the Sun and the Moon) are of like directions, there is *Vyatipāta*; when they are of unlike directions, there is *Vaidhṛta*.

When the direction of the Moon's true declination happens to differ from the direction of the Moon's own declination, then the Moon's true declination, even though larger in magnitude, should be regarded as smaller than the Sun's declination.[1]

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1. *Cf. KK*, II, i. 14; *BrSpSi*, xiv. 37; *ŚiDVṛ*, xiii. 3 (c-d); *SiŚe*, viii. 6.

The Moon's true declination will differ in direction from the Moon's own declination, when the Moon's own declination and the Moon's latitude are of unlike directions and the former is smaller in magnitude than the latter.

By the Moon's true declination is meant the declination of the actual position of the Moon in its orbit, and by the Moon's own declination is meant the declination of the Moon's position on the ecliptic.

PĀTA PAST OR TO COME

19. When the Moon is in an odd quadrant and its declination is larger than the Sun's declination, the *pāta* has already occurred; in the contrary case, it is to occur. When the Moon is in an even quadrant, it is just the reverse.[1]

CALCULATION OF TIME OF *PĀTA*

20. In the case of *Vyatipāta*, find the difference or sum of the declinations of the Sun and Moon, and in the case of *Vaidhṛta*, find the sum or difference, according as they are of like or unlike directions. Thus is obtained the "first" quantity (*prathama-rāśi*). Find a (similar) result from the *iṣṭanāḍīs*, i.e., arbitrarily chosen *nāḍīs*, elapsed (if the *pāta* has already occurred) or to elapse (if the *pāta* is to occur).[2] **This is the "second" quantity (*anya*).**

Method 1 : When *iṣṭaghaṭīs* are not used.

21. (Severally) multiply all the (tabular) Rsines by the Rsine of 24° and divide (each product) by the radius : this will give the Rsines of the (corresponding) declinations (*kramajīvāḥ*). Set up the minutes of the (corresponding) declinations (*kramaliptāḥ*) and the (corresponding) arcs (of the ecliptic) [*taccāpāni*] (as well as the declination-differences) (in columns) separated by a distance.

22-24. From the "first" quantity (*prathama-rāśi*) and the position of the Moon find whether the *pāta* has already occurred or is to occur. (Suppose the *pāta* has occurred and the Moon is in the odd quadrant).

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1. *Cf. KK*, II, i. 15; *BrSpSi*, xiv. 38; *ŚiDVṛ*, xiii. 5; *SūSi*, xi. 7-8; *SiŚe*, viii. 7; *SiŚi*, I, xii. 10 (c-d)-11.
2. *Cf. KK*, II, i. 16; *SiŚi*, I, xii. 11 (c-d)-12.

Then find the minutes traversed by the Moon of the elemental arc occupied by it (*dhanukalikāḥ*). Then find the minutes of the corresponding declination-difference (*khaṇḍakrāntikalāḥ*), and subtract them from the "first". Set down the minutes that are left over after subtracting the minutes of the *khaṇḍakrāntidhanu*. Now add together (the minutes of) the arcs (of the ecliptic) corresponding to the declination-differences and parts thereof (contained in the "first"), taken in the reverse order, and divide the sum by (the minutes of) the difference between the daily motions of the Sun and Moon : the result gives the days etc. (that have elapsed since the occurrence of the *pāta*. This process is to be adopted when the *pāta* has occurred (and the Moon is in the odd quadrant). When the Moon is in the even quadrant and the *pāta* is to occur, even then the process is the same as described above.

25-28. (When the Moon is in the even quadrant and the *pāta* has occurred, proceed as follows:) Find the minutes of the declination-difference corresponding to the minutes traversed by the Moon of the elemental arc occupied by it (*khaṇḍadhanuḥkrāntikalāḥ*). Subtract from the "first" those minutes as also the other declination-differences (taken in the serial order) that can be subtracted. Note down (the minutes of) the arc left over after this subtraction. To (the minutes of) the arc (of the ecliptic) corresponding to this arc obtained by subtracting the declination-differences (from the first), add the minutes of the arcs (of the ecliptic) corresponding to the declination-differences subtracted from the "first." From that find out the days etc. as before and subtract them from the time of calculation. Thus is obtained the time of occurrence of the *pāta*, provided the Moon is in the even quadrant (and the *pāta* has occurred).

When the minutes of the declination-difference corresponding to the minutes traversed by the Moon of the elemental arc occupied by it cannot be subtracted from the "first", then multiply the minutes of the "first" by 225 and divide by (four times) the minutes of the declination-difference of the elemental arc occupied by the Moon, (and then divide that by the minutes of motion-difference of the Sun and Moon).

This method has been indicated by my own intellect. By using it one may calculate the true time of occurrence of the *pāta* without the use of *iṣṭaghaṭīs*.

In the above rule the author first finds the arc of the ecliptic corresponding to the "first" and then the time taken by the Moon in traversing that arc relative to the Sun.

Method 2 : When *iṣṭaghaṭīs* are used

29. When the "first" quantity and the "second" quantity both relate to *pāta* past or *pāta* to come, then their difference otherwise (i. e., when one relates to *pāta* past and the other to *pāta* to come) their sum, is the divisor of the product of the "first" quantity (*ādya* or *prathama-rāśi*) and the *iṣṭanāḍīs* (i.e., arbitrarily chosen *nāḍīs*, see vs. 20). (This division gives the *madhyanāḍīs*, i.e., the *nāḍīs* lying between the time of calculation and the time of the middle of the *pāta*). By so many *nāḍīs* (before or after), depending on whether the "first" quantity (*prathama*) relates to *pāta* past or to *pāta* to come, occurs the middle of the *pāta*.

30. (Taking these *nāḍīs* as the *iṣṭanāḍīs*) calculate the planets (Sun, Moon, and Moon's ascending node) for the middle of *pāta*, and iterate the process (until the *madhyanāḍīs* are fixed). Then multiply the sum of the semi-diameters of the Sun and the Moon by the *madhyanāḍīs* and divide (the product) by the "first" quantity (*prathama*). (Thus are obtained the *nāḍīs* of the *sthityardha*.) By so many *nāḍīs* before or after (the middle of *pāta*) occur the beginning and end of the *pāta*.[1]

31. Until the declination of any point of the Moon's disc does not differ from the declination of any point of the Sun's disc, so long is the Moon supposed to have the same declination as the Sun, and so long does the performance of the deeds prescribed in connection with the *pāta* bear fruit.[2]

Let the *iṣṭanāḍīs* be *I*. The *iṣṭanāḍīs* being evidently the *nāḍīs* between the *iṣṭakāla* and the time of calculation, the *iṣṭakāla* being taken as an approximation for the time of middle of the *pāta* when the declinations of the Sun and the Moon are supposed to be equal.

Let *F* be the *ādya* or "first" quantity (i.e., the algebraic difference of declinations of the Sun and Moon) for the time of calculation and F' the *ādya* or "first" quantity for the *iṣṭakāla*, the *ādya* or "first" quantity for the middle of the *pāta* being evidently zero.

We have therefore the proportion : When to *ādya*-difference $F-F'$ correspond the *iṣṭanāḍīs* *I*, what will correspond to *ādya*-difference equal

1. The rule given in vss. 20, 29-31 occurs also in *KK*, II, i. 16-20; *SīŚe*, viii. 8-11; *SiŚi* I, xii. 13-16.

2. *Cf. SiŚe*, viii. 14; *SiŚi*, I, xii. 17; *KKau*, xiii. 12.

to $F-0$? The result is the *madhyanāḍīs*, i.e., the *nāḍīs* between the time of calculation and the middle of the *pāta*.

Hence the formula :

$$madhyanādis = \frac{I \times F}{F-F'}.$$

The process of iteration is obvious.

Let the *madhyanāḍīs* obtained by the process of iteration be *M nāḍīs*. Then the *ādya* for the middle of the *pāta* = 0, the *ādya* for the time of calculation is *F*, and the time-interval between the two epochs is *M nāḍīs*; also the *ādya* for the middle of the *pāta* = 0, the *ādya* for the beginning or end of the *pāta* is $S+S'$ (where S, S' are the semi-diameters of the Sun and Moon respectively, and the time-interval between the two epochs is *sthityardha-nāḍīs*.

We have therefore the following proportion: When to *ādya*-difference *F* correspond *M nāḍīs*, what will correspond to *ādya*-difference equal to $S+S'$? The result is the *sthityardhanāḍīs*, i.e., the time-interval between the middle of the *pāta* and the beginning or end of the *pāta*.

Hence the formula:

$$sthityardhanāḍīs = \frac{M \times (S+S')}{F}.$$

Bhāskara II has iterated the process of finding the ***sthityardhanāḍīs***. The process of iteration is as follows: Calculate the *ādya*, say F'', for the beginning of the *pāta*, then the second approximation for the *sthityardha-nāḍīs* will be given by

$$sthityardhanāḍīs = \frac{\text{previous } sthityardhanāḍīs \times (S+S')}{F''},$$

because when

ādya for the beginning of the *pāta* = F'',

ādya for the middle of the *pāta* = 0,

then their difference = F'', and the time-difference = previous *sthityardhanāḍīs* ;

and when

$\bar{a}dya$ for the beginning of the $p\bar{a}ta = S+S'$,

$\bar{a}dya$ for the middle of the $p\bar{a}ta = 0$,

their difference $= S+S'$, and the time-difference = required *sthityardha-nāḍīs*.

The third and subsequent approximations are obtained similarly. The process of iteration relating to the end of the *pāta* is similar.

Mallikārjuna Sūri (A.D. 1178) gives the following method for calculating the *sparśa* and *mokṣa sthityardhas* :

Calculate the *ādya* for 60 *nāḍīs* before the time of the middle of the *pāta*. If this be denoted by D, then

$$\textit{sparśa-sthityardha} = \frac{(S+S')\times 60}{D} \textit{ nāḍīs}.$$

Similarly, if D' denote the *ādya* for 60 *nāḍīs* after the occurrence of the middle of the *pāta*, then

$$\textit{mokṣa-sthityardha} = \frac{(S+S')\times 60}{D'} \textit{ nāḍīs}.$$

The same method is given in the *Karaṇa-kaustubha* of Kṛṣṇadaivajña (A.D. 1653).

TRUE *TITHI* FROM MEAN *TITHI*

32. Calculate the corrections for the Sun and the Moon and apply them to the minutes of the (given) mean *tithi*, the correction for the Sun being applied reversely. The result thus obtained should be (reduced to degrees and) divided by the degrees of the difference between the Sun and the Moon corresponding to a *tithi*. Thus is obtained the true *tithi*. The correction for the Sun's ascensional difference should also be applied as in the case of a planet.

By "the minutes of the *tithi*" is meant "Moon's longitude minus Sun's longitude, in terms of minutes".

EQUALISATION OF SUN AND MOON

First Method

33. (Severally) multiply the daily motions of the Sun and the Moon by the *ghaṭīs* elapsed or to elapse of the current *tithi* and divide (each product) by 60; (in the former case) subtract the resulting minutes from or (in the latter case) add them to the longitudes of the Sun and the Moon. Then are obtained the longitudes of the Sun and Moon for the end of the *tithi*, agreeing in minutes.

Second Method

34. Or, (severally) multiply the daily motions of the Sun and the Moon by the minutes elapsed or to elapse of the current *tithi* and divide (each) product by the difference between the daily motions of the Sun and the Moon; and subtract the resulting minutes from or add them to the longitudes of the Sun and the Moon, as before. Then too are obtained the longitudes of the Sun and Moon agreeing in minutes.

Third Method

35. Or, subtract as many minutes as there are *ghaṭīs* elapsed of the current *tithi* from or add as many minutes as there are *ghaṭīs* to elapse of the current *tithi* to the longitudes of the Sun and Moon; and in the case of the Moon's longitude further subtract or add as many minutes as there are in the *tithi* (i. e., in Moon's longitude minus Sun's longitude). Thus too are obtained the longitudes of the Sun and Moon aggeeing in minutes.[1]

SUN AND MOON AT THE ENDS OF *TITHI*, *KARAṆA*, FULL MOON AND NEW MOON

36. The longitudes of the Sun and Moon agree in minutes at the end of a *tithi* or *karaṇa*; up to degrees at the end of a full moon day; and up to signs at the end of a (lunar) month.[2]

OCCURRENCE OF OMITTED DAYS

37. The number of civil days corresponding to the (eleven) omitted

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1. See *LBh*, iv. 1; *KR*, 1. 60.

2. *Cf. BrSpSi*, ii. 64, *SiŚe*, iii. 84.

days that fall in that period is equal to the number of minutes in a *tithi* minus one-twentyfifth thereof.

(The number of lunar days elapsed divided by the time-interval between the fall of two omitted days gives the number of omitted days elapsed). The remainder of the division gives the lunar days elapsed since the fall of the previous omitted day; and the remainder subtracted from 64 gives the number of lunar days to elapse before the fall of the next omitted day.

There are 11 omitted days in 692 civil days; moreover,

no. of minutes in a *tithi*— 1/25 of that = 720 − 28 = 692.

Hence the rule stated in the first part of the stanza.

The rule in the second half of the stanza follows from the fact that there are 11 omitted days in 703 lunar days, so that the omitted days occur at the interval of 703/11 or 64 lunar days.

TIME TAKEN BY SUN'S DISC IN TRANSITING A SIGN-END

38. (The minutes in the diameter of) a planet's disc divided by the degrees of the planet's daily motion, give the time of transit of a sign-end by the planet, in terms of *ghaṭīs*, etc. In the case of the Sun, this is of the highest merit and virtue; for this is the time in which the Sun's disc crosses the end of a sign.[1]

The time during which the Sun crosses the end of a sign is called holy time (*puṇya-kāla*). Since the Sun's diameter is roughly equal to 32′ and the Sun's daily motion is roughly 60′, therefore the time taken by the Sun in crossing the end of a sign is equal to 32′ × 60/60′ or 32 *ghaṭīs*. So 16 *ghaṭīs* preceding the *saṅkrānti* and 16 *ghaṭīs* following the *saṅkrānti* constitute the holy time (*puṇya-kāla*), *saṅkrānti* being the instant when the Sun's centre is crossing the end of the sign.

The author of the *Bṛhajjyotiḥsāra* writes: "16 *ghaṭīs* preceding the Sun's *saṅkrānti* and 16 *ghaṭīs* following the Sun's *saṅkrānti* constitute the *ghaṭīs* of holy time. When the *saṅkrānti* occurs sometime before midnight, then the latter half of the preceding day constitutes the holy

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1. *Cf. BrSpSi*, xiv. 29-30; *PSi*, iii. 26; *ŚiDVṛ*, xii. 11; *SiŚe*, iii. 85 (a-b); *SiŚi*, I, ii. 76.

time; and when the *saṅkrānti* occurs sometime after midnight, then the former half of the next day constitutes the holy time."

The author of the *Vṛddha-vaśiṣṭha-siddhānta* is rightly of the opinion that in finding the time of *saṅkrānti*, one should consider tropical signs.[1]

TIME TAKEN BY MOON'S DISC IN TRANSITING THE END OF *TIṬHI, KARAṆA* OR *YOGA*

39-40. (The minutes in the diameter of) the Moon's disc multiplied by 60 and divided by (the minutes in) the difference between the daily motions of the Sun and Moon gives the time of transit of a *tithi*-end or a *karaṇa*-end by the Moon's disc, (in terms of *ghaṭīs*). The same (i e, the minutes in the diameter of the Moon's disc multiplied by 60) divided by (the minutes of) the sum of the daily motions of the Sun and Moon gives the time of transit of a *yoga*-end by the Moon's disc, (in terms of *ghaṭīs*).[2] As long as a planet stays at these (end) points, it yields mixed results. This is why the beginnings and ends of the inauspicious *tithis*, *karaṇas* and *yogas* are malignant. So also are the *viṣṭi* day and the day which touches three *tithis*.[3]

PERIODS OF OCCURRENCE OF INTERCALARY MONTHS AND OMITTED DAYS

41-42. Divide the residue of the intercalary months by the number of intercalary months in a *yuga* : the quotient gives the solar days etc. elapsed since the fall of the previous intercalary month. Subtract the (same) residue of the intercalary months from the number of solar days in a *yuga* and divide (the difference thus obtained) by the number of intercalary months in a *yuga* : the quotient denotes the solar days etc. to elapse before the occurrence of the next intercalary month. The sum of the two gives the period of occurrence of intercalary months, in days etc. (i.e., the period, in solar days etc., from the occurrence of one intercalary month to the next).

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1. See *VVSi*, i, 68.
2. *Cf. BrSpSi*, xiv. 31-32; *SiŚe*, iii. 85 (c-d); *SiŚi*, I, ii. 77. Lalla instead of taking the minutes of the diameter of the Moon's disc takes the minutes of half the sum of the diameters of the Moon's and Sun's discs in deciding the durations of transists of of the end-points of *tithi*, *karaṇa* and *yoga*. See *ŚiDVṛ*, xii. 12.
3. The *tithi* which touches three days is also regarded as malignant. See *Jyotiścandrārka*, i. 60 and commentary.

Similar is the method of calculating the time, in terms of lunar days, elapsed since the occurrence of an omitted day or to elapse before the occurrence of the next omitted day or the period from the occurrence of one omitted day to the next.

The fractions of the current intercalary month elapsed and to elapse are respectively equal to

$$\frac{\text{residue of intercalary months}}{\text{solar days in a } yuga} \tag{1}$$

and

$$\frac{\text{solar days in a } yuga - \text{residue of intercalary months}}{\text{solar days in a } yuga}. \tag{2}$$

Multiplying (1) and (2) by solar days in a *yuga* and dividing by intercalary months in a *yuga*, we get the corresponding solar days as

$$\frac{\text{residue of intercalary months}}{\text{intercalary months in a } yuga} \tag{3}$$

and

$$\frac{\text{solar days in a } yuga - \text{residue of intercalary months}}{\text{intercalary months in a } yuga}. \tag{4}$$

Hence the above rule.

Section 7 : Examples on Chapter II

1. I now give a chapter on problems relating to the true motion (of the planets) which is like the Moon for the lily-like intellect of the learned astronomers and like the lion for the elephants in the guise of ill-versed astronomers.

2. One who finds the degrees of the *bhuja* from the degrees of the *koṭi*, the degrees of the *koṭi* from the degrees of the *bhuja*, the *bhuja* from the *kendra* and the mean planet from the planet's *kendra*, knows the true motion of the planets.[1]

3. One who finds the Rsine of the *bhuja* from the degrees of the *koṭi*, the Rsine of the *koṭi* from the degrees of the *bhuja*, the *koṭijyā* from the *bāhujyā*, and the *bāhujyā* from the *koṭijyā* knows the true motion of the planets.

4. One who finds the corresponding Rversed-sine from the Rsine, the Rsine from the Rversed-sine, the *karṇa* without using the *koṭijyā*, the *koṭijyā* from the *bhujajyā* and the *karṇa*, and the *bhujajyā* from the *koṭijyā* (and the *karṇa*), is endowed with flawless intellect.

5. One who corrects, in many ways, the (mean) planets, the (mean) motions, as well as the *manda* and *śīghra kendras*, with the help of their own corrections, converts the true planet into the corresponding mean planet, knows the true motion of the planets and is indeed an astronomer.

6. One who derives the true planet from its *ucca*, and the true motion of the Moon for the preceding, succeeding and current days from that of its *ucca*, knows the true motion of the planets.

7. If the true motion of a planet, calculated from its *mandakarṇa*, be equal to the mean motion, what is the *mandakendra* there ? Say, if you are aware of this fact.

8. One who finds the time elapsed corresponding to the *avamaśeṣa* (i.e., residue of the omitted days) and the period of occurrence of the omitted days with the help of civil days and planetary revolutions in a *yuga*, etc., is a learned astronomer, proficient in the subject of *avamapāta*.

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1. *Cf. BrSpSi*, xiv. 1.

9. One who finds the true planet from the *ahargaṇa*,[1] or for the time of rising of a given heavenly body, or for the time of rising of the *nakṣatra* Aśvinī (ζ Piscium), knows the flawless true motion (of the planets).[2]

10. One who, without making use of the (tabular) Rsines, calculates the *bhujajyā* and the *koṭijyā*, and the arc (corresponding to the *bhujajyā* or *koṭijyā*);[3] who, with the help of the longitude of the *ucca*, converts the true planet into the mean planet; and who finds the true motion (of a planet) with the help of the motion of the *ucca* and the mean motion (of the planet); knows the motion of the heavenly bodies (as if submitted to the eye) like an emblic myrobalan placed on (the palm of) the hand.

11. Say, if you know the true motion of the planets, what the *śīghrakendra* is when the (*śīghra*)*karṇa* is equal to the radius or the *bhujajyā* or when equal to the *koṭijyā* or *antyaphalajyā*.

12. One who finds the *kendra* from the given *phala* (i. e., *manda phala* or *śīghraphala*); obtains the (*śīghra*) *kendra* for the heliacal rising or setting of a planet; or one who finds the (*śīghra*) *kendra* for the beginning of retrograde or re-retrograde motion (of a planet) or the corresponding days (i.e., the days of retrograde or direct motion) is designated as Gaṇaka (astronomer).

13. One who knows the true *bhogas* of the *nakṣatras*, the *bhoga* of Abhijit as well as the true location of that malignant (*nakṣatra*), and the true *nāḍīs* of *saṅkrāntikāla* (i.e., the time in *nāḍīs*, during which the Sun crosses the end of a sign), is an astronomer well-versed in Gaṇita and the true motions (of the heavenly bodies).

14. One who knows (how to find) the times when *Vyatipāta* and *Vaidhṛta* begin and end, the times when the new moon and full moon days, *tithi*, *karaṇa*, *yoga* and *nakṣatra* end, the longitudes of the Sun and Moon agreeing to minutes, degrees, signs, etc., as well as the lord of the day which touches three *tithis* is a Gaṇaka having no one to match him.

15-16. One who correctly knows the eight varieties of planetary motion, viz. very fast, fast, natural (or mean), slow, very slow, retrograde, very retrograde, and re-retrograde, along with the corresponding (*śīghra*) *kendras*, is a good astronomer.[4]

1. *Cf. BrSpSi*, xiv. 4 (a-b).
2. *Cf. BrSpSi*, xiv. 5.
3. *Cf. BrSpSi*, xiv. 4; *SiŚe*, xx. 5 (a-b).
4. Same example occurs in *SiŚe*, xx. 6.

Chapter III

THREE PROBLEMS

Section 1 : Cardinal Directions and Equinoctial Midday Shadow

INTRODUCTION

1. Since the entire science of astronomy contained in the eight chapters (of this book) is based on what is stated in the chapter on "Three Problems" (*Tripraśna*), therefore I now set out the chapter on Three Problems in clear terms.

LATITUDE-TRIANGLES

Before proceeding further we shall describe certain right-angled triangles which are formed within the Celestial Sphere and are known as latitude-triangles (*akṣakṣetra*) in Hindu astronomy. The three angles of such triangles are equal to ϕ (the latitude of the place), $90° - \phi$ (the colatitude of the place), and 90°. The side facing the angle ϕ is called the base (*bhuja* or *bāhu*); that facing the angle $90 - \phi$, the upright (*koṭi*); and that facing the right angle, the hypotenuse (*karṇa*). The radius R of the Celestial Sphere is supposed to be equal to 3438′ or, more correctly, 3437′44″ (which is the value of one radian).

Let S be the Sun (or any other heavenly body) on the Celestial Sphere at any given time; SA the perpendicular dropped from S on the plane of the celestial horizon; SB the perpendicular dropped from S on its rising-setting line; and AB the perpendicular dropped from A on the same rising-setting line, SA is equal to the Rsine of the altitude of S (i.e., Rsin a); it is called *śaṅku*. SB is called *iṣṭahṛti*; Vaṭeśvara has called it *dhṛti*, *svadhṛti*, *iṣṭadhṛti*, *nijadhṛti*, etc.[1] AB is called *śaṅkutala* or *śaṅkvagra*. Since the angle ASB is equal to ϕ, therefore the right-angled triangle SAB is a latitude-triangle. The base, upright and hypotenuse of this triangle are:

Base	Upright	Hypotenuse	
śaṅkutala	*śaṅku* or Rsin a	*svadhṛti* or *iṣṭadhṛti*	(A)

When S is on the prime vertical, SA is called *samaśaṅku*,, AB *agrā*, and SB *samadhṛti* or *taddhṛti*. The triangle SAB is a latitude-triangle. The base, upright and hypotenuse of this triangle are:

Base	Upright	Hypotenuse	
agrā	*samaśaṅku*	*taddhṛti*	(*B*)

When S is on the prime vertical, then if a perpendicular AC is dropped from A on the *taddhṛti* SB, two more latitude-triangles ACB and ACS are formed. AC is equal to the Rsine of the declination of S (i.e., Rsin δ), CB is called earthsine (*kujyā*, *kṣitijyā*, *bhūjyā*, *mahījīvā*, etc.), and SC is equal to *taddhṛti* minus earthsine. The base, upright and hypotenuse of the latitude-triangles ACB and ACS are respectively:

Base	Upright	Hypotenuse	
earthsine	Rsin δ	*agrā*	(*C*)
Rsin δ	*taddhṛti* — earthsine	*samaśaṅku*	(*D*)

When the Sun is on the equator and S its position on the Celestial Sphere at midday, SA the perpendicular on the plane of the celestial horizon, and O the centre of the Celestial Sphere, then the triangle SAO is again a latitude-triangle. The base, upright and hypotenuse of this triangle are:

Base	Upright	Hypotenuse	
Rsin φ	Rcos φ	R	(*E*)

When the Sun is on the equator, then at midday the gnomon, its shadow called the equinoctial midday shadow (*palabhā*, *akṣabhā*, *palacchāyā*, *viṣuvacchāyā*, etc.), and hypotenuse of that equinoctial midday shadow (called *palakarṇa*, *palaśravaṇa*, *palaśruti*, *akṣakarṇa*, *akṣaśruti*, etc.) also form a latitude-triangle. Its base, upright and hypotenuse are:

Base	Upright	Hypotenuse	
palabhā	gnomon or 12	*palakarṇa*	(*F*)

The gnomon is supposed to be of 12 *aṅgulas* and the *palabhā* and *palakarṇa* are also measured in *aṅgulas*. Therefore, gnomon and 12 are taken as synonyms.

Several other latitude-triangles have been defined by Bhāskara II and other Hindu astronomers. But the six latitude-triangles stated above are sufficient for our purpose. We shall see that most of the rules stated below in this chapter can be derived simply by their comparison. We shall refer to them as latitude-triangles (*A*), (*B*), (*C*), (*D*), (*E*) and (*F*) respectively.

Besides these latitude-triangles, there are altitude-triangles also for different positions of the Sun :

Base	Upright	Hypotenuse	
śaṅku (Rsin *a*)	*dṛgjyā* or *natajyā* (Rsin *z*)	R	(*G*)
gnomon or 12	shadow	hypotenuse of shadow	(*H*)

When the Sun is on the meridian, *śaṅku* is called *madhyaśaṅku* or *madhyāhnaśaṅku* ; shadow is called *madhyāhnacchāyā*; and hypotenuse of shadow *madhyāhnacchāyā-karṇa*.

When the Sun is on the prime meridian (*samamaṇḍala*), *śaṅku* is called *samaśaṅku*; shadow *samacchāyā*;. and hypotenuse of shadow *samacchāyā-karṇa*. And so on.

These altitude-triangles have also been used.

DETERMINATION OF CARDINAL DIRECTIONS

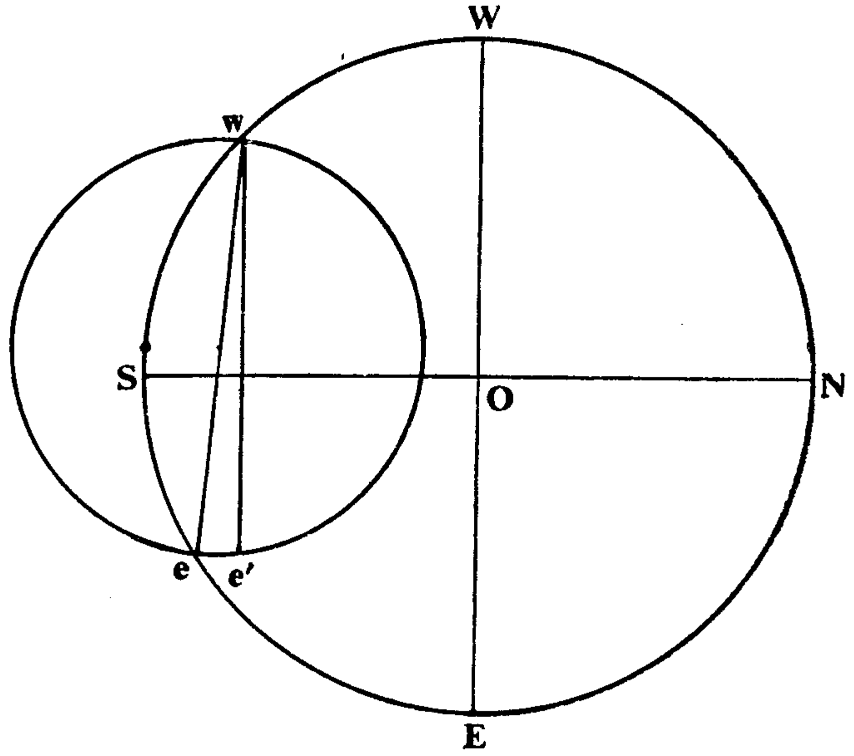
Method 1

2. (The points) where the shadow of the (vertical) gnomon, set up at the centre of a circle drawn on level ground, enters into (the circle in the forenoon) and passes out (of the circle in the afternoon), give (respectively) the west and east directions (with respect to each other), when due allowance is made for the variation of the Sun's declination.[1]

Let ESWN be the circle drawn on level ground, and O the centre of the circle where a vertical gnomon is set up. Let w be the point where the tip of the shadow of the gnomon enters into the circle in the forenoon. and e the point where the tip of the shadow of the gnomon goes out of the circle in the afternoon. Had the declination of the Sun been the same at

1. *Cf. BrSpSi*, xxii. 27; *ŚiDVr*, iv. 1; *MSi*, iv. 1-2 (a-b) ; *SiŚe*, iv. 1-3; *SiŚi*, I, iii. 8.

both the times ew would have been the east-west line but since the declination of the Sun goes on changing a correction to ew is necessary.



The distance of the shadow-tip from the east-west line is defined as the *chāyā-bhuja* ("*bhuja* or base of shadow"). Let the difference between the *chāyā-bhuja* when the tip of the shadow enters into the circle and the *chāyā-bhuja* when the tip of the shadow passes out of the circle be *d*. Let δ be the Sun's declination when the tip of the shadow enters into the circle in the forenoon and δ' the Sun's declination when the tip of the shadow passes out of the circle in the afternoon. Then

$$d = \frac{(\text{Rsin } \delta' \sim \text{Rsin } \delta) \times \text{hypotenuse of shadow}}{\text{Rcos } \phi},$$

where ϕ is the local latitude. This *d* denotes the correction which is applied as follows:

Construct a circle with ew as diameter, and with centre e and radius *d* draw an arc cutting this circle at e' towards the north if the Sun's *ayana* is north, or towards the south if the Sun's *ayana* is south. Then e'w is the true orientation of the east-west line.

Now, through O, draw a line EW parallel to e'w. Then, relative to the point O, E is the east and W the west. The line NS, drawn through O, at right-angles to EW is the north-south line, N being the north and S the south relative to O.

Method 2

3. Or, put down points at the extremities of two equal shadows, one (in the forenoon) when the Sun is in the eastern half of the celestial sphere and the other (in the afternoon) when the Sun is in the western half of the celestial sphere. These, too, give (respectively) the west and east directions, provided due allowance is made for the change in the (Sun's) declination.[1]

This method is essentially the same as the previous one.

Method 3

4. When the Sun enters the circle called the prime vertical, the shadow (of a vertical gnomon) falls exactly east to west. Towards the north pole lies the north direction.[2]

Method 4

5. As long as the shadow (of a vertical gnomon), for the desired time, is equal (in magnitude and direction) to the hypotenuse of the right-angled (shadow) triangle formed by that shadow and the *bhuja* (base) and *koṭi* (upright) for that shadow, so long is the *koṭi* (upright) directed east to west.[3]

The *bhuja* (base) of the shadow triangle is the perpendicular dropped from the tip of the shadow on the east-west line; and the *koṭi* (upright) of the shadow triangle is the projection of the shadow on the east-west line. Hence the above rule.

Bhāskara II explains the above method more explicitly. Writes he: "At the desired time, put down a mark at the tip of the gnomonic shadow; then calculate the *bhuja* (base) and *koṭi* (upright) for that shadow in the prescribed manner; and then take two bamboo strips, one equal to the *bhuja* (base) and the other equal to the *koṭi* (upright). Then lay off, on the ground, the *koṭi* strip from the centre in its own direction and the *bhuja* strip from the tip of the shadow in the reverse direction in such a way that the extremities of the two strips meet. This being done, the *koṭi* strip will be directed east-west and the *bhuja* strip north-south."[4]

1. *Cf. BrSpSi*, iii. 1.
2. *Cf. SiŚi*, I, iii. 9 (a).
3. *Cf. SiŚi*, I, iii. 9 (b-d).
4. See Bhāskara II's commentary on *SiŚi*, I, iii, 9.

Method 5

6. (The points of the horizon) where any heavenly body, with zero declination, rises and sets are (respectively) the east and west directions (relative to the observer).

Method 6 : Ancient Method

7. (The point of the horizon) where the star Revatī (ζ Piscium) or Śravaṇa (Altair or α Aquilae) rises is the east direction. Or, as stated grossly by the learned, it is that point (of the horizon) which lies midway between the points of rising of Citrā (Spica) and Svātī (Arcturus).

The second alternative, according to Nārāyaṇa, the author of the *Muhūrtamārtaṇḍa*, was meant for people living south of Ujjayinī. For he writes:

"To the south of Ujjayinī, the east cardinal point is to be determined by the point lying midway between (the rising points of) Citrā and Svātī."[1]

The observation of rising Citrā or Svātī was, however, made in practice when it was 86 *aṅgulas* above the horizon. Perhaps it was not possible to observe them when they were lower than this height.

A similar observation is made in the *Devayajanadīpikā* :

"The point lying midway between (the rising points of) Citrā and Svātī is the east for the people living south of Ujjayinī; for those living to the north, the rising point of Kṛttikā is the east."[2]

Similar statements are made in *Kātyāyana-śulba, Mānava-śulba, Trikāṇḍamaṇḍana, Kuṇḍasiddhi, Kuṇḍadarpaṇa*, etc.[3]

The statements made in vs. 7, however, will be true only when the stars Revatī and Śravaṇa as also the point lying midway between the rising points of Citrā and Svātī are on the celestial equator, because only those stars rise

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1. प्राक्साध्योज्जयिनीस्थलाद्यमदिशि त्वाष्ट्रानिलाभ्यन्तरात् । See *Muhūrta-mārtaṇḍa, gṛhaprakaraṇa*, vs. 5 (a-b).
2. चित्रास्वात्यन्तरे श्रोणाद् दक्षिणापथवासिनाम् ।
   प्राची तु कृत्तिका ज्ञेया उत्तरापथवासिनाम् ॥
3. See *Digmīmāṁsā* by Sudhakar Dvivedi, pp. 34-35.

in the east and set in the west which lie on the celestial equator and likewise have zero declination.[1] See *supra*, vs. 6.

Method 7

8. The junction of the two threads which pass through the two fish-figures that are constructed with the extremities of three shadows (taken two at a time) as centre is the south or north relative to the foot of the gnomon, according as the Sun is in the northern or southern hemisphere.[2]

9. With the junction of the (two) threads as centre, draw a circle passing through the extremities of the three shadows. (The tip of) the shadow (of the gnomon) does not leave this circle in the same way as a lady born in a noble family does not discard the customs and traditions of the family.[3]

The statement made in vs. 9 is not quite correct, because the locus of the shadow-tip is not exactly a circle unless the observer is at the north or south pole. Bhāskara II has rightly criticised it. See *SiŚi*, II, *yantrādhyāya*, vs. 38 (c-d).

Method 8

10. The midday shadow of the gnomon lies on the north-south line between the circle (denoting the locus of the shadow-tip) and (the foot of) the gnomon (situated at the centre).[4] When the Sun is at the first point of Aries or Libra, the equinoctial midday shadow, too, lies south to north.

HYPOTENUSE OF SHADOW

11. The square-root of the sum of the squares of the length of the gnomon and the length of the shadow is the hypotenuse of shadow. The square-root of the difference between the squares of the hypotenuse of shadow and the gnomon is the shadow.

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1. Sudhakar Dvivedi has shown that Śravaṇa, whose celestial latitude is about 30°N cannot rise in the east. See *Digmīmāṁsā*, p. 32.
2. *Cf. BrSpSi*, iii. 2; *ŚiDVṛ*, iv. 2; *SiŚe*, iv. 4.
3. *Cf. MBh*, iii. 52; *ŚiDVṛ*, iv. 3; *SiŚe*, iv. 5.
   For finding the locus of the shadow-tip of the gnomon with the help of two *bhujas* and the midday shadow, see infra, chap. III, sec. 14, vss. 5-8.
4. A similar idea is expressed in *BrSpSi*, iii. 3; *SiŚe*, iv. 6 (a-b).

DETERMINATION OF EQUINOCTIAL MIDDAY SHADOW

Method 1

12. One should build an earthen platform which should be large, circular, as high as one's shoulders, with surface levelled with water, with circumference graduated with signs and degrees, and with well-ascertained cardinal points.

13. Let a person, standing on the western side of that (platform) observe the rising Sun through the centre of the circle. Then the Rsine of the degrees of that point of the circle where he sees the rising Sun is the Sun's *agrā*.[1]

14. The (Sun's) *agrā* multiplied by 12 and divided by the Rsine of the (Sun's) declination is the hypotenuse of the equinoctial midday shadow (*palaśravaṇa* or *palakarṇa*). By the difference between the hypotenuse of the equinoctial midday shadow and the gnomon multiply their sum and take the square root (of the product): the result is the equinoctial midday shadow (*akṣābhā* or *palabhā*).[2]

The circumference of the circular platform is supposed to be graduated with the zero mark at the east point, so that the mark at the Sun's rising point indicates the Sun's arcual distance from the east point and its Rsine the distance of the Sun's rising point from the east-west line (i.e., the Sun's *agrā*).

The *agrā* and the Rsine of the Sun's declination being known, the equinoctial midday shadow and its hypotenuse are obtained by the following formulae :

$$\text{hypotenuse of equinoctial midday shadow} = \frac{agr\bar{a} \times 12}{\text{R}\sin\delta}, \qquad (1)$$

and

$$\text{equinoctial midday shadow} = \sqrt{(H-12)(H+12)}, \qquad (2)$$

where δ is the Sun's declination and H the hypotenuse of the equinoctial midday shadow, the gnomon being supposed to be of 12 *aṅgulas*, as usual.

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1. *Cf. BrSpSi*, xv. 45 (a-b) ; *SiŚe*, iv. 111.

2. For the second rule, see *SiŚi*, I, iii. 11 (c-d); in its ordinary form it occurs in *ŚiDVr*, iv. 4 (c).

Formula (1) may be obtained by comparing the latitude-triangles (*F*) and (*C*), viz.

	Base	Upright	Hypotenuse
(*F*)	equi. midday shadow	gnomon (= 12 *aṅgulas*)	hyp. equi. midday shadow
(*C*)	earthsine	Rsin δ	*agrā*

Formula (2) follows from triangle (*F*).

The equinoctial midday shadow, as defined above in vs. 10 and by Bhāskara II,[1] is the midday shadow of the gnomon when the Sun is at the vernal equinox or at the autumnal equinox. Mahāvīra, Āryabhaṭa II and Śrīpati, however, define it as half the sum of (i) the midday shadow of the gnomon when the Sun is at the first point of Aries, and (ii) the midday shadow of the gnomon when the Sun is at the first point of Libra.[2]

Method 2

15. The square-root of the difference between the squares of the Rsine of the (Sun's) declination and the *agrā* is the earthsine (*kujyā*), which lies in the plane of the (Sun's) diurnal circle. The earthsine multiplied by 12 and divided by the Rsine of the (Sun's) declination is also the equinoctial midday shadow (*palābhā* or *palabhā*).[3]

$$\text{Earthsine} = \sqrt{(agr\bar{a})^2 - (\text{Rsin}\ \delta)^2},$$

and equinoctial midday shadow $= \dfrac{\text{earthsine} \times 12}{\text{Rsin}\ \delta}$.

These formulae follow from the comparison of the latitude-triangles (*F*) and (*C*) mentioned above.

Method 3

16. One should hold a Yaṣṭi, equal to the radius of the celestial sphere, pointing towards the Sun in such a way that it may not cast any shadow. Then the perpendicular (dropped on the ground from the upper

1. See *SiŚi*, I, ii. 46 (d).
2. See *GSS*, ix. 4 1/2; *MSi*, iv. 3; *SiŚe*, iv. 69.
3. This second rule occurs also in *BrSpSi*, xv. 34 (c-d)-35(a-b).

extremity of the Yaṣṭi), which is called upright, is the *Śaṅku* (or Rsine of the Sun's altitude).[1]

17. The distance between (the foot of) that (*śaṅku*) and the east-west line is (called) the *bāhu* or (base). The shadow of that *śaṅku-yaṣṭi* is equal to the Rsine of the (Sun's) zenith distance. The Yaṣṭi is the hypotenuse. The *bāhu* for the middle of the day is equal to the Rsine of the Sun's (meridian) zenith distance.[2]

18. The sum or difference of the *bāhu* and the *agrā*, according as they are of unlike or like directions, is the *śaṅkutala*. That *śaṅkutala* multiplied by 12 and divided by the upright (= Rsine of the Sun's altitude) gives the equinoctial midday shadow.[3]

The *bāhu* (base) and the *agrā* are measured from the east-west line, and the *śaṅkutala* from the rising-setting line of the Sun (or the heavenly body concerned). Thus the *bāhu* and *agrā* are north or south according as they are towards the north or south of the east-west line. The *śaṅkutala*, is south during the day and north during the night.[4]

The Sun's altitude and *śaṅkutala* being known, the equinoctial midday shadow is obtained by the formula :

$$\text{equi. midday shadow} = \frac{\textit{śaṅkutala} \times 12}{\text{Rsin } a},$$

where a is the Sun's altitude.

This formula easily follows from the comparison of the latitude-triangles (F) and (A), viz.

	Base	Upright	Hypotenuse
(F)	equi. midday shadow	12	hyp. of equi. midday shadow
(A)	*śaṅkutala*	Rsin a	*svadhṛti*

where a is the Sun's altitude.

1. and 2. *Cf.* *SiŚe*, iv. 112; *SiŚi*, II, *yantrādhyāya*, 28-31.

3. *Cf.* vss. 16-18 with *BrSpSi*, xv. 46-47.

4. See *BrSpSi*, iii. 65; *SiŚe*, iv. 91(c-d).

Method 4

19. The equinoctial midday shadow (*palabhā*) is also equal to the Rsine of latitude multiplied by 12 and divided by the Rsine of colatitude.[1]

$$\text{Equinoctial midday shadow} = \frac{\text{Rsin } \phi \times 12}{\text{Rcos } \phi},$$

where ϕ is the latitude of the local place.

This follows from the comparison of the latitude-triangles (F) and (E), viz.

	Base	Upright	Hypotenuse
(F)	equi. midday shadow	12	hyp. of equi. midday shadow
(E)	Rsin ϕ	Rcos ϕ	R

Method 5

20. Multiply the Sun's *agrā* by the midday shadow and divide by the Rsine of the Sun's own (i.e., meridian) zenith distance. The resulting quotient being added to or subtracted from the midday shadow, in the same way as the *agrā* is added to or subtracted from the *bāhu*, also gives the equinoctial midday shadow.

$$\text{Equi. midday shadow} = \text{midday shadow} \pm \frac{agr\bar{a} \times \text{midday shadow}}{\text{Rsin } z},$$

where z is the Sun's meridian zenith distance.

Rationale. Comparing the latitude-triangles (F) and (A), viz.

	Base	Upright	Hypotenuse
(F)	equi. midday shadow	12	hyp. of equi. midday shadow
(A)	midday *śaṅkutala*	Rsin a	midday *dhṛti*

where a is the Sun's altitude at midday, we have

$$\frac{\text{equi. midday shadow}}{\text{midday } \acute{s}a\dot{n}kutala} = \frac{12}{\text{Rsin } a} \qquad (1)$$

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1. *Cf. SiŚe*, iv. 94 (d).

And comparing the similar right-angled triangles :

Base	Upright	Hypotenuse
midday shadow	12	hyp. of midday shadow
Rsin z	Rsin a	R

where a and z are respectively the Sun's altitude and zenith distance at midday, we have

$$\frac{12}{\text{Rsin } a} = \frac{\text{midday shadow}}{\text{Rsin } z}. \qquad (2)$$

From (1) and (2),

$$\frac{\text{equi. midday shadow}}{\text{midday } \acute{s}a\dot{n}kutala} = \frac{\text{midday shadow}}{\text{Rsin } z}.$$

$$\therefore \text{ equi. midday shadow} = \frac{\text{midday } \acute{s}a\dot{n}kutala \times \text{midday shadow}}{\text{Rsin } z}$$

$$= \frac{(\text{Rsin } z \pm agr\bar{a}) \times \text{midday shadow}}{\text{Rsin } z}$$

$$= \text{midday shadow} \pm \frac{agr\bar{a} \times \text{midday shadow}}{\text{Rsin } z}.$$

Method 6

21. Find the difference or sum of the two given *bhujas* (of shadow) according as they are of like or unlike directions. Multiply (the difference or sum thus obtained) by 12 and divide by the difference between the Rsines of the Sun's altitudes corresponding to the two *bhujas* : the result is the *aṅgulas* of the equinoctial midday shadow (*akṣabhā* or *palabhā*).

Let a be the Sun's altitude and t and b the corresponding *śaṅkutala* and *bhuja* respectively; let a' be the Sun's altitude at another time and t' and b' the corresponding *śaṅkutala* and *bhuja* respectively. Then from vs. 18, we have

$$\text{equi. midday shadow} = \frac{t \times 12}{\text{Rsin } a}.$$

Therefore,

$$\text{Rsin } a \times (\text{equi. midday shadow}) = t \times 12. \qquad (1)$$

Similarly,

$$\text{Rsin } a' \times (\text{equi. midday shadow}) = t' \times 12. \qquad (2)$$

Taking the difference of (1) and (2),

$$(\text{Rsin } a - \text{Rsin } a') \times (\text{equi. midday shadow}) = (t - t').\ 12,$$

whence

$$\text{equi. midday shadow} = \frac{(t - t').\ 12}{\text{Rsin } a - \text{Rsin } a'}$$

$$= \frac{(b \pm b').\ 12}{\text{Rsin } a \sim \text{Rsin } a'}.$$

where $\pm$ denotes $+$ or $\sim$.

Method 7

22. Multiply each of the two given *bhujas* of shadow by the hypotenuse of shadow corresponding to the other *bhuja*, and divide (both the products) by the difference between the two hypotenuses of shadow. The difference or sum of the two results, according as they are of like or unlike directions, is the equinoctial midday shadow.[1]

The *bhuja* of shadow is the *bhuja* for the sphere of radius equal to the hypotenuse of shadow. The *śaṅkutala* for this sphere is equal to the equinoctial midday shadow.

Let b, b' be the two given *bhujas* of shadow and h, h' the corresponding hypotenuses of shadow. Also let A, A' be the *agrās* corresponding to the spheres of radii h, h'. Then denoting the equinoctial midday shadow by P, and $+$ or $\sim$ by $\pm$, we have

$$A = P \pm b, \quad A' = P \pm b',$$

Making A, A' correspond to the sphere of radius R, we have

$$agr\bar{a} = (P \pm b)\ \text{R}/h = (P \pm b')\ \text{R}/h'.$$

Therefore,

$$(P \pm b)/h = (P \pm b')/h',$$

giving

$$P = (hb' \pm h'b)/(h \sim h').$$

Hence the rule.

1. *Cf. BrSpSi*, iii. 57; *SiŚe*, iv. 94 (a-c); *SiŚi*, I, iii, 76; II, xiii (chapter on problems), 48.

Method 8

23. Or, the *agrā* multiplied by 12 and divided by the Rsine of the Sun's prime vertical altitude, gives the equinoctial midday shadow.

Also, the *taddhṛti* multiplied by 12 and divided by the Rsine of the Sun's prime vertical altitude gives the hypotenuse of the equinoctial midday shadow.

$$\text{Equinoctial midday shadow} = \frac{agr\bar{a} \times 12}{\text{Rsin } a},$$

$$\text{hypotenuse of equinoctial midday shadow} = \frac{taddh\d{r}ti \times 12}{\text{Rsin } a},$$

where a is the Sun's prime vertical altitude.

When the Sun is on the prime vertical the distance of the Sun from its rising-setting line is called *taddhṛti.*

Method 9

24. Or, the hypotenuse of the equinoctial midday shadow is equal to the radius multiplied by 12 and divided by the Rsine of colatitude; and the equinoctial midday shadow is equal to the earthsine multiplied by the hypotenuse of the prime vertical shadow and divided by the Rsine of latitude.

$$\text{Hyp. equi. midday shadow} = \frac{\text{radius} \times 12}{\text{Rcos } \phi},$$

$$\text{equi. midday shadow} = \frac{\text{earthsine} \times \text{hyp. prime vertical shadow}}{\text{Rsin } \phi}.$$

The first result is obvious; the second follows from the following relations:

(1) $\text{equi. midday shadow} = \dfrac{12 \times \text{Rsin } \phi}{\text{Rcos } \phi}$

(2) $\text{hyp. prime vertical shadow} = \dfrac{12 \times \text{radius}}{\text{Rsin } a}$

(3) $\text{Rsin } a = \dfrac{agr\bar{a} \times \text{Rcos } \phi}{\text{Rsin } \phi}$.

(4) $agr\bar{a} = \dfrac{\text{earthsine} \times \text{radius}}{\text{Rsin}\,\phi}$,

where a is the Sun's prime vertical altitude.

LATITUDE

25. The Sun's zenith distance for midday increased or diminished by the Sun's declination according as the Sun is in the six signs beginning with the first point of Aries or in the six signs beginning with the first point of Libra, gives the latitude. (But when at midday the Sun is to the north of the zenith) the Sun's declination diminished by its northern zenith distance gives the latitude.

Method 10

26. One should observe the Pole Star towards the north along the hypotenuse of the triangle-instrument, assuming its base to be equal to the gnomon; then the upright (of the triangle-instrument), which lies between the line of vision and the base, will be equal to the equinoctial midday shadow.

The triangle-instrument, referred to here, is of the shape of a right-angled triangle. When it is held in the meridian plane towards the north with its base horizontal, its hypotenuse points to the Pole Star.

Since the angle between the sides meeting at the eye is equal to ϕ, the latitude of the place, therefore the upright of the triangle-instrument is equal to 12tan ϕ, the length of the equinoctial midday shadow, 12 being the assumed length of the base of the triangle-instrument.

Method 11

27. When one, with one of his eyes raised up, observes towards the south the star Revatī as clinging to the tip of a (vertical) gnomon, then the distance between the foot of the gnomon and the eye equals the equinoctial midday shadow.

This is true only when the star Revatī is on the equator.

Method 12

28. The square-root of the difference between the squares of the radius and the *agrā*, multiplied by 2, gives the length of the rising-setting

line.[1] The distance from the rising-setting line to the (upper) extremity of the (great) gnomon is the *svadhṛti*.

The great gnomon is the Rsine of altitude. In the case of the Sun it is the perpendicular dropped from the Sun on the plane of the celestial horizon.

Method 13

29. The distance between the foot of the (great) gnomon and the rising-setting line, multiplied by 12 and divided by the (great) gnomon (i.e., the Rsine of the Sun's altitude) is also the equinoctial midday shadow. And the *svadhṛti* multiplied by 12 and divided by the (great) gnomon gives the hypotenuse of the equinoctial midday shadow.

$$\text{Equinoctial midday shadow} = \frac{\textit{śaṅkutala} \times 12}{\text{Rsin } a},$$

$$\text{hyp. equi. midday shadow} = \frac{\textit{svadhṛti} \times 12}{\text{Rsin } a},$$

where a is the Sun's altitude.

Methods 14 and 15

30. Or, the *śaṅkutala* multiplied by the given shadow of the gnomon and divided by the Rsine of the Sun's zenith distance gives the equinoctial midday shadow; the same (*śaṅkutala*) multiplied by the hypotenuse of the given shadow and divided by the *yaṣṭi* (i.e., the radius) also gives the equinoctial midday shadow.

$$\text{Equinoctial midday shadow} = \frac{\textit{śaṅkutala} \times \text{shadow}}{\text{Rsin } z} \qquad (1)$$

$$= \frac{\textit{śaṅkutala} \times (\text{hyp. of shadow})}{\text{radius}}, \qquad (2)$$

where z is the Sun's zenith distance.

Rationale. Since

$$\frac{\text{equi. midday shadow}}{\textit{śaṅkutala}} = \frac{12}{\text{Rsin } a},$$

and

$$\frac{\text{shadow}}{\text{Rsin } z} = \frac{12}{\text{Rsin } a},$$

1. *Cf. SiŚe*, iv. 90.

therefore, $\dfrac{\text{equi. midday shadow}}{\textit{śaṅkutala}} = \dfrac{\text{shadow}}{\text{Rsin } z}$.

$$\therefore \quad \text{equi. midday shadow} = \frac{\textit{śaṅkutala} \times \text{shadow}}{\text{Rsin } z}. \qquad (1)$$

Again, since

$$\frac{\text{shadow}}{\text{Rsin } z} = \frac{\text{hyp. of shadow}}{\text{radius}}$$

$$\therefore \text{ equinoctial midday shadow} = \frac{\textit{śaṅkutala} \times (\text{hyp. of shadow})}{\text{radius}}. \qquad (2)$$

Method 16

31. **Multiply the** *agrā* **by the given shadow and divide by the Rsine of the Sun's zenith distance : (the result is the** *chāyākarṇāgrī agrā*)**. The difference or sum of that "result" (viz. the** *chāyākarṇāgrī agrā*) **and the** *bhuja* **for the given shadow (***iṣṭabhābhuja* or *chāyākarṇāgrī bhuja*)**, according as they are of like or unlike directions, is the equinoctial midday shadow.**

Equi. midday shadow = *chāyākarṇāgrī agrā* + or ~ *chāyākarṇāgrī-bhuja*, (1)

where $\quad \textit{chāyākarṇāgrī agrā} = \dfrac{\textit{agrā} \times \text{hypotenuse of shadow}}{\text{R}}$

$$= \frac{\textit{agrā} \times \text{shadow}}{\text{Rsin } z},$$

and $\textit{chāyākarṇāgrī bhuja} = \dfrac{\textit{bhuja} \times \text{shadow}}{\text{Rsin } z}$,

z being the Sun's zenith distance.

Rationale. We know that

$$\textit{śaṅkutala} = \textit{agrā} + \text{or} \sim \textit{bhuja}.$$

Therefore, multiplying both sides by the length of shadow and dividing by the Rsine of the Sun's zenith distance (z), we get

$$\frac{\textit{śaṅkutala} \times \text{shadow}}{\text{Rsin } z} = \frac{\textit{agrā} \times \text{shadow}}{\text{Rsin } z} + \text{or} \sim \frac{\textit{bhuja} \times \text{shadow}}{\text{Rsin } z}.$$

$$= \textit{chāyākarṇāgrī agrā} + \text{or} \sim \textit{chāyākarṇāgrī bhuja}. \qquad (2)$$

Now, comparing the similar triangles :

Base	Upright	Hypotenuse
Rsin z	Rsin a	R
shadow	12	hyp. of shadow

and

śaṅkutala	Rsin a	*svadhṛti*
palabhā	12	*palakarṇa*

where a denotes the Sun's altitude, we have

$$\frac{\text{Rsin } z}{\text{shadow}} = \frac{\text{Rsin } a}{12} = \frac{\textit{śaṅkutala}}{\textit{palabhā}},$$

whence $\dfrac{\textit{śaṅkutala} \times \text{shadow}}{\text{Rsin } z} = \textit{palabhā}$ ("equi. midday shadow").

Therefore, from (2), we have

equi. midday shadow = *chāyākarṇāgrī agrā* + or ~ *chāyākarṇāgrī bhuja.*

Method 17

32-33. **The *agrā* multiplied by the hypotenuse of shadow and divided by the radius gives the *agrā* for the sphere of radius equal to the hypotenuse of shadow.**[1] **Similarly, the *bhuja* multiplied by the hypotenuse of shadow and divided by the radius gives another *bhuja* which corresponds to the sphere of radius equal to the hypotenuse of shadow. From these (*chāyākarṇāgrī agrā* and *chāyākarṇāgrī bhuja*) the equinoctial midday shadow is obtained as before.** (See vs. 31)

The rationale is already given above (under vs. 31).

Method 18

34. **The square-root of the difference between the squares of the "result" stated above (in vs. 31) (i.e., *chāyākarṇāgrī agrā*) and the length of the shadow, gives (half the length of) the rising-setting line (in the shadow sphere). The distance between this rising-setting line and (the gnomon's position in) the circle forming the locus of the gnomon, is the equinoctial midday shadow in the shadow circle.**

1. *Cf. BrSpSi,* iii. 4 (a-b); *SiŚe,* iv. 6 (c-d); *SiŚi,* I, iii. 72 (a-b).

This is so because in the shadow-circle the *śaṅkutala* (i.e., the distance of the moving gnomon from the rising-setting line) is equal to the equinoctial midday shadow. For, we have already shown that

$$\frac{\textit{śaṅkutala} \times \text{shadow}}{\text{Rsin } z} \text{ or } \frac{\textit{śaṅkutala} \times h}{\text{R}} = \text{equi. midday shadow,}$$

where h denotes the hypotenuse of shadow.

Method 19

35-36. The result obtained by multiplying the distance (of the local place from the equator) along the meridian of Ujjayinī by 5, or the distance of the local equatorial place or the equator from the local place, as multiplyed by 5, when divided by 46 gives the degrees of the (local) latitude, and when divided by 5×40 gives the equinoctial midday shadow (at the local place, in terms of *aṅgulas*).[1]

Let the distance of the local place from the equator be Y *yojanas*. Then the degrees of the local latitude are equal to

$$\frac{360 \times Y}{\text{Earth's circumference}}$$

$$= \frac{360 \times Y}{1054 \times 3{\cdot}1416}$$

$$= \frac{5\,Y}{46},$$

because, according to our author, Earth's diameter = 1054 *yajanas*.[2]

Now, we know that at Ujjayinī, latitude =24° and equinoctial midday shadow = 5·5 *aṅgulas*. So we apply the proportion : When the latitude is equal to 24° the equinoctial midday shadow is equal to 5·5 *aṅgulas*, what will be the equinoctial midday shadow when the latitude is equal to $5Y/46$ degrees ? The result, viz.

$$\frac{5{\cdot}5 \times 5\,Y}{24 \times 46}, \text{ i.e., } \frac{Y}{40} \textit{ aṅgulas},$$

is the equinoctial midday shadow at the local place.

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1. A similar rule is stated in *KR*, i. 33.
2. See ch. 1, sec. 8, vs. 3.

Section 2 : Latitude and Colatitude

This section gives a number of formulae for the Rsine of the latitude and the Rsine of the colatitude. A rule for the *ayana-calana* or precession of the equinoxes is also given at the end.

1. (Severally) multiply the square of the radius by the square of the *palabhā* (i.e., equinoctial midday shadow) and by the square of 12 and divide (each product) by the square of the *palakarṇa* (i.e., hypotenuse of the equinoctial midday shadow) : the square-roots of the resulting quotients are the Rsine of the latitude and the Rsine of the colatitude, (respectively).[1]

$$\text{Rsin}\,\phi = \sqrt{\frac{(\text{radius})^2 \times (palabh\bar{a})^2}{(palakar\d{n}a)^2}} \tag{1}$$

$$\text{Rcos}\,\phi = \sqrt{\frac{(\text{radius})^2 \times 12^2}{(palakar\d{n}a)^2}} \tag{2}$$

where ϕ is the latitude of the local place.

2. Or, multiply the radius (severally) by the square of the *palabhā* and by the square of 12, and divide (the resulting products) by *palabhā* into *palakarṇa* and 12 into *palakarṇa*, (respectively); or, multiply the radius (severally) by the *palabhā* and by 12, and divide (both the resulting products) by the *palakarṇa* only. (The results in both the cases are the Rsine of the latitude and the Rsine of the colatitude, respectively).[2]

$$\text{Rsin}\,\phi = \frac{\text{radius} \times (palabh\bar{a})^2}{palabh\bar{a} \times palakar\d{n}a} \tag{3}$$

$$\text{Rcos}\,\phi = \frac{\text{radius} \times 12^2}{12 \times palakar\d{n}a} \tag{4}$$

or $$\text{Rsin}\,\phi = \frac{\text{radius} \times palabh\bar{a}}{palakar\d{n}a} \tag{5}$$

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1. *Cf. BrSpSi*, iii. 9; *SiŚe*, iv. 10 (a-b).
2. *Cf. BrSpSi*, iii. 10; *KK* (BC), iii. 11; *ŚiDVr*, iv. 5 (a-b); *SūSi*, iii. 13-14; *LBh*, iii. 2-3; *MBh*, iii. 5; *SiŚe*, iv. 7; *SiŚi*, I, iii. 13-18; *SiSā*, iii. 49; *KP*, viii. 1, 2 (a-b).

$$\text{Rcos}\,\phi = \frac{\text{radius} \times 12}{palakarṇa}. \tag{6}$$

Bhāskara II has given a number of similar formulae. See *SiŚi*, I, iii. 13-18.

3. Or, multiply the *palabhā* and 12 by the square of the radius and divide (the resulting products) by radius into *palakarṇa*; or, multiply (the same) by the radius and divide by the *palakarṇa*. The results, as before, are the Rsine of the latitude and the Rsine of the colatitude, respectively.

$$\text{Rsin}\,\phi = \frac{palabhā \times (\text{radius})^2}{\text{radius} \times palakarṇa} \tag{7}$$

$$\text{Rcos}\,\phi = \frac{12 \times (\text{radius})^2}{\text{radius} \times palakarṇa}; \tag{8}$$

or

$$\text{Rsin}\,\phi = \frac{palabhā \times \text{radius}}{palakarṇa} \tag{9}$$

$$\text{Rcos}\,\phi = \frac{12 \times \text{radius}}{palakarṇa}. \tag{10}$$

4. Or, multiply the radius (severally) by the *palabhā* and by 12 and divide (the resulting products) by the *palakarṇa*: the results are the Rsines of the latitude and colatitude respectively. These multiplied by 12 and the *palabhā* (respectively) and divided by the *palabhā* and 12 (respectively) give the other.[1]

$$\text{Rsin}\,\phi = \frac{\text{Radius} \times palabhā}{palakarṇa} \tag{11}$$

$$\text{Rcos}\,\phi = \frac{\text{Radius} \times 12}{palakarṇa}; \tag{12}$$

and

$$\text{Rcos}\,\phi = \frac{\text{Rsin}\,\phi \times 12}{palabhā} \tag{13}$$

$$\text{Rsin}\,\phi = \frac{\text{Rcos}\,\phi \times palabhā}{12}. \tag{14}$$

It will be noted that formulae (1) and (2), (3) and (4), (5) and (6), (7) and (8), (9) and (10), and (11) and (12) are the same. The difference, if any, exists in form only.

1. *Cf.* this latter rule with *BrSpSi*, iii. 11 (c-d); *SiŚe*, iv. 9,

5. Or, the Rsine of the latitude is equal to the square-root of the square of the radius minus the square of the Rsine of the colatitude; and the Rsine of the colatitude is the square-root of the difference between the squares of the radius and the Rsine of the latitude.[1]

$$\text{Rsin}\,\phi = \sqrt{(\text{Radius})^2 - (\text{Rcos}\,\phi)^2} \tag{15}$$

and

$$\text{Rcos}\,\phi = \sqrt{(\text{Radius})^2 - (\text{Rsin}\,\phi)^2}. \tag{16}$$

6. Or, the Rsine of the latitude is equal to the result obtained by multiplying the earthsine by the hypotenuse of shadow and dividing (the resulting product) by the *agrā* corresponding to the shadow-circle; and the Rsine of the colatitude is equal to the result obtained by multiplying the Rsine of the Sun's longitude by the Rsine of 24° and dividing (the product) by the *agrā*.

$$\text{Rsin}\,\phi = \frac{\text{earthsine} \times \text{hypotenuse of shadow}}{\textit{agrā} \text{ corresponding to shadow-circle}} \tag{17}$$

and

$$\text{Rcos}\,\phi = \frac{\text{Rsin (Sun's longitude)} \times \text{Rsin } 24^\circ}{\textit{agrā}}. \tag{18}$$

Rationale. Comparing the latitude-triangles (*C*) and (*E*), given on page 275, we have

$$\text{Rsin}\,\phi = \frac{\text{earthsine} \times \text{R}}{\textit{agrā}} \tag{i}$$

and

$$\text{Rcos}\,\phi = \frac{\text{Rsin}\,\delta \times \text{R}}{\textit{agrā}}. \tag{ii}$$

But

$$\frac{\text{R}}{\textit{agrā}} = \frac{\text{hypotenuse of shadow}}{\textit{chāyākarṇāgrī agrā}} \tag{iii}$$

and

$$\text{Rsin}\,\delta = \frac{\text{Rsin}\,\lambda \times \text{Rsin } 24^\circ}{\text{R}}, \tag{iv}$$

where λ is the Sun's tropical longitude and δ the Sun's declination.

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1. *Cf. SiŚe*, iv. 10 (c-d). Similar rules for general argument have been already given above. See *supra*, chap. 2, sec. 1, vs. 56 (a-b). Also *cf. BrSpSi*, iii. 12 (a-b).

Therefore, from (i) and (iii),

$$\text{Rsin } \phi = \frac{\text{earthsine} \times \text{hypotenuse of shadow}}{\textit{chāyākarṇāgrī agrā}}$$

and from (ii) and (iv)

$$\text{Rcos } \phi = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{\textit{agrā}}.$$

7. **Or, the Rsine of the latitude is the square-root of the product of the results obtained by diminishing and increasing the radius by the Rsine of the colatitude; and the other one (i.e., the Rsine of the colatitude) is the square-root of the product of the results obtained by diminishing and increasing the radius by the Rsine of the latitude.**[1]

$$\text{Rsin } \phi = \sqrt{(\text{R} - \text{Rcos } \phi)(\text{R} + \text{Rcos } \phi)} \tag{19}$$

and $$\text{Rcos } \phi = \sqrt{(\text{R} - \text{Rsin } \phi)(\text{R} + \text{Rsin } \phi)} \tag{20}$$

8. **The same (i.e., the Rsines of the latitude and colatitude) are obtained also on multiplying the earthsine and the Rsine of the declination by the radius and dividing (the products) by the *agrā*;**[2] **or, on multiplying the *agrā* and the *samaśaṅku* by the radius and dividing (the products) by the *taddhṛti*.**

$$\text{Rsin } \phi = \frac{\text{earthsine} \times \text{radius}}{\textit{agrā}} \tag{21}$$

and $$\text{Rcos } \phi = \frac{\text{Rsin } \delta \times \text{radius}}{\textit{agrā}} \tag{22}$$

or, $$\text{Rsin } \phi = \frac{\textit{agrā} \times \text{radius}}{\textit{taddhṛti}} \tag{23}$$

and $$\text{Rcos } \phi = \frac{\textit{samaśaṅku} \times \text{radius}}{\textit{taddhṛti}}. \tag{24}$$

The rationale of formulae (21) and (22) has already been given above (under vs. 6). Formulae (23) and (24) follow from the comparison of the latitude-triangles (*B*) and (*E*), given on page 275.

1. Similar rules have already been given. See *supra*, ch. 2, sec. 1, vs. 56 (b).

2. *Cf. BrSpSi*, xv. 35 (c-d)-36 (a-b), also 43 (c-d)-44 (a-b); *SiŚe*, iv. 92 (b-c).

9. The Rsines of the latitude and the colatitude are also obtained on multiplying the radius (severally) by the *śaṅkutala* and the *śaṅku* and dividing (the products) by the *svadhṛti*.[1] **Also, the Rsine of three signs diminished by the latitude or the colatitude gives the other.**[2]

$$\text{Rsin}\ \phi = \frac{\textit{śaṅkutala} \times \text{R}}{\textit{svadhṛti}} \tag{25}$$

$$\text{Rcos}\ \phi = \frac{\textit{śaṅku} \times \text{R}}{\textit{svadhṛti}} \tag{26}$$

and

$$\text{Rsin}\ \phi = \text{Rsin}\ (90^\circ - \text{colatitude}) \tag{27}$$

$$\text{Rcos}\ \phi = \text{Rsin}\ (90^\circ - \text{latitude}). \tag{28}$$

Formulae (27) and (28) are obvious. Formulae (25) and (26) follow from the comparison of the latitude-triangles (*A*) and (*E*), given on pages 274 and 275.

10. Or, multiply the Rsine of the latitude (severally) by the *samaśaṅku*, the Rsine of the declination and the Rsine of the altitude and divide (the resulting products) by the *agrā*, the earthsine and the *śaṅkutala*, respectively : the result (in each case) is the Rsine of the colatitude.

11. (Similarly) multiply the Rsine of the colatitude (severally) by the earthsine, *agrā* and *śaṅkutala* and divide (the resulting products) by the Rsine of the declination, the *samaśaṅku* and *sveṣṭaśaṅku*, respectively : the result (in each case) is the Rsine of the latitude.

$$\text{Rcos}\ \phi = \frac{\text{Rsin}\ \phi \times \textit{samaśaṅku}}{\textit{agrā}} \tag{29}$$

$$= \frac{\text{Rsin}\ \phi \times \text{Rsin}\ \delta}{\text{earthsine}} \tag{30}$$

$$= \frac{\text{Rsin}\ \phi \times \text{Rsin}\ a}{\textit{śaṅkutala}} \tag{31}$$

$$\text{Rsin}\ \phi = \frac{\text{Rcos}\ \phi \times \text{earthsine}}{\text{Rsin}\ \delta} \tag{32}$$

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1. *Cf. SiŚe*, iv. 115.
2. Similar rules have already been given above. See *supra*, ch. 2, sec. 1, vs. 57 (c-d). Also compare the rule stated in vs. 9 (c-d) with that in *BrSpSi*, iii. 11 (a-b) and *SiŚe*, iv. 8 (a-d).

$$= \frac{\text{Rcos } \phi \times agr\bar{a}}{sama\acute{s}a\dot{n}ku} \tag{33}$$

$$= \frac{\text{Rcos } \phi \times \acute{s}a\dot{n}kutala}{\text{Rsin } a}. \tag{34}$$

These formulae follow from the comparison of the similar right-angled triangles already mentioned above.

Sveṣṭaśaṅku is the Rsine of the own altitude for the desired time, or simply, the Rsine of the altitude.

12. Alternatively, the product of the Rsine of 24° and the Rsine of the Sun's *bhuja* divided by the *samaśaṅku* is equal to the Rsine of the latitude.[1] Or else, the product of the Rsine of the (Sun's) declination and the Rsine of three signs divided by the *samaśaṅku* is equal to the Rsine of the latitude.

$$\text{Rsin } \phi = \frac{\text{Rsin } \lambda \times \text{Rsin } 24°}{sama\acute{s}a\dot{n}ku} \text{ or } \frac{\text{Rsin } \delta \times \text{R}}{sama\acute{s}a\dot{n}ku}, \tag{35}$$

where λ denotes the Sun's *bhuja* (longitude) and δ the Sun's declination.

Rationale. Comparison af the latitude-triangles (*D*) and (*E*), given on page 275, yields the formula

$$\text{Rsin } \phi = \frac{\text{Rsin } \delta \times \text{R}}{sama\acute{s}a\dot{n}ku}.$$

Substitution of

$$\text{Rsin } \delta = \frac{\text{Rsin } \lambda \times \text{Rsin } 24°}{\text{R}}$$

gives the other. See *infra*, sec 3, vs. 1(c-d).

13. Multiply the difference between the earthsine and the *agrā* and the difference between the Rsine of the declination and the *agrā* (severally) by the radius and divide (each product) by the *agrā*: the results (thus obtained) are the Rversed-sines of the colatitude and the latitude, respectively.

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1. *Cf. BrSpSi*, xv. 28; *SiŚe*, iv. 103.

$$\text{Rvers}\,(90^\circ - \phi) = \frac{(agr\bar{a} - \text{earthsine}) \times R}{agr\bar{a}} \tag{36}$$

and
$$\text{Rvers}\,\phi = \frac{(agr\bar{a} - \text{Rsin}\,\delta) \times R}{agr\bar{a}}. \tag{37}$$

Rationale.
$$\frac{(agr\bar{a} - \text{earthsine}) \times R}{agr\bar{a}} = R - \frac{\text{earthsine} \times R}{agr\bar{a}}$$
$$= R - \text{Rsin}\,\phi$$
$$= \text{Rvers}\,(90^\circ - \phi);$$

and
$$\frac{(agr\bar{a} - \text{Rsin}\,\delta) \times R}{agr\bar{a}} = R - \frac{\text{Rsin}\,\delta \times R}{agr\bar{a}}$$
$$= R - \text{Rsin}\,(90^\circ - \phi)$$
$$= \text{Rvers}\,\phi.$$

14. Multiply the difference between the *palakarṇa* and 12 and the difference between the *palakarṇa* and the *palabhā* (severally) by the radius and divide (each product) by the *palakarṇa* : the results are the Rversed-sines of the latitude and the colatitude (respectively). These (results) are also equal to the difference between the radius and the Rsine of the colatitude and that between the radius and the Rsine of the latitude, respectively.

$$\text{Rvers}\,\phi = \frac{(palakar\d{n}a - 12) \times R}{palakar\d{n}a} = R - \text{Rcos}\,\phi \tag{38}$$

$$\text{Rvers}\,(90^\circ - \phi) = \frac{(palakar\d{n}a - palabh\bar{a}) \times R}{palakar\d{n}a} = R - \text{Rsin}\,\phi. \tag{39}$$

15. Multiply the differences between the *agrā* and the *taddhṛti* and between the *taddhṛti* and the *samaśaṅku* (severally) by the radius and divide (each product) by the *taddhṛti* : the results are the Rversed-sines of the colatitude and the latitude, (respectively).

$$\text{Rvers}\,(90^\circ - \phi) = \frac{(taddh\d{r}ti - agr\bar{a}) \times R}{taddh\d{r}ti} \tag{40}$$

$$\text{Rvers}\,\phi = \frac{(taddh\d{r}ti - sama\acute{s}a\dot{n}ku) \times R}{taddh\d{r}ti}. \tag{41}$$

16. Multiply the differences between the *śaṅkutala* and the *svadhṛti* and between the *svadhṛti* and the *śaṅku* (i.e., Rsine of the altitude) (severally) by the radius and divide (each product) by the *svadhṛti* : the results are the Rversed-sines of the colatitude and the latitude, (respectively).

$$\text{Rvers}\,(90° - \phi) = \frac{(svadhṛti - śaṅkutala) \times R}{svadhṛti} \tag{42}$$

$$\text{Rvers}\,\phi = \frac{(svadhṛti - śaṅku) \times R}{svadhṛti}. \tag{43}$$

17. Divide the square of the Rsine of the latitude by the Rversed-sine of the latitude and the square of the Rsine of the colatitude by the Rversed-sine of the colatitude, and diminish (the two results thus obtained) by the radius : the results are the Rsines of the colatitude and the latitude, respectively.[1]

$$\text{Rcos}\,\phi = \frac{(\text{Rsin}\,\phi)^2}{\text{Rvers}\,\phi} - R \tag{44}$$

$$\text{Rsin}\,\phi = \frac{(\text{Rcos}\,\phi)^2}{\text{Rvers}\,(90° - \phi)} - R. \tag{45}$$

18. Multiply the Rversed-sines of the latitude and the colatitude by the diameter, and diminish (the resulting products) by the squares of the self-same Rversed-sines : the square-roots thereof are the Rsines of the latitude and the colatitude respectively.

Or, (severally) diminish the diameter by the same Rversed-sines, then multiply by the same Rversed-sines, and then take the square-root : the results are the Rsines of the latitude and the colatitude respectively.[2]

$$\text{Rsin}\,\phi = \sqrt{2R \times \text{Rvers}\,\phi - (\text{Rvers}\,\phi)^2} \tag{46}$$

$$\text{Rcos}\,\phi = \sqrt{2R \times \text{Rvers}\,(90° - \phi) - \{\text{Rvers}\,(90 - \phi)\}^2} \tag{47}$$

and

$$\text{Rsin}\,\phi = \sqrt{\text{Rvers}\,\phi\,(2R - \text{Rvers}\,\phi)} \tag{48}$$

$$\text{Rcos}\,\phi = \sqrt{\text{Rvers}\,(90° - \phi)\,\{2R - \text{Rvers}\,(90° - \phi)\}} \tag{49}$$

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1. Similar rules have already been given above. See *supra*, ch. 2, sec. 1. vs. 57(a-b).
2. Similar rules have already been given above. See *supra*, ch. 2, sec. 1, vs. 56(c-d).

19. **Half of what is obtained by subtracting the square of the difference between the Rversed-sines of the latitude and the colatitude from the square of the radius, when divided by the Rsine of the latitude gives the Rsine of the colatitude, and when divided by the Rsine of the colatitude gives the Rsine of the latitude.**

$$\text{Rsin}\,(90^\circ - \phi) = \frac{[R^2 - \{\text{Rvers}\,\phi \sim \text{Rvers}\,(90^\circ - \phi)\}^2]/2}{\text{Rsin}\,\phi} \quad (50)$$

$$\text{Rsin}\,\phi = \frac{[R^2 - \{\text{Rvers}\,\phi \sim \text{Rvers}\,(90^\circ - \phi)\}^2]/2}{\text{Rcos}\,\phi} \quad (51)$$

20. **Multiply the square of the radius by two; therefrom subtract the square of the difference between the Rversed-sines of the latitude and the colatitude; extract the square-root of that; (severally) diminish and increase that by the said difference (between the Rversed-sines of the latitude and the colatitude); and then halve the resulting quantities. The results (thus obtained) are again the Rsines of the latitude and the colatitude.**

$$\text{Rsin}\,\phi = \frac{\sqrt{2R^2 - [\text{Rvers}\,(90^\circ - \phi) - \text{Rvers}\,\phi]^2} - [\text{Rvers}(90^\circ - \phi) - \text{Rvers}\,\phi]}{2} \quad (52)$$

$$\text{Rcos}\,\phi = \frac{\sqrt{2R^2 - [\text{Rvers}\,(90^\circ - \phi) - \text{Rvers}\,\phi]^2} + [\text{Rvers}\,(90^\circ - \phi) - \text{Rvers}\,\phi]}{2} \quad (53)$$

21. **The same (square-root) when diminished by the Rsine of the latitude gives the Rsine of the colatitude, and when diminished by the Rsine of the colatitude gives the Rsine of the latitude. The difference between the radius and the Rversed-sine of the latitude gives the Rsine of the colatitude, and the difference between the radius and the Rversed-sine of the colatitude gives the Rsine of the latitude.**[1]

$$\text{Rcos}\,\phi = \sqrt{2R^2 - \{\text{Rvers}\,\phi \sim \text{Rvers}\,(90^\circ - \phi)\}^2} - \text{Rsin}\,\phi \quad (54)$$

$$\text{Rsin}\,\phi = \sqrt{2R^2 - \{\text{Rvers}\,\phi \sim \text{Rvers}\,(90^\circ - \phi)\}^2} - \text{Rcos}\,\phi; \quad (55)$$

and

$$\text{Rsin}\,(90^\circ - \phi) = R - \text{Rvers}\,\phi \quad (56)$$

$$\text{Rsin}\,\phi = R - \text{Rvers}\,(90^\circ - \phi). \quad (57)$$

1. This latter rule occurs also in *BrSpSi*, iii. 10(c-d) and *SiŚe*, iv. 8(c-d).

22. Or, the product of the Rsine of the ascensional difference and the day-radius, divided by the *agrā*, is the Rsine of the latitude; or, the product of the *samakarṇa* (i.e., hypotenuse of the gnomonic shadow when the heavenly body is on the prime vertical) and the Rsine of the declination, divided by 12, is the Rsine of the latitude.

$$\text{Rsin}\,\phi = \frac{\text{Rsin (asc. diff.)} \times \text{Rcos}\,\delta}{agr\bar{a}} \tag{58}$$

$$\text{Rsin}\,\phi = \frac{samakar\d{n}a \times \text{Rsin}\,\delta}{12}, \tag{59}$$

δ being the declination.

Rationale. Since

$$\text{Rsin}\,\phi = \frac{\text{R} \times \text{earthsine}}{agr\bar{a}}$$

$$\text{and earthsine} = \frac{\text{Rsin (asc. diff.)} \times \text{Rcos}\,\delta}{\text{R}},$$

$$\text{therefore,} \quad \text{Rsin}\,\phi = \frac{\text{Rsin (asc. diff.)} \times \text{Rcos}\,\delta}{agr\bar{a}}.$$

Again, since

$$\text{Rsin}\,\phi = \frac{\text{R} \times \text{Rsin}\,\delta}{sama\acute{s}a\dot{n}ku}$$

$$\text{and} \quad sama\acute{s}a\dot{n}ku = \frac{12 \times \text{R}}{samakar\d{n}a},$$

$$\text{therefore,} \quad \text{Rsin}\,\phi = \frac{samakar\d{n}a \times \text{Rsin}\,\delta}{12}.$$

23. The product of the *śaṅku* (i.e., Rsine of the Sun's altitude), the *palabhā* and the Rsine of the latitude, divided by the *śaṅkutala*, when (further) divided by the *palabhā* gives the Rsine of the colatitude. The same result is also obtained on multiplying the Rsine of declination by the hypotenuse of shadow and dividing (that product) by the *agrā* corresponding to the shadow-sphere.

$$\text{Rsin}\,(90^\circ - \phi) = \frac{\acute{s}a\dot{n}ku \times palabh\bar{a} \times \text{Rsin}\,\phi}{\acute{s}a\dot{n}kutala \times palabh\bar{a}} = \frac{\acute{s}a\dot{n}ku \times \text{Rsin}\,\phi}{\acute{s}a\dot{n}kutala} \tag{60}$$

$$\text{Rsin}\,(90^\circ - \phi) = \frac{\text{Rsin}\,\delta \times \text{hypotenuse of shadow}}{agr\bar{a}\text{ for the shadow-sphere}}. \qquad (61)$$

The latter formula is equivalent to :

$$\text{Rsin}\,(90^\circ - \phi) = \frac{\text{Rsin}\,\delta \times \text{R}}{agr\bar{a}}.$$

LATITUDE IS ALWAYS SOUTH

24 (a-c). Their arcs (i.e., the arcs of Rsin (90° — φ) and Rsin φ) are the colatitude and the latitude;[1] the versed arcs of their Rversed-sines are also the colatitude and the latitude. The latitude is south and the equinoctial midday shadow is also south.[2]

It should be noted that in Hindu astronomy the latitude of a place is defined as the distance of the equator measured from the zenith of the place; and this distance is always south. The equinoctial midday shadow is taken to be of south direction, because it is the *śaṅkutala* reduced to the shadow-sphere and *śaṅkutala,* being always south of the rising-setting line, is taken to be of south direction.

AYANA-CALANA OR PRECESSION OF THE EQUINOXES

24 (d)-27. Multiply the Rsine of the difference between the latitudes due to the (*sāyana* and *nirayaṇa*) *meṣādi* or *tulādi* by the radius and divide (the resulting product) by the Rsine of the (Sun's) greatest declination. The arc corresponding to that (is the *ayana-calana,* which) should be added to the longitude of a planet provided the (midday) shadow of the gnomon at the time of (the Sun's next position at the *nirayaṇa*) *tulādi* is greater than the (midday) shadow of the gnomon at the time of (the Sun's previous position at the *nirayaṇa*) *meṣādi,* and subtracted in the contrary case. In the case of the Moon's ascending node, it is to be applied contrarily. This correction should be applied in all calculations pertaining to the Three Problems. The astronomer, who is proficient in trigonometry (lit. the science of arc), should calculate the *ayana-calana,* when the Sun is six signs distant from the *nirayaṇa meṣādi* or *tulādi* or at any desired time, by the application of his own intellect.

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1. *Cf. SiŚe,* iv. 10(d).
2. *Cf. BrSpSi,* iii. 4(c); xv. 41(d); *ŚiDVr,* iv. 5(d); *SiŚe,* iv. 11(a).

The midday shadow of the gnomon at the time of the Sun's position at the *nirayaṇa tulādi* is greater than the midday shadow of the gnomon at the time of the Sun's position at the *nirayaṇa meṣādi* provided the (tropical) longitude of the *nirayaṇa meṣādi* is greater than zero and the (tropical) longitude of the *nirayaṇa tulādi* is greater than 6 signs. In the contrary case, the (tropical) longitude of the *nirayaṇa meṣādi* is less than 360° and the (tropical) longitude of the *nirayaṇa tulādi* is less than 6 signs.

The above rule is correct, because the difference between the local latitudes derived from the midday shadows corresponding to the Sun's positions at the *sāyana* and *nirayaṇa meṣādi* or *tulādi* is equal to the difference between the Sun's declinations at those positions of the Sun.

It is noteworthy that the author Vaṭeśvara prescribes the application of the *ayanacalana* (precession of the equinoxes) to celestial longitudes in all calculations pertaining to the Three Problems. The reader should note that the *ayancalana* has to be applied to celestial longitudes whenever they are to be reckoned from the first point of Aries. When they are to be reckoned from the first point of the *nakṣatra* Aśvinī, this correction is not to be applied.

Section 3 : The Sun's declination

1(a-b). The greatest declination (of the Sun) is 24 degrees and the Rsine of the (Sun's) greatest declination is stated to be the Rsine of 24 degrees.

1(c-d)-2. The Rsine of the Sun's *bhuja* multiplied by that (Rsine of the Sun's greatest declination) and divided by the radius gives the Rsine of the (Sun's) desired declination.[1]

The Rsine of the Sun's *bhuja* multiplied by 416 and divided by 1023 (also) gives the Rsine of the (Sun's) desired declination.[2] The arc of that is the (Sun's) desired declination.

$$\text{Rsin } \delta = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{\text{R}} \tag{1}$$

$$= \frac{416 \times \text{Rsin } \lambda}{1023}, \tag{2}$$

where λ is the Sun's (tropical) longitude[3] (reduced to *bhuja*), δ the Sun's declination, and R= 3437′ 44″.

Formulae (1) and (2) are equivalent, because

$$\frac{\text{Rsin } 24^\circ}{\text{R}} = \frac{1398'\ 13''}{3437'\ 44''} = \frac{416}{1023}.$$

The rationale of formula (1) is as follows :

Let O be the centre of the Celestial Sphere; and TSC the ecliptic, T being the first point of Aries, S the Sun and C the first point of Cancer, so that TC=90°. Let SA be the perpendicular from S on OT; and SB the perpendicular from S and CD the perpendicular from C on the plane of the celestial equator. Then in the triangle SAB, SA = Rsin λ, SB = Rsin δ, $\angle$SAB = 24° and $\angle$SBA = 90° ; and in the triangle COD, CO = R, the radius of the Celestial Sphere, CD = Rsin 24°, $\angle$COD = 24° and $\angle$CDO = 90°. Since the triangles SAB and COD are similar, and SA and SB are

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1. *Cf. BrSpSi*, ii, 55; *ŚiDVr*, ii. 17; *MSi*, iii. 11(c-d); *SiŚe*, iii. 63; *SiŚi*, I, ii. 47 (c-d).
2. Similar rules occur in *MBh*, iv. 25; *ŚiDVr*, ix. 1; *SiŚe*, iii. 64.
3. That is, longitude corrected for *ayanacalana* (precession of the equinoxes).

parallel to CO and CD respectively, therefore SB/CD = SA/CO, i.e.,

$$\frac{\text{Rsin } \delta}{\text{Rsin } 24^\circ} = \frac{\text{Rsin } \lambda}{\text{R}}.$$

3. Or, the Rsine computed (from the minutes of the Sun's declination) with the help of the (ninety six) Rsines stated before is the Rsine of the (Sun's) declination. (From the Rsine of the Sun's declination) the minutes of the (Sun's) declination should be calculated as before.

4. Or, the earthsine (severally) multiplied by the Rsine of the colatitude, the Rsine of the desired altitude (*iṣṭanṛ* or *iṣṭaśaṅku*), the Rsine of the prime vertical altitude (*samanara* or *samaśaṅku*), and 12 and divided by the Rsine of the latitude, the *śaṅkutala* (*nṛtala*), the *agrā*, and the equinoctial midday shadow respectively gives (in each case) the Rsine of the declination.

$$\text{Rsin } \delta = \frac{\text{earthsine} \times \text{Rcos } \phi}{\text{Rsin } \phi} \tag{3}$$

$$= \frac{\text{earthsine} \times \text{Rsin (altitude)}}{\textit{śaṅkutala}} \tag{4}$$

$$= \frac{\text{earthsine} \times \text{Rsin (prime vertical altitude)}}{\textit{agrā}} \tag{5}$$

$$= \frac{\text{earthsine} \times 12}{\textit{palabhā}}. \tag{6}$$

Formula (6) occurs also in *BrSpSi*, xv. 44 (c-d); *SiŚe*, iv. 92 (d).

These formulae and those that follow may be easily derived from the comparison of the latitude-triangles.

5. Or, the Rsine of the *agrā* severally multiplied by 12, the Rsine of the colatitude, the Rsine of the desired altitude (*iṣṭaśaṅku*) and the Rsine of the prime vertical altitude (*samaśaṅku*), and divided by the hypotenuse of the equinoctial midday shadow (*akṣaśruti* or *palakarṇa*), the radius, the *iṣṭadhṛti* (*nijadhṛti* or *svadhṛti*) and the *taddhṛti* respectively yields the Rsine of the declination.

$$\text{Rsin } \delta = \frac{\textit{agrā} \times 12}{\textit{palakarṇa}} \tag{7}$$

$$= \frac{\textit{agrā} \times \text{Rcos } \phi}{\text{R}} \tag{8}$$

$$= \frac{agr\bar{a} \times i\d{s}\d{t}a\acute{s}a\dot{n}ku}{i\d{s}\d{t}adh\d{r}ti} \tag{9}$$

$$= \frac{agr\bar{a} \times \text{Rsin (prime vertical altitude)}}{taddh\d{r}ti}. \tag{10}$$

6. **The *samaśaṅku* (severally) divided by the *agrā*, the hypotenuse of the equinoctial midday shadow, the *iṣṭadhṛti* and the radius and multiplied by the earthsine, the *palabhā*, the *iṣṭaśaṅkutala* and the Rsine of the latitude respectively, yields (in each case) the Rsine of the declination.**

$$\text{Rsin } \delta = \frac{sama\acute{s}a\dot{n}ku \times \text{earthsine}}{agr\bar{a}} \tag{11}$$

$$= \frac{sama\acute{s}a\dot{n}ku \times palabh\bar{a}}{palakar\d{n}a} \tag{12}$$

$$= \frac{sama\acute{s}a\dot{n}ku \times i\d{s}\d{t}a\acute{s}a\dot{n}kutala}{i\d{s}\d{t}adh\d{r}ti} \tag{13}$$

$$= \frac{sama\acute{s}a\dot{n}ku \times \text{Rsin } \phi}{\text{R}}. \tag{14}$$

7. **The *taddhṛti* multiplied by (the product of) the Rsine of the latitude and the Rsine of the colatitude and divided by the square of the radius is also the same. The *taddhṛti* multiplied by (the product of) the *śaṅkutala* and the *śaṅku* and divided by the square of the *iṣṭadhṛti* is also the same.**

$$\text{Rsin } \delta = \frac{\text{Rsin } \phi \times \text{Rcos } \phi \times taddh\d{r}ti}{\text{R} \times \text{R}} \tag{15}$$

$$= \frac{\acute{s}a\dot{n}kutala \times \acute{s}a\dot{n}ku \times taddh\d{r}ti}{(i\d{s}\d{t}adh\d{r}ti)^2}. \tag{16}$$

8. **The Rsine of the latitude and the Rsine of the colatitude multilied by 12 and the *palabhā* (respectively) and divided by the *samakarṇa* (i.e., the hypotenuse of the prime vertical shadow) give the Rsine of the declination.**[1]

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1. *Cf. BrSpSi*, xv. 27; *SiŚe*, iv. 66, 102 (repeated). Also see *SūSi*, iii. 25(c-d)-26(a-b).

The square-root of the difference between the squares of the earth-sine and the *agrā* is also the same.

$$\text{Rsin } \delta = \frac{\text{Rsin } \phi \times 12}{samakarṇa} \tag{17}$$

$$= \frac{\text{Rcos } \phi \times palabhā}{samakarṇa} \tag{18}$$

$$\text{Rsin } \delta = \sqrt{(agrā)^2 - (\text{earthsine})^2} \tag{19}$$

9. Multiply the Rsine of the meridian altitude (lit. altitude for midday) by the *palakarṇa* and divide by 12 : (the result is the *dhṛti* for midday). Of that result and the earthsine, take the sum or difference according as the hemisphere is southern or northern : (the result is the day-radius). The square-root of the difference between the squares of that and the radius is the Rsine of the declination.

$$\text{Rsin } \delta = \sqrt{R^2 - (\text{Rcos } \delta)^2}, \tag{20}$$

where Rcos δ = *dhṛti* for midday + or $\sim$ earthsine,

and *dhṛti* for midday $= \dfrac{\text{Rsin (meridian altitude)} \times palakarṇa}{12}$,

+ or $\sim$ sign being taken according as the hemisphere is southern or northern.

10. The product of the *palabhā* and the (*iṣṭa*) *dhṛti* divided by 12 gives the *cheda* (divisor). The *śaṅkutala* multiplied by the *agrā* and divided by the *cheda* gives the Rsine of the declination. The same is also obtained by dividing the square of the *agrā* by the *samaccheda*.

11. The *dhṛti* (i.e., *iṣṭadhṛti*) multiplied by the earthsine and divided by the *cheda* is again the Rsine of the declination. The same is obtained also by multiplying the Rsine of the altitude by the product of the *palabhā* and the *agrā* and dividing that by 12 times the *hāra* (i.e., *cheda*).

$$\text{Rsin } \delta = \frac{śaṅkutala \times agrā}{cheda} \tag{21}$$

$$= \frac{(agrā)^2}{samaccheda} \tag{22}$$

$$= \frac{\text{earthsine} \times iṣṭadhṛti}{cheda} \tag{23}$$

$$= \frac{palabh\bar{a} \times agr\bar{a} \times \text{Rsin (altitude)}}{12 \times cheda}, \tag{24}$$

where

$$cheda = \frac{palabh\bar{a} \times i\d{s}\d{t}adh\d{r}ti}{12}, \text{ and } samaccheda = \frac{palabh\bar{a} \times taddh\d{r}ti}{12}.$$

The *samaccheda* is the *cheda* for the prime vertical (*samavṛtta*).

12. The square-root of the difference between the squares of the day-radius and the radius is also the Rsine of the declination. The square-root of the result obtained by multiplying the sum of the radius and the day-radius by their difference is also the same.

$$\text{Rsin } \delta = \sqrt{R^2 - (\text{day-radius})^2} \tag{25}$$

$$= \sqrt{(R + \text{day-radius})(R - \text{day-radius})}. \tag{26}$$

13. The Rsine of the ascensional difference multiplied by the product of the day-radius and 12 and divided by the product of the *palabhā* and the radius, too, gives the Rsine of the own desired declination.

$$\text{Rsin } \delta = \frac{\text{Rcos } \delta \times 12 \times \text{Rsin (asc. diff.)}}{palabh\bar{a} \times R}. \tag{27}$$

The Rsine of the declination may also be obtained from the ascensional difference by the formula:[1]

$$\text{Rsin } \delta = \frac{R}{D},$$

where $$D = \sqrt{\left[\frac{(R \times palabh\bar{a})^2 + [\,12 \times \text{Rsin (asc. diff.)}\,]^2}{[12 \times \text{Rsin (asc. diff.)}]^2}\right]}.$$

1. See *BrSpSi*, xv. 36(c-d)-38; *SiŚe*, iv. 106; *SiŚi*, I, iii. 99.

Section 4

Day-radius or Radius of the Diurnal Circle

1. The day-radius is (equal to) the square-root of the difference obtained by subtracting the square of the Rsine of the declination from the square of the radius,[1] or the square-root of the product of the difference and sum of the radius and the Rsine of the declination.

Let δ be the declination. Then

$$\text{day-radius} = \text{R}\cos\delta$$

$$= \sqrt{\text{R}^2 - (\text{R}\sin\delta)^2} \tag{1}$$

$$= \sqrt{(\text{R} - \text{R}\sin\delta)\,(\text{R} + \text{R}\sin\delta)} \tag{2}$$

2. The day-radius is also equal to the result obtained by subtracting the radius from the square of the Rsine of the declination as divided by the Rversed sine of the declination. It is also equal to the difference between the radius and the Rversed-sine of the declination.[2]

$$\text{Day-radius} = \frac{(\text{R}\sin\delta)^2}{\text{Rvers}\,\delta} - \text{R} \tag{3}$$

$$= \text{R} - \text{Rvers}\,\delta. \tag{4}$$

3. The Rsine of the difference between three signs and the declination is also equal to the day-radius.[3] The radius multiplied by the earth-sine and divided by the Rsine of the ascensional difference also gives the day-radius.

$$\text{Day-radius} = \text{R}\sin(3\text{ signs} - \delta) \tag{5}$$

$$= \frac{\text{R} \times \text{earthsine}}{\text{R}\sin(\text{asc. diff.})}. \tag{6}$$

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1. *Cf. ŚiDVr*, ii. 18; *MSi*, iii. 17(c-d); *SiŚe*, iii. 65(a-b); *SiŚi*, I, ii. 47. (d)-48(a-b).
2. *Cf. SiŚe*, iii. 66(a-b).
3. *Cf. SiŚe*, iii. 66(c-d).

4. The radius multiplied by the *dhṛti* and divided by the *antyā* also gives the day-radius. The same is obtained also by dividing the product of the Rsine of the altitude, the radius, and the hypotenuse of the equinoctial midday shadow by 12 times the *antyā*.

$$\text{Day-radius} = \frac{R \times dhṛti}{antyā} \tag{7}$$

$$= \frac{\text{Rsin } a \times R \times palakarṇa}{12 \times antyā}, \tag{8}$$

where a is the altitude.

5. The day-radius is obtained also on dividing the product of the radius, the *śaṅkutala* and the hypotenuse of the equinoctial midday shadow by the product of the equinoctial midday shadow and the *antyā*, or by dividing the product of the Rsine of the latitude and the *agrā* by the Rsine of the ascensional difference.

$$\text{Day-radius} = \frac{R \times śaṅkutala \times palakarṇa}{palabhā \times antyā} \tag{9}$$

$$= \frac{\text{Rsin } \phi \times agrā}{\text{Rsin (asc. diff.)}}. \tag{10}$$

6. The day-radius is obtained also by dividing the product of the Rsine of the declination, the equinoctial midday shadow and the radius by twelve times the Rsine of the ascensional difference, or by dividing the product of the equinoctial midday shadow, the Rsine of the latitude, and the Rsine of the prime vertical altitude by twelve times the Rsine of the ascensional difference.

$$\text{Day-radius} = \frac{\text{Rsin } \delta \times palabhā \times R}{12 \times \text{Rsin (asc. diff.)}} \tag{11}$$

$$= \frac{palabhā \times \text{Rsin } \phi \times samaśaṅku}{12 \times \text{Rsin (asc. diff.)}} \tag{12}$$

Both are eqivalent because

$$\text{Rsin } \delta \times R = \text{Rsin } \phi \times samaśaṅku.$$

7. The product of the equinoctial midday shadow, the Rsine of the latitude and the *taddhṛti*, divided by (the product of) the hypotenuse of the equinoctial midday shadow and the Rsine of the ascensional difference

is also equal to the day-radius. The *dhṛti* for midday diminished by the earthsine when the hemisphere is northern and increased by the earthsine when the hemisphere is southern, also gives the day-radius.

$$\text{Day-radius} = \frac{palabh\bar{a} \times \text{Rsin}\,\phi \times taddh\d{r}ti}{palakar\d{n}a \times \text{Rsin (asc. diff.)}} \tag{13}$$

$$= dh\d{r}ti \text{ for midday} \pm \text{earthsine}, \tag{14}$$

$-$ or $+$ sign being taken according as the hemisphere is northern or southern.

8. The same *dhṛti* (i.e., *dhṛti* for midday) when diminished or increased by the result obtained by dividing the *agrā* as multiplied by the *śaṅkutala* for midday, by the *dhṛti* for midday, according as the hemisphere is northern or southern, also gives the day-radius.

$$\text{Day-radius} = dh\d{r}ti \text{ for midday} \pm \frac{agr\bar{a} \times (\acute{s}a\dot{n}kutala \text{ for midday})}{dh\d{r}ti \text{ for midday}}, \tag{15}$$

$-$ or $+$ sign being taken according as the hemisphere is northern or southern.

Section 5 : Earthsine

1. The Rsine of the declination multiplied by the Rsine of the latitude and divided by the Rsine of the colatitude gives the earthsine. The Rsine of the declination multiplied by the equinoctial midday shadow and divided by 12, too, yields the same.[1]

$$\text{Earthsine} = \frac{\text{Rsin}\,\delta \times \text{Rsin}\,\phi}{\text{Rcos}\,\phi} \tag{1}$$

$$= \frac{\text{Rsin}\,\delta \times \textit{palabhā}}{12}, \tag{2}$$

where δ is the declination and ϕ the local latitude.

2. Or, the product of the Rsine of the declination and the *agrā*, divided by the Rsine of the prime vertical altitude (*samanara* or *samaśaṅku*), gives the earthsine. Or, the *agrā* multiplied by the equinoctial midday shadow and divided by the hypotenuse of the equinoctial midday shadow is the earthsine.

$$\text{Earthsine} = \frac{\text{Rsin}\,\delta \times \textit{agrā}}{\textit{samaśaṅku}} \tag{3}$$

$$= \frac{\textit{agrā} \times \textit{palabhā}}{\textit{palakarṇa}}. \tag{4}$$

3. Or, the square of the *agrā* being divided by the *samadhṛti*, the result is the earthsine. Or, the *agrā* multiplied by the *śaṅkutala* and divided by the *svadhṛti* gives the earthsine.

$$\text{Earthsine} = \frac{(\textit{agrā})^2}{\textit{samadhṛti}} \tag{5}$$

$$= \frac{\textit{agrā} \times \textit{śaṅkutala}}{\textit{svadhṛti}}. \tag{6}$$

Bhāskara II states a number of other equivalent formulae involving the *agrā*.[2]

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1. *Cf. BrSpSi*, ii. 57(a-b); *MSi*, iii. 17(a-b); *SiŚe*, iii. 65(c-d); *SiŚi*, I. ii. 48(c-d).
2. See *SiŚi*, I, iii. 27(a-b).

Note. In the rules stated in vss. 4 to 10(a-b) the author has inadvertently interchanged the multipliers and divisors. While translating these verses this error has been rectified.

4. Or, it is equal to what is obtained on dividing the *samaśaṅku* by the product of the Rsine of the colatitude and the radius and multiplying by the square of the Rsine of the latitude; or, to what is obtained by multiplying the *samaśaṅku* by the product of the equinoctial midday shadow and the Rsine of the latitude and dividing by the product of the radius and 12.

$$\text{Earthsine} = \frac{samaśaṅku \times (\text{Rsin } \phi)^2}{\text{Rcos } \phi \times \text{R}} \tag{7}$$

$$= \frac{samaśaṅku \times (palabhā \times \text{Rsin } \phi)}{12 \times \text{R}}. \tag{8}$$

5. Or, the earthsine is equal to the result obtained on dividing the *samaśaṅku* by (the product of) the hypotenuse of the equinoctial midday shadow and 12 and multiplying (the resulting quotient) by the square of the equinoctial midday shadow; or, to the result obtained on dividing the *samaśaṅku* by the *taddhṛti* multiplied by the *samaśaṅku* and multiplying (the resulting quotient) by the square of the *agrā*.

$$\text{Earthsine} = \frac{samaśaṅku \times (palabhā)^2}{12 \times palakarṇa} \tag{9}$$

$$= \frac{samaśaṅku \times (agrā)^2}{taddhṛti \times samaśaṅku}. \tag{10}$$

Formula (10) reduces to formula (5).

6. Or, to the *samaśaṅku* multiplied by the square of the *śaṅkutala* and divided by the product of the *śaṅku* and *svadhṛti*; or, to the *samaśaṅku* multiplied by the product of the *agrā* and the Rsine of the latitude and divided by the product of the *taddhṛti* and the Rsine of the colatitude.

$$\text{Earthsine} = \frac{samaśaṅku \times (śaṅkutala)^2}{śaṅku \times svadhṛti} \tag{11}$$

$$= \frac{samaśaṅku \times (agrā \times \text{Rsin } \phi)}{\text{Rcos } \phi \times taddhṛti}. \tag{12}$$

7. Or, to the *samaśaṅku* multiplied by the product of the *agrā* and the Rsine of the latitude and divided by the product of the radius and

the *samaśaṅku*; or, to the *samaśaṅku* divided by the product of the *taddhṛti* and 12 and multiplied by the product of the *agrā* and the *palabhā*.

$$\text{Earthsine} = \frac{samaśaṅku \times (agrā \times \text{Rsin}\,\phi)}{\text{R} \times samaśaṅku} \text{ or } \frac{agrā \times \text{Rsin}\,\phi}{\text{R}} \tag{13}$$

$$= \frac{samaśaṅku \times (agrā \times palabhā)}{12 \times taddhṛti}. \tag{14}$$

8. Or, to the *samaśaṅku* multiplied by the product of the equinoctial midday shadow and the *agrā* and divided by the product of the hypotenuse of the equinoctial midday shadow and the *samaśaṅku*; or, to the *samaśaṅku* multiplied by the product of the Rsine of the latitude and the *śaṅkutala* and divided by the product of the Rsine of the colatitude and the *svadhṛti*.

$$\text{Earthsine} = \frac{samaśaṅku \times (palabhā \times agrā)}{palakarṇa \times samaśaṅku} \tag{15}$$

$$= \frac{samaśaṅku \times (\text{Rsin}\,\phi \times śaṅkutala)}{\text{Rcos}\,\phi \times svadhṛti}. \tag{16}$$

Formula (15) reduces to formula (4).

9. Or, to the *samaśaṅku* multiplied by the product of the *śaṅkutala* and the Rsine of the latitude and divided by the product of the radius and the Rsine of the altitude (*śaṅku*); or, to the *samaśaṅku* multiplied by the product of the equinoctial midday shadow and the *śaṅkutala* and divided by the product of the *svadhṛti* and twelve.

$$\text{Earthsine} = \frac{samaśaṅku \times (śaṅkutala \times \text{Rsin}\,\phi)}{\text{R} \times śaṅku} \tag{17}$$

$$= \frac{samaśaṅku \times (palabhā \times śaṅkutala)}{12 \times svadhṛti}. \tag{18}$$

10(a-b). Or, the *samaśaṅku* multiplied by the product of the *agrā* and the *śaṅkutala* and divided by the product of the *samaśaṅku* and the *svadhṛti*.

$$\text{Earthsine} = \frac{samaśaṅku \times (agrā \times śaṅkutala)}{svadhṛti \times samaśaṅku}. \tag{19}$$

This formula reduces to formula (6).

10(c-d). Or, the earthsine is equal to: the *taddhṛti* multiplied by the square of the latitude and divided by the square of the radius;

11. Or, the *taddhṛti* multiplied by the square of the equinoctial midday shadow and divided by the square of the hypotenuse of the equinoctial midday shadow; or, the *taddhṛti* multiplied by the square of the *śaṅkutala* and divided by the square of the *svadhṛti*.

$$\text{Earthsine} = \frac{taddhṛti \times (\text{R}\sin\phi)^2}{\text{R}^2} \tag{20}$$

$$= \frac{taddhṛti \times (palabhā)^2}{(palakarṇa)^2} \tag{21}$$

$$= \frac{taddhṛti \times (śaṅkutala)^2}{(svadhṛti)^2}. \tag{22}$$

These formulae are equivalent to formula (7) because

$$\frac{taddhṛti}{\text{R}} = \frac{samaśaṅku}{\text{R}\cos\phi} \text{ and } \frac{\text{R}\sin\phi}{\text{R}} = \frac{palabhā}{palakarṇa} = \frac{śaṅkutala}{svadhṛti}.$$

12-13(a-b). Or, the product of the *agrā* and the *iṣṭaśaṅku* (i.e., Rsine of the given altitude) multiplied by the *palabhā* and divided by (the product of) the *svadhṛti* and 12, is the earthsine; or, the same product (of the *agrā* and the *iṣṭaśaṅku*) multiplied by the Rsine of the latitude and divided by the product of the Rcosine of the latitude and the *svadhṛti* is the earthsine; or, the same product multiplied by the *agrā* and divided by the product of the *svadhṛti* and the *samaśaṅku* is the earthsine.

$$\text{Earthsine} = \frac{(agrā \times iṣṭaśaṅku) \times palabhā}{svadhṛti \times 12} \tag{23}$$

$$= \frac{(agrā \times iṣṭaśaṅku) \times \text{R}\sin\phi}{\text{R}\cos\phi \times svadhṛti} \tag{24}$$

$$= \frac{(agrā \times iṣṭaśaṅku) \times agrā}{svadhṛti \times samaśaṅku}. \tag{25}$$

13 (c). Or, the difference between the *dhṛti* for midday and the day-radius is the earthsine.

$$\text{Earthsine} = \text{midday } dhṛti \sim \text{day-radius}. \tag{26}$$

13(d)-14. Multiply the *unnatajyā* by the day-radius and divide by the radius; the difference between that and the *svadhṛti* is the earth-

sine. The Rsine of the ascensional difference multiplied by the day-radius and divided by the radius is also the earthsine.

$$\text{Earthsine} = \frac{\textit{unnatajyā} \times \text{day-radius}}{\text{R}} \sim \textit{svadhṛti} \qquad (27)$$

$$= \frac{\text{Rsin (asc. diff.)} \times \text{day-radius}}{\text{R}}. \qquad (28)$$

The *unnatakāla* is defined as the time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon; and *unnatajyā* is defined by the formula:

$$\textit{unnatajyā} = \text{Rsin } (\textit{unnatakāla} \sim \text{or} + \text{asc. diff.}),$$

where $\sim$ or $+$ sign is taken according as the Sun is in the northern or southern hemisphere. See *infra*, sec. 10, vs. 2.

In other words, the *unnatajyā* is the Rsine of the complement of the hour angle.

15. Or, the product of the Rsine of the latitude and the *agrā* corresponding to the shadow-sphere divided by its own hypotenuse (i. e., the hypotenuse of the corresponding shadow) is the earthsine. Or, the square-root of the difference between the squares of the Rsine of the declination and the *agrā* is the earthsine.[1]

$$\text{Earthsine} = \frac{\text{Rsin } \phi \times \textit{bhāvṛttāgrā}}{\text{hyp. of shadow}} \qquad (29)$$

$$= \sqrt{[(\textit{agrā})^2 \sim (\text{Rsin } \delta)^2]}. \qquad (30)$$

Bhāvṛttāgrā is also known as *chāyāvṛttāgrā*, more generally as *chāyākarṇāgrā*.

1. Same rule occurs in *BrSpSi*, xv. 43(a-b); *SiŚe*, iv. 92(a-b).

Section 6

Agrā or Rsine of amplitude at rising

1. The Rsine of the Sun's *bhuja* multiplied by the Rsine of the (Sun's) greatest declination and divided by the Rsine of the colatitude is the (Sun's) *agrā*. The Rsine of the (Sun's) declination multiplied by the radius and divided by the Rsine of the colatitude is also the same.[1]

Let λ be the *bhuja* of the Sun's (tropical) longitude, δ the Sun's declination, and ϕ the latitude of the station. Then

$$agr\bar{a} = \frac{\text{Rsin}\ \lambda \times \text{Rsin}\ 24^\circ}{\text{Rcos}\ \phi}. \tag{1}$$

$$= \frac{\text{Rsin}\ \delta \times \text{R}}{\text{Rcos}\ \phi} \tag{2}$$

2. Or, the Rsine of the declination multiplied by the *palakarṇa* and divided by 12 is the *agrā*;[2] **or, the Rsine of the declination multiplied by the *taddhṛti* and divided by the *samaśaṅku* is the *agrā*.**

$$Agr\bar{a} = \frac{\text{Rsin}\ \delta \times palakar\d{n}a}{12} \tag{3}$$

$$= \frac{\text{Rsin}\ \delta \times taddh\d{r}ti}{sama\acute{s}a\dot{n}ku}. \tag{4}$$

3. Or, the Rsine of the declination multiplied by the *svadhṛti* and divided by the own *iṣṭaśaṅku* is the *agrā*; or, the square-root of the sum of the squares of the earthsine and the Rsine of the declination is the *agrā*.[3]

$$Agr\bar{a} = \frac{\text{Rsin}\ \delta \times svadh\d{r}ti}{sve\d{s}\d{t}a\acute{s}a\dot{n}ku}. \tag{5}$$

$$= \sqrt{(\text{earthsine})^2 + (\text{Rsin}\ \delta)^2}. \tag{6}$$

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1. Same rule occurs in *BrSpSi*, iii. 64(a-b); *SiŚe*, iv. 58, 59.
2. *Cf. ŚiDVr*, iv. 6; *SiŚe*, iv. 57(a-b).
3. The latter rule occurs also in *BrSpSi*, xv. 35.

4. Or, the earthsine multiplied by the radius and divided by the Rsine of the latitude is the *agrā*; or, the earthsine multiplied by the *palakarṇa* and divided by the *palabhā* is the *agrā*.

$$Agr\bar{a} = \frac{\text{earthsine} \times R}{R\sin\phi} \tag{7}$$

$$= \frac{\text{earthsine} \times palakar\d{n}a}{palabh\bar{a}}. \tag{8}$$

5. Or, the square-root of the product of the *taddhṛti* and the earthsine is the *agrā* on the eastern or western horizon; or, the earthsine multiplied by the *svadhṛti* and divided by the *śaṅkutala* is the *agrā*.

$$Agr\bar{a} = \sqrt{taddh\d{r}ti \times \text{earthsine}} \tag{9}$$

$$= \frac{\text{earthsine} \times svadh\d{r}ti}{\acute{s}a\dot{n}kutala}. \tag{10}$$

Formula (9) follows from multiplication of the following results :

$$\frac{taddh\d{r}ti}{agr\bar{a}} = \frac{R}{R\sin\phi} \quad \text{and} \quad \frac{\text{earthsine}}{agr\bar{a}} = \frac{R\sin\phi}{R}.$$

6. Or, the *samaśaṅku* multiplied by the Rsine of the latitude and divided by the Rsine of the colatitude is the *agrā*; or, the *samaśaṅku* multiplied by the *palabhā* and divided by 12 is the *agrā*.

$$Agr\bar{a} = \frac{sama\acute{s}a\dot{n}ku \times R\sin\phi}{R\cos\phi} \tag{11}$$

$$= \frac{sama\acute{s}a\dot{n}ku \times palabh\bar{a}}{12}. \tag{12}$$

7. Or, the *samaśaṅku* multiplied by the earthsine and divided by the Rsine of the declination is the *agrā*; or the *samaśaṅku* multiplied by the *śaṅkutala* and divided by the *śaṅku* (i.e., Rsine of the altitude) is the *agrā*.

$$Agr\bar{a} = \frac{sama\acute{s}a\dot{n}ku \times \text{earthsine}}{R\sin\delta} \tag{13}$$

$$= \frac{sama\acute{s}a\dot{n}ku \times \acute{s}a\dot{n}kutala}{\acute{s}a\dot{n}ku}. \tag{14}$$

8. Or, the *taddhṛti* multiplied by the Rsine of the latitude and divided by the radius is the *agrā*; or, the *taddhṛti* multiplied by the *palabhā* and divided by the *palakarṇa* is the *agrā*.

$$Agr\bar{a} = \frac{taddh\d{r}ti \times \text{Rsin}\,\phi}{\text{R}} \tag{15}$$

$$= \frac{taddh\d{r}ti \times palabh\bar{a}}{palakar\d{n}a}. \tag{16}$$

9. Or, the *taddhṛti* multiplied by the *śaṅkutala* and divided by the *svadhṛti* is the *agrā*; or, the product of the *taddhṛti* and the *śaṅku* multiplied by the Rsine of the latitude and divided by the product of the *svadhṛti* and the Rsine of the colatitude (is the *agrā*).

$$Agr\bar{a} = \frac{taddh\d{r}ti \times \acute{s}a\dot{n}kutala}{svadh\d{r}ti} \tag{17}$$

$$= \frac{(taddh\d{r}ti \times \acute{s}a\dot{n}ku) \times \text{Rsin}\,\phi}{svadh\d{r}ti \times \text{Rcos}\,\phi}. \tag{18}$$

10. Or, the product (of the *taddhṛti* and the *śaṅku*) multiplied by the earthsine and divided by the product of the Rsine of the declination and the *svadhṛti* is the *agrā*; or, the same product (of the *taddhṛti* and the *śaṅku*) multiplied by the *palabhā* and divided by 12 times the *svadhṛti* is the *agrā*.

$$Agr\bar{a} = \frac{(taddh\d{r}ti \times \acute{s}a\dot{n}ku) \times \text{earthsine}}{\text{Rsin}\,\delta \times svadh\d{r}ti} \tag{19}$$

$$= \frac{(taddh\d{r}ti \times \acute{s}a\dot{n}ku) \times palabh\bar{a}}{12 \times svadh\d{r}ti}. \tag{20}$$

10.* Or, the product of the earthsine and the *śaṅku* multiplied by the radius and divided by the product of the Rsine of the colatitude and the *śaṅkutala* is the *agrā*; or the same product (of the earthsine and the *śaṅku*) multiplied by the *palakarṇa* and divided by the product of the *śaṅkutala* and 12 is the *agrā*.[1]

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1. This verse does not occur in the original and has been inserted by the translator. Such a verse is needed here, for the word *ghāta* in verse 10 means the product of earthsine and *śaṅku*. The reading *kujyāśaṅkvorghāto* in place of *taddhṛtiśaṅkvorghāto* in the original text of verse 9 proves the existence of a verse beginning with *kujyāśaṅkvorghāto* in the original.

$$Agr\bar{a} = \frac{(\text{earthsine} \times \textit{śaṅku}) \times R}{R\cos\phi \times \textit{śaṅkutala}} \tag{21}$$

$$= \frac{(\text{earthsine} \times \textit{śaṅku}) \times \textit{palakarṇa}}{12 \times \textit{śaṅkutala}}. \tag{22}$$

11. Or, the product (of the earthsine and the *śaṅku*) multiplied by the *svadhṛti* and divided by the product of the *śaṅku* and the *śaṅkutala* is the *agrā*; or, the product of the day-radius and the Rsine of the ascensional difference divided by the Rsine of the latitude is the *agrā*.

$$Agr\bar{a} = \frac{(\text{earthsine} \times \textit{śaṅku}) \times \textit{svadhṛti}}{\textit{śaṅku} \times \textit{śaṅkutala}} \tag{23}$$

$$= \frac{\text{day-radius} \times \text{Rsin (asc. diff.)}}{\text{Rsin}\,\phi}. \tag{24}$$

12. Or, the square-root of the difference between the squares of the *taddhṛti* and the *samaśaṅku* is the *agrā* on the horizon;[1] or, the difference or the sum of the *bhuja* and the *śaṅkutala*, according as they are of like or unlike directions, is the *agrā*.

$$Agr\bar{a} = \sqrt{(\textit{taddhṛti})^2 - (\textit{samaśaṅku})^2} \tag{25}$$

$$= \textit{śaṅkutala} \sim \text{or} + \textit{bhuja}. \tag{26}$$

13. Or, the radius multiplied by the *palabhā* and divided by the *samamaṇḍalakarṇa* is the *agrā*; or, when the Sun is in the northern hemisphere, the *śaṅkutala* corresponding to the *samaśaṅku* is the *agrā*.

$$Agr\bar{a} = \frac{R \times \textit{palabhā}}{\textit{samamaṇḍalakarṇa}} \tag{27}$$

$$= \textit{śaṅkutala} \text{ corresponding to the } \textit{samaśaṅku} \text{ (in the northern hemisphere).} \tag{28}$$

Formula (27) is true because

$$Agr\bar{a} = \frac{\textit{samaśaṅku} \times \textit{palabhā}}{12}$$

and $$\textit{samaśaṅku} = \frac{12 \times R}{\textit{samamaṇḍalakarṇa}}.$$

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1. *Cf. SiŚe*, iv. 60(c-d).

Samamaṇḍalakarṇa is the hypotenuse of the shadow of the gnomon when the Sun is on the prime vertical (*samamaṇḍala*).

14. Or, the product of the radius and the *agrā* corresponding to the shadow-circle (*bhāvṛttāgrā*), divided by the hypotenuse of shadow, is the *agrā*;[1] or, the product of the *agrā* corresponding to the shadow-circle and the Rsine of the zenith distance divided by the shadow gives the *agrā*.

$$Agr\bar{a} = \frac{\text{R} \times bhav\d{r}tt\bar{a}gr\bar{a}}{\text{hyp. of shadow}} \qquad (29)$$

$$= \frac{bh\bar{a}v\d{r}tt\bar{a}gr\bar{a} \times \text{Rsin } z}{\text{shadow}}. \qquad (30)$$

The *bhāvṛttāgrā* (i.e., the *agrā* for the shadow-circle) is the *agrā* calculated by taking the hypotenuse of shadow for the radius of the celestial sphere. It is also known as *chāyākarṇāgrī agrā* or briefly *karṇāgrī agrā* or *karṇāgrā*.

Formulae (29) and (30) are equivalent, because

$$\frac{\text{R}}{\text{hypotenuse of shadow}} = \frac{\text{Rsin } z}{\text{shadow}}.$$

1. For the converse of this rule see *BrSpSi*, iii. 4(a-b) and also xv. 49(a-b).

Section 7 : Ascensional Difference

1. The earthsine multiplied by the radius and divided by the day-radius is the Rsine of the ascensional difference.[1] **The earthsine multiplied by the *antyā* and divided by the *dhṛti* is also the Rsine of the ascensional difference.**

$$\text{Rsin (asc. diff.)} = \frac{\text{earthsine} \times \text{R}}{\text{day-radius}} \tag{1}$$

$$= \frac{\text{earthsine} \times antyā}{dhṛti}. \tag{2}$$

Let A, B, C be the points of the celestial equator where it is intersected by the Sun's hour circle at the given time, at sunrise and at sunset, respectively. Then the distance of the point A from the line BC is defined as the Sun's *antyā* for the given time. The Sun's *dhṛti* is the distance of the Sun from its rising-setting line.

2. The remainder obtained by taking the difference of the *unnatajyā* and the *antyā* is the Rsine of the ascensional difference. The Rsine of the *nāḍīs* of the difference between the semi-duration of the day obtained from the (*ghaṭī*) *yantra* and 15 *ghaṭīs* is also the same.[2]

$$\text{Rsin (asc. diff.)} = antyā \sim unnatajyā, \tag{3}$$

where *unnatajyā* is the Rsine of the complement of the hour angle.

$$\text{Rsin (asc. diff.)} = \text{Rsin (semi-duration of day} \sim 15\ ghaṭīs). \tag{4}$$

3. Or, the *agrā* multiplied by the Rsine of the latitude and divided by the day-radius is the Rsine of the ascensional difference. Or, the product of the Rsine of the declination and the radius multiplied by the *palabhā* and divided by 12 times the day-radius (is the Rsine of the ascensional difference).[3]

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1. *Cf. BrSpSi*, ii. 57(c-d)-58(a-b); *ŚiDVr*, ii. 18(c-d); *MSi*, iii. 18(a-b); *SiŚe*, iii. 67(a-b); *SiŚi*, 1, ii. 49(a-b).
2. A similar rule occurs in *BrSpSi*, xv. 54(c-d).
3. *Cf. SiŚe*, iii. 68; *Karaṇottama* (of Acyuta), iii. 4.

$$\text{Rsin (asc. diff.)} = \frac{agr\bar{a} \times \text{Rsin}\,\phi}{\text{day-radius}} \tag{5}$$

$$= \frac{(\text{Rsin}\,\delta \times \text{R}) \times palabh\bar{a}}{12 \times \text{day-radius}}. \tag{6}$$

Formula (5) may be derived from formula (1) by substituting earthsine = (*agrā* × Rsin ϕ)/R; and formula (6) by substituting earthsine = (Rsin δ × *palabhā*) /12.

4. The product of the *antyā* and the Rsine of the declination multiplied by the *agrā* and divided by (the product of) the *dhṛti* and the *samaśaṅku* is the Rsine of the ascensional difference. Or, the (same) product (of the *antyā* and the Rsine of the declination) multiplied by the Rsine of the latitude and divided by the product of the Rsine of the colatitude and the *dhṛti* (is the Rsine of the ascensional difference).

$$\text{Rsin (asc. diff.)} = \frac{(anty\bar{a} \times \text{Rsin}\,\delta) \times agr\bar{a}}{dh\d{r}ti \times sama\acute{s}a\dot{n}ku} \tag{7}$$

$$= \frac{(anty\bar{a} \times \text{Rsin}\,\delta) \times \text{Rsin}\,\phi}{\text{Rcos}\,\phi \times dh\d{r}ti}. \tag{8}$$

Formula (7) is derived from formula (2) by substituting earthsine = (Rsin δ × *agrā*)/*samaśaṅku*; and formula (8) by substituting earthsine = (Rsin δ × Rsin ϕ)/Rcos ϕ.

5. The product of the *antyā* and the *agrā* multiplied by the Rsine of the latitude and divided by the product of the radius and the *dhṛti* is also the same. The same product (of the *antyā* and the *agrā*) multiplied by the earthsine and divided by the product of the *agrā* and the *dhṛti* is also the same.

$$\text{Rsin (asc. diff.)} = \frac{(anty\bar{a} \times agr\bar{a}) \times \text{Rsin}\,\phi}{\text{R} \times dh\d{r}ti} \tag{9}$$

$$= \frac{(anty\bar{a} \times agr\bar{a}) \times \text{earthsine}}{agr\bar{a} \times dh\d{r}ti}. \tag{10}$$

Formula (9) may be derived from formula (2) by substituting earthsine = (*agrā* × Rsin ϕ)/R. Formula (10) is a trivial equivalent of formula (2).

6. The multiplication of the *agrā* and the product (of the *antyā* and the *agrā*) divided by the multiplication of the *taddhṛti* and the *dhṛti* is also the same.

$$\text{Rsin (asc. diff.)} = \frac{agr\bar{a} \times (anty\bar{a} \times agr\bar{a})}{taddh\d{r}ti \times dh\d{r}ti}. \qquad (11)$$

This formula may be derived from formula (2) by substituting
earthsine = (*agrā* × *agrā*)/*taddhṛti*.

7. Or, the product of the *agrā* and the radius multiplied by the Rsine of the latitude and divided by the multiplication of the day-radius and the radius (is the Rsine of the ascensional difference). The (same) product (of the *agrā* and the radius) multiplied by the *agrā* and divided by the multiplication of the day-radius and the *taddhṛti*, too, gives the same.

$$\text{Rsin (asc. diff.)} = \frac{(agr\bar{a} \times \text{R}) \times \text{Rsin}\,\phi}{\text{day-radius} \times \text{R}} \qquad (12)$$

$$= \frac{(agr\bar{a} \times \text{R}) \times agr\bar{a}}{\text{day-radius} \times taddh\d{r}ti}. \qquad (13)$$

Formula (13) may be derived from formula (1) by substituting earthsine = (*agrā* × *agrā*)/*taddhṛti*. Formula (12) is equivalent to formula (5).

8. Or, the (same) product (of the *agrā* and the radius) multiplied by the *śaṅkutala* and divided by the multiplication of the day-radius and the *svadhṛti* is the Rsine of the ascensional difference. Or, the product of the Rsine of the declination, the Rsine of the latitude and the *palakarṇa* divided by the product of the day-radius and 12 is also the same.

$$\text{Rsin (asc. diff.)} = \frac{(agr\bar{a} \times \text{R}) \times \acute{s}a\dot{n}kutala}{\text{day-radius} \times svadh\d{r}ti} \qquad (14)$$

$$= \frac{\text{Rsin}\,\delta \times \text{Rsin}\,\phi \times palakar\d{n}a}{\text{day-radius} \times 12}. \qquad (15)$$

Formula (14) may be derived from formula (1) by replacing earthsine by (*agrā* × *śaṅkutala*)/*svadhṛti*; and formula (15) by replacing earthsine by (Rsin δ × Rsin ϕ)/Rcos ϕ and then R/Rcos ϕ by *palakarṇa*/12.

9. Or, the product of the radius and the *samaśaṅku* when multiplied by the square of the Rsine of the latitude and divided by the day-radius as multiplied by the product of the Rsine of the colatitude and the radius, the result is the Rsine of the ascensional difference.

$$\text{Rsin (asc. diff.)} = \frac{(\text{R} \times sama\acute{s}a\dot{n}ku) \times (\text{Rsin}\,\phi)^2}{\text{day-radius} \times (\text{Rcos}\,\phi \times \text{R})}. \qquad (16)$$

This formula may be derived from (1) by replacing earthsine by (*agrā* × Rsin φ)/R, and then *agrā* by (*samaśaṅku* × Rsin φ)/Rcos φ.

10. Just as the earthsine is obtained from the *samaśaṅku* in 16 ways, the Rsine of the ascensional difference may be obtained in 16 ways from the product of the *antyā* and the *samaśaṅku*, divided by the *dhṛti*.

The earthsine is obtained from the *samaśaṅku* in 16 ways. For,

$$\text{earthsine} = \frac{\text{Rsin}\,\delta \times \text{N}}{\text{D}},$$

$$\text{where Rsin}\,\delta = \frac{samaśaṅku \times \text{earthsine}}{agrā}$$

$$= \frac{samaśaṅku \times palabhā}{palakarṇa}$$

$$= \frac{samaśaṅku \times iṣṭaśaṅkutala}{iṣṭadhṛti}$$

$$= \frac{samaśaṅku \times \text{Rsin}\,\phi}{\text{R}},$$ [vide *supra*, sec. 3 formulae (11) to (14)]

$$\text{and } \frac{\text{N}}{\text{D}} = \frac{\text{Rsin}\,\phi}{\text{Rcos}\,\phi} = \frac{palabhā}{12} = \frac{agrā}{samaśaṅku} = \frac{śaṅkutala}{iṣṭaśaṅku}.$$

11-12. The product of the Rsines of the declination, local latitude and of three signs when divided by the product of the Rsine of the colatitude and the day-radius also gives the Rsine of the ascensional difference. Or, the product of the Rsine of the declination, the Rsine of the latitude and the *samadhṛti* divided by the product of the day-radius and the *samaśaṅku* gives the same. The product of the *svadhṛti*, the earthsine and the Rsine of the latitude divided by the product of the day-radius and the *śaṅkutala* also gives the same.

$$\text{Rsin (asc. diff.)} = \frac{\text{Rsin}\,\delta \times \text{Rsin}\,\phi \times \text{R}}{\text{Rcos}\,\phi \times \text{Rcos}\,\delta} \quad (17)$$

$$= \frac{\text{Rsin}\,\delta \times \text{Rsin}\,\phi \times samadhṛti}{\text{Rcos}\,\delta \times samaśaṅku} \quad (18)$$

$$= \frac{svadhṛti \times \text{earthsine} \times \text{Rsin}\,\phi}{\text{Rcos}\,\delta \times śaṅkutala}. \quad (19)$$

Formula (17) may be derived from formula (1) by replacing earthsine by (Rsin δ $\times$ Rsin ϕ)/Rcos ϕ. Formula (18) is equivalent to formula (17), because

$$\frac{\text{R}}{\text{Rcos } \phi} = \frac{samadhṛti}{samaśaṅku}.$$

Formula (19) is equivalent to formula (1) because

$$\frac{svadhṛti}{śaṅkutala} = \frac{\text{R}}{\text{Rsin } \phi}.$$

13-14. The product of the *samaśaṅku* and the square of the Rsine of the latitude divided by the product of the Rsine of the colatitude and the day-radius, or the product of the *taddhṛti* and the Rsine of the latitude multiplied by the *palabhā* and divided by the product of the *palakarṇa* and the day-radius, or the product (of the *taddhṛti* and the Rsine of the latitude) multiplied by the earthsine and divided by the product of the *agrā* and the day-radius is the Rsine of the ascensional difference. The (same) product (of the *taddhṛti* and the Rsine of the latitude) multiplied by the *śaṅkutala* and divided by the product of the *svadhṛti* and the day-radius is also the same.

$$\text{Rsin (asc. diff.)} = \frac{samaśaṅku \times (\text{R}\sin \phi)^2}{\text{day-radius} \times \text{Rcos}\, \phi} \quad (20)$$

$$= \frac{(taddhṛti \times \text{Rsin } \phi) \times palabhā}{\text{day-radius} \times palakarṇa} \quad (21)$$

$$= \frac{(taddhṛti \times \text{Rsin } \phi) \times \text{earthsine}}{\text{day-radius} \times agrā} \quad (22)$$

$$= \frac{(taddhṛti \times \text{Rsin } \phi) \times śaṅkutala}{\text{day-radius} \times svadhṛti}. \quad (23)$$

Formula (20) may be derived from formula (1) by replacing earthsine by (*agrā* $\times$ Rsin ϕ)/R, and then replacing *agrā* by

(*samaśaṅku* $\times$ Rsin ϕ)/Rcos ϕ.

Formula (21) may be derived from formula (1) by replacing earthsine by (*agrā* $\times$ Rsin ϕ)/R, then replacing *agrā* by (*taddhṛti* $\times$ Rsin ϕ)/R, and then again replacing Rsin ϕ/R by *palabhā*/*palakarṇa*.

Formulae (21), (22) and (23) are equivalent, because

$$\frac{palabhā}{palakarṇa} = \frac{\text{earthsine}}{agrā} = \frac{śaṅkutala}{svadhṛti}.$$

15. The results obtained by multiplying the product of the *sama-śaṅku* and the Rsine of the latitude (severally) by the Rsine of the latitude, the earthsine, the *palabhā* and the *śaṅkutala* and dividing by the Rsine of the colatitude, the Rsine of the declination, 12 and the *śaṅku*, each multiplied by the day-radius, respectively, are also the same.

$$\text{Rsin (asc. diff.)} = \frac{(\textit{samaśaṅku} \times \text{Rsin } \phi) \times \text{Rsin } \phi}{\text{day-radius} \times \text{Rcos } \phi} \tag{24}$$

$$= \frac{(\textit{samaśaṅku} \times \text{Rsin } \phi) \times \text{earthsine}}{\text{day-radius} \times \text{Rsin } \delta} \tag{25}$$

$$= \frac{(\textit{samaśaṅku} \times \text{Rsin } \phi) \times \textit{palabhā}}{\text{day-radius} \times 12} \tag{26}$$

$$= \frac{(\textit{samaśaṅku} \times \text{Rsin } \phi) \times \textit{śaṅkutala}}{\text{day-radius} \times \textit{śaṅku}}. \tag{27}$$

Formula (24) is another form of formula (20). Formulae (25), (26) and (27) are equivalent to formula (24), because

$$\frac{\text{Rsin } \phi}{\text{Rcos } \phi} = \frac{\text{earthsine}}{\text{Rsin } \delta} = \frac{\textit{palabhā}}{12} = \frac{\textit{śaṅkutala}}{\textit{śaṅku}}.$$

16. The product of the square of the Rsine of the latitude and the *taddhṛti* divided by the product of the radius and the day-radius, or the product of the *taddhṛti*, the *antyā* and the square of the latitude divided by the product of the square of the radius and the *dhṛti*, is also the same.

$$\text{Rsin (asc. diff.)} = \frac{\textit{taddhṛti} \times (\text{Rsin } \phi)^2}{\text{R} \times \text{day-radius}} \tag{28}$$

$$= \frac{\textit{taddhṛti} \times \textit{antyā} \times (\text{Rsin } \phi)^2}{\text{R}^2 \times \textit{dhṛti}}. \tag{29}$$

Formula (28) is equivalent to formula (21), because

$$\frac{\textit{palabhā}}{\textit{palakarṇa}} = \frac{\text{Rsin } \phi}{\text{R}}.$$

Formula (29) may be derived from formula (28) by replacing R/day-radius by *antyā*/*dhṛti*.

17. The product of the radius and the *taddhṛti* (severally) multiplied by the squares of the *palabhā*, the earthsine, and the *śaṅkutala* and

divided by the squares of the *palakarṇa*, the *agrā* and the *svadhṛti*, each multiplied by the day-radius, respectively, also give the same (Rsine of the ascensional difference).

$$\text{Rsin (asc. diff.)} = \frac{(\text{R} \times \textit{taddhṛti}) \times (\textit{palabhā})^2}{\text{day-radius} \times (\textit{palakarṇa})^2} \quad (30)$$

$$= \frac{(\text{R} \times \textit{taddhṛti}) \times (\text{earthsine})^2}{\text{day-radius} \times (\textit{agrā})^2} \quad (31)$$

$$= \frac{(\text{R} \times \textit{taddhṛti}) \times (\textit{śaṅkutala})^2}{\text{day-radius} \times (\textit{svadhṛti})^2}. \quad (32)$$

Formula (30) may be derived from formula (1) by first replacing earthsine by (*agrā* × Rsin ϕ)/R, then *agrā* by (*taddhṛti* × Rsin ϕ)/R, and finally (Rsin ϕ/R)^2 by (*palabhā*/*palakarṇa*)^2.

Formulae (30), (31) and (32) are equivalent, because

$$\frac{\textit{palabhā}}{\textit{palakarṇa}} = \frac{\text{earthsine}}{\textit{agrā}} = \frac{\textit{śaṅkutala}}{\textit{svadhṛti}}.$$

18-19. Or, the Rsine of the ascensional difference is obtained by multiplying the product of the *antyā*, the earthsine and the Rsine of the (prime vertical) altitude (*nṛ* or *śaṅku*) by the multipliers viz. the *palabhā*, the Rsine of the latitude, the *agrā*, or the earthsine, and dividing (respectively) by the divisors viz. 12, the Rsine of the colatitude, the *samaśaṅku*, or, the Rsine of the declination, each multiplied by the product of the *agrā* and the *dhṛti*; or by multiplying the product of the radius, the earthsine and the Rsine of the (prime vertical) altitude by the abovementioned multipliers and dividing by the corresponding divisors (as stated above), each multiplied by the product of the *agrā* and the day-radius.

$$\text{Rsin (asc. diff.)} = \frac{\textit{antyā} \times \text{earthsine} \times \textit{samaśaṅku}}{\textit{agrā} \times \textit{dhṛti}} \times \frac{\text{M}}{\text{D}} \quad (33)$$

$$= \frac{\text{R} \times \text{earthsine} \times \textit{samaśaṅku}}{\textit{agrā} \times \text{day-radius}} \times \frac{\text{M}}{\text{D}}, \quad (34)$$

$$\text{where } \frac{\text{M}}{\text{D}} = \frac{\textit{palabhā}}{12} = \frac{\text{Rsin}\,\phi}{\text{Rcos}\,\phi} = \frac{\textit{agrā}}{\textit{samaśaṅku}} = \frac{\text{earthsine}}{\text{Rsin}\,\delta}.$$

Since *samaśaṅku*/*agrā* = D/M, the above formulae (33) and (34) are obviously equivalent to formulae (2) and (1) respectively.

20. (The same is also equal to) the product of the *samaśaṅku*, the *śaṅkutala* and the Rsine of the latitude, divided by the product of the *iṣṭaśaṅku* and the day-radius; or the product of the radius, the *agrā* and the *śaṅkutala*, divided by the product of the day-radius and the *dhṛti*.

$$\text{Rsin (asc. diff.)} = \frac{\textit{samaśaṅku} \times \textit{śaṅkutala} \times \text{Rsin}\,\phi}{\textit{iṣṭaśaṅku} \times \text{day-radius}} \tag{35}$$

$$= \frac{\text{R} \times \textit{agrā} \times \textit{śaṅkutala}}{\text{day-radius} \times \textit{dhṛti}}. \tag{36}$$

Formula (35) is the same as formula (27); and formula (36) the same as formula (14).

21. The product of the *antyā*, the *agrā* and the *śaṅkutala*, divided by the square of the *dhṛti*, is also the Rsine of the ascensional difference. So also is the product of the *śaṅkutala*, the Rsine of the declination and the radius, divided by the product of the *iṣṭaśaṅku* and the day-radius.

$$\text{Rsin (asc. diff.)} = \frac{\textit{antyā} \times \textit{agrā} \times \textit{śaṅkutala}}{(\textit{dhṛti})^2} \tag{37}$$

$$= \frac{\textit{śaṅkutala} \times \text{Rsin}\,\delta \times \text{R}}{\textit{iṣṭaśaṅku} \times \text{day-radius}}. \tag{38}$$

These reduce to formula (2) and formula (1) respectively, because

$$\text{earthsine} = \frac{\textit{agrā} \times \textit{śaṅkutala}}{\textit{dhṛti}} = \frac{\textit{śaṅkutala} \times \text{Rsin}\,\delta}{\textit{iṣṭaśaṅku}}.$$

22. The product of the *śaṅkutala*, the *antyā* and the Rsine of the declination, divided by the product of the *iṣṭaśaṅku* and the *dhṛti*, is the Rsine of the ascensional difference, So also is the product of the *dhṛti*, the earthsine and the Rsine of the latitude divided by the product of the *śaṅkutala* and the day-radius.

$$\text{Rsin (asc. diff.)} = \frac{\textit{śaṅkutala} \times \textit{antyā} \times \text{Rsin}\,\delta}{\textit{iṣṭaśaṅku} \times \textit{dhṛti}} \tag{39}$$

$$= \frac{\textit{dhṛti} \times \text{earthsine} \times \text{Rsin}\,\phi}{\textit{śaṅkutala} \times \text{day radius}}. \tag{40}$$

Formula (39) reduces to formula (2) by replacing ($\textit{śaṅkutala} \times \text{Rsin}\,\delta$)/*iṣṭaśaṅku* by earthsine; and formula (40) reduces to formula (1) by replacing *dhṛti*/*śaṅkutala* by R/Rsin ϕ.

23. The product of the Rsine of the declination, the Rsine of the latitude and the *dhṛti*, divided by (the product of) the day-radius and the *śaṅku*, is the Rsine of the ascensional difference. So is also the difference between the result obtained by dividing the product of the radius and the *dhṛti* by the day-radius, and the *unnatajyā*.

$$\text{Rsin (asc. diff.)} = \frac{\text{Rsin } \delta \times \text{Rsin } \phi \times \textit{dhṛti}}{\text{day-radius} \times \textit{śaṅku}} \tag{41}$$

$$= \frac{\text{R} \times \textit{dhṛti}}{\text{day-radius}} \sim \textit{unnatajyā}. \tag{42}$$

Formula (41) is analogous to formula (18), while formula (42) is another form of formula (3).

LENGTHS OF DAY AND NIGHT

24. The arc of that (Rsine of the Sun's ascensional difference) gives the *asus* of the (Sun's) ascensional difference. When the Sun is in the northern hemisphere, 5400 *asus* are respectively increased and diminished by those *asus*; when the Sun is in the southern hemisphere, 5400 *asus* are respectively diminished and increased by those *asus*. The results (in both cases) are the semi-durations of the day and night, respectively.[1]

That is :

semi-duration of day $=$ 5400 *asus* $\pm$ *asus* of asc. diff.

semi-duration of night $=$ 5400 *asus* $\mp$ *asus* of asc. diff.,

the upper or lower sign is taken according as the Sun is in the northern or southern hemisphere.

GENERAL INSTRUCTION

25. One should find the *asus* of the ascensional difference from each Rsine in the manner stated above; or, in the case of the Sun, which moves on the ecliptic (lit. in the signs), one should find the *asus* of the ascensional difference from the Rsine of the Sun's *bhuja*.

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1. *Cf. BrSpSi*, ii. 60; *KK* (BC), iii. 3; *ŚiDVr*, ii. 20-21; *SiŚe*, iii. 60; *SiŚi*, I, ii. 52.

ASCENSIONAL DIFFERENCES OF THE SIGNS

26. The *asus* of the ascensional difference corresponding to the signs Aries, Taurus and Gemini, etc, should be determined in the manner stated. The *asus* of the ascensional difference corresponding to a fraction of those signs should be obtained (by proportion) with the help of the number of minutes in a sign.

27. Or, (severally) multiply 714, 1294 and 1530 by (the *aṅgulas* of) the equinoctial midday shadow and divide (each product) by 12: the resulting arcs are, as stated, the *asus* of the ascensional difference (for the end points of the signs Aries, Taurus and Gemini, respectively).[1]

$$\textit{Asus} \text{ of asc. diff. of Aries} = \frac{714 \times \textit{palabhā}}{12}$$

$$\textit{Asus} \text{ of asc. diff. of Aries and Taurus} = \frac{1294 \times \textit{palabhā}}{12}$$

$$\textit{Asus} \text{ of asc. diff. of Aries, Taurus and Gemini} = \frac{1530 \times \textit{palabhā}}{12}$$

the *palabhā* being measured in *aṅgulas.*

The ascensional differences of the individual signs Aries, Taurus and Gemini, therefore, are as follows:

$$\text{Asc. diff. of Aries} = \frac{714P}{12}\ \textit{asus} = 10P\ \textit{vināḍīs}$$

$$\text{Asc. diff. of Taurus} = \frac{580P}{12}\ \textit{asus} = 8P\ \textit{vināḍīs}$$

$$\text{Asc. diff. of Gemini} = \frac{236P}{12}\ \textit{asus} = \frac{10P}{3}\ \textit{vināḍīs},$$

where *P* stands for *palabhā* (equinoctial midday shadow).

CONCLUDING STANZA

28. The methods of determining the cardinal points, the length of the equinoctial midday shadow and the Rsines (of the latitude and colatitude, of the Sun's declination, of the earthsine, of the *agrā* and of the ascensional difference) have been stated (above) by indications only. It is not possible to describe them in their entirety like the (number of) showers of rain.

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1. *Cf. PSi*, iii. 10; *KK* (BC), iii. 1; *ŚiDVṛ*, xiii. 9; *SiŚi*, I, ii. 50-51.

Section 8

Lagna or Rising Point of the Ecliptic

RIGHT ASCENSIONS OF THE SIGNS OR TIMES OF RISING OF THE SIGNS AT LAṄKA

Method 1

1. (Severally) multiply the Rsines of (the longitudes of) the last points of Aries, Taurus and Gemini by the day-radius for the last point of Gemini and divide (the resulting products) by the day-radii for the last points of Aries, Taurus and Gemini, respectively. Reduce the resulting Rsines to the corresponding arcs and diminish each arc by the preceding arc (if any) : the results (in terms of minutes) are the *asus* of rising (of Aries, Taurus and Gemini) at Laṅkā.[1]

Let λ_1 (= 30°), λ_2 (= 60°), λ_3 (= 90°) be the (tropical) longitudes, δ_1, δ_2, δ_3 (= 24°) the declinations and α_1, α_2, α_3 the right ascensions of the last points of Aries, Taurus and Gemini, respectively. Then

$$\text{Rsin } \alpha_r = \frac{\text{Rsin } \lambda_r \times \text{Rcos } \delta_3}{\text{Rcos } \delta_r}, \ r = 1, 2, 3; \qquad (1)$$

and Time of rising of Aries at Laṅkā = α_1

Time of rising of Taurus at Laṅkā = $\alpha_2 - \alpha_1$

Time of rising of Gemini at Laṅkā = $\alpha_3 - \alpha_2$.

Method 2

2. Or, (severally) diminish the day-radii for the last points of Aries, Taurus and Gemini by the day-radius of the last point of Gemini (lit. day-radius for 3 signs of longitude), and multiply (the resulting differences) by the Rsines of the corresponding longitudes and divide by the corresponding day-radii; and then subtract them from the Rsines of the corresponding longitudes.

$$\text{Rsin } \alpha_r = \text{Rsin } \lambda_r - \frac{(\text{Rcos } \delta_r - \text{Rcos } \delta_3) \times \text{Rsin } \lambda_r}{\text{Rcos } \delta_r}, \ r = 1, 2, 3. \qquad (2)$$

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1. *Cf. SūSi*, iii. 42(c-d)-43; *Ā*, iv. 25; *MBh*, iii. 9; *BrSpSi*, iii. 15; *ŚiDVr*, iv. 8; *MSi*, iv. 38(c-d)-39; *SiŚe*, iv. 15; *SiŚi*, I, ii. 57.

Method 3

3. Or, diminish the products of the Rsines of the longitudes (of the last points of Aries, Taurus and Gemini) and their own day-radii, by the (corresponding) products of (i) the differences between the Rversed sines of their own declinations and the (Sun's) greatest declination and (ii) the Rsine of their own longitudes; and divide (the resulting differences) by their own day-radii.

$$\text{Rsin}\,\alpha_r = \frac{\text{Rsin}\,\lambda_r \times \text{Rcos}\,\delta_r - (\text{Rvers}\,24^\circ - \text{Rvers}\,\delta_r) \times \text{Rsin}\,\lambda_r}{\text{Rcos}\,\delta_r},$$

$$r = 1, 2, 3. \qquad (3)$$

One can easily see that formulae (2) and (3) reduce to formula (1) on simplification.

Method 4

4 Or, multiply the radius (severally) by the square-roots of the differences between the squares of the Rsines of the longitudes of (the last points of) Aries etc. and the squares of the Rsines of the corresponding declinations, and divide (the products) by the corresponding day-radii. Reduce (the resulting Rsines) to the corresponding arcs, and diminish each arc by the preceding arc (if any). Then are obtained the times of of rising at the equator of the signs (Aries, etc.).[1]

$$\text{Rsin}\,\alpha_r = \frac{\text{R} \times \sqrt{(\text{Rsin}\,\lambda_r)^2 - (\text{Rsin}\,\delta_r)^2}}{\text{Rcos}\,\delta_r}, \quad r = 1, 2, 3. \qquad (4)$$

and, as before.

Time of rising of Aries at the equator $= \alpha_1$

Time of rising of Taurus at the equator $= \alpha_2 - \alpha_1$

Time of rising of Gemini at the equator $= \alpha_3 - \alpha_2$.

Method 5

5. Or, find the product of the difference and sum of the radius and the Rsine of the longitude (for the last points of Aries, Taurus and Gemini); then divide (each result) by the corresponding day-radius and subtract the quotient from the corresponding day-radius; then multiply that by the

1. *Cf. BrSpSi*, iii. 16; *SiŚe*, iv. 16; *SiŚi*, I, ii. 54-55.

square of the radius and divide by the corresponding day-radius; then take the square-root and reduce the resulting Rsine to the corresponding arc. The successive differences of the results (for the last points of Aries, Taurus and Gemini) are the times of rising of the signs (Aries, Taurus and Gemini) at the equator.

$$\text{Rsin}\ \alpha_r = \sqrt{\frac{R^2}{\text{Rcos}\ \delta_r}\left[\text{Rcos}\ \delta_r - \frac{(R - \text{Rsin}\ \lambda_r)(R + \text{Rsin}\ \lambda_r)}{\text{Rcos}\ \delta_r}\right]},$$
$$r = 1, 2, 3. \qquad (5)$$

Method 6

6. Or, multiply the sum of the Rsine of the declination and the Rsine of the longitude for the last points of Aries etc. (i.e., Aries, Taurus and Gemini) by their difference and take the square-root (of the resulting product). Then multiply by the radius and divide by the corresponding day-radius. Then reduce (the resulting Rsines) to arc and obtain the successive differences. (Then are also obtained the times of rising of the signs Aries, Taurus and Gemini at the equator).

$$\text{Rsin}\ \alpha_r = \frac{\sqrt{(\text{Rsin}\ \lambda_r + \text{Rsin}\ \delta_r)(\text{Rsin}\ \lambda_r - \text{Rsin}\ \delta_r)} \times R}{\text{Rcos}\ \delta_r},$$
$$r = 1, 2, 3. \qquad (6)$$

Method 7

7. Or, multiply the sum of the Rsine of the declination and the Rsine of the longitude for the last points of Aries etc. (i. e , Aries, Taurus and Gemini) by their difference; then multiply (the resulting products) by the square of the radius and divide by the square of the corresponding day-radius; and then take the square-root. Reduce (the resulting Rsines) to arc and take the successive differences (of those arcs). (Then too are obtained the times of rising of the signs Aries, Taurus and Gemini at the equator),

$$\text{Rsin}\ \alpha_r = \sqrt{\frac{(\text{Rsin}\ \lambda_r + \text{Rsin}\ \delta_r)(\text{Rsin}\ \lambda_r - \text{Rsin}\ \delta_r)\ R^2}{(\text{Rcos}\ \delta_r)^2}},$$
$$r = 1, 2, 3. \qquad (7)$$

OBLIQUE ASCENSIONS OF THE SIGNS OR TIMES OF RISING OF THE SIGNS AT THE LOCAL PLACE

8. Those times of rising (in *asus*) (i. e., the times of rising, in *asus*, of Aries, Taurus and Gemini at the equator) are 1669, 1796 and 1935, respectively. These diminished by the corresponding (*asus* of the) ascensio-

nal differences are the times of rising of Aries, Taurus and Gemini at the local place. The same (times of rising of Aries, Taurus and Gemini at the equator) set down in the reverse order and increased by the corresponding (*asus* of the) ascensional differences are the times of rising of the signs Cancer, Leo and Virgo at the local place. And the times of rising of the signs Aries, Taurus, Gemini, Cancer, Leo and Virgo at the local place, in the reverse order, are the times of rising of the signs Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces, respectively, at the local place.[1]

Let *a*, *b*, *c* be the *asus* of the ascensional differences of the signs Aries, Taurus, and Gemini respectively, for the local place. Then the times of rising, in *asus*, of the various signs at the local place are as exhibited in the following table.

Times of rising in *asus* of the signs Aries, etc. at the local place

Sign	Time of rising in *asus*	Sign
1. Aries	$1669 - a$	12. Pisces
2. Taurus	$1796 - b$	11. Aquarius
3. Gemini	$1935 - c$	10. Capricorn
4. Cancer	$1935 + c$	9. Sagittarius
5. Leo	$1796 + b$	8. Scorpio
6. Virgo	$1669 + a$	7. Libra

TIME OF SETTING AND TRANSITING THE MERIDIAN BY THE SIGNS AT THE LOCAL PLACE

9. A sign, as a rule, takes as many *asus* in setting as the seventh sign takes in rising;[2] **and it takes as many *asus* in crossing the meridian as it takes in rising at Laṅkā.**

1. *Cf. SūSi*, iii. 44-45; *MBh*, iii. 10; *BrSpSi*, iii. 17; xv. 32-33(a); *KK*(BC), iii. 4; *ŚiDVr*, iv. 9; *SiŚe*, iv. 17; 15(c-d); also 29(a-b); *SiŚi*, I, ii. 58-59(a-b).
2. See *SiŚi*, I, ii. 59(c-d); II, vii. 24.

Time taken in setting and crossing the meridian by the signs

Sign	Time of setting in *asus*	Time of crossing the meridian, in *asus*	Sign
1. Aries	1669 + *a*	1669	12. Pisces
2. Taurus	1796 + *b*	1796	11. Aquarius
3. Gemini	1935 + *c*	1935	10. Capricorn
4. Cancer	1935 − *c*	1935	9. Sagittarius
5. Leo	1796 − *b*	1796	8. Scorpio
6. Virgo	1669 − *a*	1669	7. Libra

LAGNA OR RISING POINT OF THE ECLIPTIC

General instruction

10(a-b). One should determine the longitude of the rising point of the ecliptic (*vilagna* or *lagna*) with the help of the day elapsed (i.e., with the help of the time elapsed since sunrise) during the day (and with the help of the night elapsed, i.e., with the help of the time elapsed since sunset, during the night). The result obtained in the case of the night should be increased by six signs. The longitude of the rising point of the ecliptic may also be obtained with the help of the night to elapse (during the night) or with the help of the day to elapse (during the day).

Computation of *Lagna* : Method 1

10(c-d)-12. Or, to be precise, by the untraversed minutes of the sign occupied by the instantaneous Sun multiply the time of rising at the local place of that sign and divide (the product) by the number of minutes in a sign (i. e., by 1800); subtract the resulting *asus* from the given *asus* and add the untraversed portion of the current sign to the Sun's longitude. Thereafter add as many signs to the Sun's longitude as have their times of rising subtractable from the remaining *asus*. Then multiply the *asus* still remaining by 30 and divide by the time of rising of the next unsubtracted sign; and add the resulting degrees, etc., to the Sun's longitude. Then is obtained the horizon-ecliptic point towards the east.[1]

1. Same rule occurs in *BrSpSi*, iii. 18-20; *KK* (BC), iii. 5(a-b); *MBh*, iii. 30-32; *LBh*, iii. 17-19; *SūSi*, iii. 46-48; *ŚiDVr*, iv. 11-12; *MSi*, iv. 42-45; *SiŚe*, iv. 18-20(a-b); *SiŚi*, I, iii. 2-4.

13. Similarly, from the times of rising of the signs and that (longitude of the rising point of the ecliptic) one may determine the instantaneous longitude of the Sun.[1]

(Thus) from the given time elapsed since sunrise one may determine, by proportion, the portion of the ecliptic (which rises during that time) as well as the longitude of the horizon-ecliptic point in the east.

Verses 10 (c-d)-12 give the method for finding the longitude of the rising point of the ecliptic when the Sun's instantaneous longitude and the time (in *asus*) elapsed since sunrise are given. The first half of verse 13 gives the method for finding the instantaneous longitude of the Sun when the longitude of the rising point of the ecliptic and the time elapsed since sunrise are given.

Method 2

14. Multiply the Rsine of the Sun's altitude by the radius and divide by the Rsine of the altitude of the meridian-ecliptic point; and add the arc corresponding to the resulting Rsine to the Sun's longitude. Then is also obtained the longitude of the horizon-ecliptic point in the east.

Longitude of rising point of the ecliptic

$$= \text{Sun's longitude} + \text{arc } \frac{\text{Rsin } a \times \text{R}}{\text{Rsin } a_m},$$

where a and a_m are the altitudes of the Sun and the meridian-ecliptic point.

This formula is gross. The correct formula will be obtained when the altitude of the meridian-ecliptic point is replaced by the altitude of the central ecliptic point.

Method 3

15. One may obtain the longitude of the rising point of the ecliptic also by reversing the methods for the day and night, by taking the signs Libra etc. in place of the signs Aries etc., in order, and reversing the rule of adding 6 signs for the day and night.

For example, if the Sun is in the sign Taurus and the time is 3 *ghaṭīs* elapsed since sunrise, then one should assume that the Sun is in the sign Scorpio and the time is 3 *ghaṭīs* past sunrise; and in the end one should add six signs to the resulting longitude.

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1. For details of this method see *infra*, vs. 16.

And if the Sun is in the sign Gemini and the time is 3 *ghaṭīs* past sunset, then one should assume that the Sun is in the sign Sagittarius and that the time is 3 *ghaṭīs* past sunrise. In this case 6 signs should not be added.

This rule, too, is approximate.

LAGNAKĀLA (*IṢṬAKĀLA*) OR TIME CORRESPONDING TO THE GIVEN *LAGNA*

Method 1

16. (Severally) multiply the traversed degrees of the sign occupied by the rising point of the ecliptic and the untraversed degrees of the sign occupied by the Sun by their own *asus* of rising and divide (each result) by 30. To the *asus* (thus obtained) add those (of rising) of the intervening signs. Then is obtained the first approximation to the *lagnāsu*.[1] To obtain the nearest approximation one should apply the process of iteration.[2]

The term *lagnāsu* means "the *asus* of the *lagnakāla*" or "*lagnakāla* in terms of *asus*". The *lagnakāla* is usually called *iṣṭakāla.*

Method 2

17. Or, subtract the oblique ascension of the Sun (*kālātmaka sahasrāṁśu* or *kāla-sahasrāṁśu*) from the oblique ascension of the rising point of the ecliptic (*kālātmaka vilagna* or *kāla-vilagna*): the remainder is the first approximation for the *vilagnakāla.*[3] To obtain the nearest approximation, apply the process of iteration, using the (Sun's) motion.

The abovementioned two methods relate to finding the civil time elapsed since sunrise with the help of (i) the longitude of the rising point of the ecliptic and (ii) the Sun's longitude at sunrise.

When, however, one has to find the civil time measured since sunrise with the help of (i) the Sun's instantaneous longitude and (ii) the longitude of the rising point of the ecliptic; or the sidereal time elapsed since sunrise with the help of (i) the Sun's longitude at sunrise and (ii) the longitude of the rising point of the ecliptic, the process of iteration should not be applied.

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1. See *KK* (BC), iii. 5(c-d).

2. Similar rules occur in *BrSpSi*, iii. 21-23; *MBh*, iii. 34-36; *LBh*, iii. 20; *ŚiDVr*, iv. 13; *SūSi*, iii. 50; *MSi*, iv. 46-47; *SiŚe*, iv. 20(c-d)-22(a-b); *SiŚi*, I, iii. 5-7(a-b).

3. *Cf. SiŚe*, iv. 25, 26, 28.

ASTALAGNA OR SETTING POINT OF THE ECLIPTIC

18. The longitude of the rising point of the ecliptic increased by six signs is called the longitude of the setting point of the ecliptic by the learned.[1]

LAGNA AND *LAGNAKĀLA* (*IṢṬAKĀLA*) WHEN SUN AND *LAGNA* ARE IN THE SAME SIGN

19. When the given time (measured since sunrise) is less than the time of rising of the untraversed portion of the sign occupied by the Sun, then the given time should be multiplied by 30 and divided by the time of rising of the (Sun's) own sign : the result, in degrees etc., should be added to the longitude of the Sun. Then is obtained the longitude of the rising point of the ecliptic.[2]

20. When the rising point of the ecliptic and the Sun are situated in the same sign, then the degrees intervening between them, multiplied by the time of rising of that sign and divided by 30, yield the (desired) time.[3]

MADHYALAGNA AND TIME ELAPSED SINCE SUNRISE

21. The longitude of the Sun diminished by the traversed portion of its sign and by the other signs traversed by it (as determined from the given hour angle) is the longitude of the meridian-ecliptic point. The time by which the semi-duration of the day is in excess of the *asus* of the eastern hour angle gives, as before, the *lagnakāla* (or *iṣṭakāla*).

Lagnasamaravau kālaḥ means the same thing as *lagnakāla* (or *iṣṭakāla*).

It is to be noted that in finding the meridian-ecliptic point, use is to be made of the times of rising of the signs at Laṅkā.

LAGNA AND *LAGNAKĀLA* AT NIGHT

22. When the night is yet to elapse, the longitude of the rising point of the ecliptic should be obtained as before by subtracting (the signs and parts thereof lying between the Sun and the rising point of the ecliptic) from the longitude of the Sun. Adding together the oblique ascensions of the intervening signs and parts thereof is obtained the time which is to elapse until the Sun coincides with the rising point of the ecliptic.

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1. *Cf. MBh*, iii. 33(a-b); *ŚiDVṛ*, iv. 13(a-b); *MSi*, iv. 50(a); *SiŚe*, iv. 23(d).
2. *Cf. ŚiDVṛ*, iv. 14(a-b); *SiŚe*, iv. 23(a-c); *SiŚi*, I, iii. 4.
3. *Cf. ŚiDVṛ*, iv. 14(c-d); *SiŚe*, iv. 24; *SiŚi*, I, iii. 5(c-d).

LAGNAKĀLA WITHOUT THE USE OF OBLIQUE ASCENSIONS OF THE SIGNS

23-25. Calculate the *asus* of the Sun's right ascension in the manner taught before; also calculate the *asus* of the Sun's ascensional difference. Take their difference if the Sun is in the six signs beginning with Capricorn, or their sum if the Sun is in the six signs beginning with Cancer.

When the Sun is in the three signs beginning with Aries, these *asus* (of the difference or sum) themselves give the *asus* of oblique ascension of the part of the ecliptic lying between the first point of Aries and the Sun; when the Sun is in the three signs beginning with Cancer, the same *asus* should be subtracted from the *asus* corresponding to six signs (i.e., from 10800); when the Sun is in the three signs beginning with Libra, those *asus* should be increased by the *asus* corresponding to six signs; and when the Sun is in the three signs beginning with Capricorn, those *asus* should be subtracted from the *asus* in a circle (i.e, from 21600). (Thus are obtained the *asus* of oblique ascension of the part of the ecliptic lying between the first point of Aries and the Sun).

In the same way calculate the *asus* of oblique ascension from the rising point of the ecliptic, and diminish them by the *asus* of oblique ascension calculated from the Sun. If the *asus* obtained from the rising point of the ecliptic are less than the *asus* obtained from the Sun, increase the former by 21600 and then subtract. Thus is obtained the *lagnakāla* (i.e., time corresponding to the given *lagna*, in terms of *asus*).[1]

The *asus* of the oblique ascension of that part of the ecliptic that lies between the first point of Aries and the Sun is technically called *samaya-sūrya*, *kālārka*, *kālāditya*, etc., all these terms meaning "Sun or Sun's longitude in terms of time". Similarly, the *asus* of the oblique ascension of that part of the ecliptic that lies between the first point of Aries and the *lagna* is called *kālalagna* (= *kālātmaka lagna*), meaning "*lagna* in terms of time". Thus

$$lagnakāla = kālalagna - kālasūrya.$$

Also see *supra*, vs. 17.

KĀLĀṀŚA OR TIME-DEGREES

26. The *asus* of oblique ascension divided by 60 or the *nāḍīs* (of oblique ascension) multiplied by 6 are defined as time-degrees (*kālāṁśa*).[2] **In other words, 10 *vighaṭīs* or 60 *asus* make one time-degree.**

1. This rule occurs also in *BrSpSi*, xv. 29-31; *SiŚe*, iv. 30-31.
2. Also see *BrSpSi*, vi. 6(c-d); *SiŚe*, iv. 29(a-b).

Section 9 : Midday Shadow

ALTITUDE TRIANGLE FOR MIDDAY

1-2. The sum or difference of the declination and the (local) latitude, according as they are of like or unlike directions, is the *natāṁśa* ("zenith distance" for midday, or "meridian zenith distance"): this is also known as *khākṣa*. The Rsine of that is called the *dṛgjyā* or *dorjyā* :[1] (this is the base). The degrees corresponding to three signs (i.e., 90°) diminished by the degrees of the *natāṁśa* are the degrees of the *unnata* or *unnatāṁśa* ("altitude"):[2] this is the *koṭi*. The Rsine of that is called *unnatajīvā*, *ujjyā*, *nara* or *śaṅku* :[3] (this is the upright). The radius is the hypotenuse : this is also known as *yaṣṭi* or *nalaka*.

Let O be the centre of the celestial sphere, S the Sun and SA the perpendicular dropped from the Sun on the plane of the horizon. Then in the triangle SAO, right-angled at A,

$$\text{base AO} = \text{Rsin (Sun's zenith distance)}$$
$$\text{upright SA} = \text{Rsin (Sun's altitude)}$$
$$\text{hypotenuse SO} = \text{R, the radius of the celestial sphere.}$$

This is the altitude triangle for any position of the Sun. The text describes the altitude triangle for the midday Sun.

DIRECTION OF MIDDAY SHADOW

3. When the local latitude is less than the northern declination, the midday shadow falls towards the south. The same happens even when the Rsine of colatitude is greater than the Rsine of codeclination. In the contrary case, the midday shadow falls towards the north.

That is, the midday shadow of the gnomon falls towards the south or north according as

$$\phi \lesseqgtr \delta$$

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1. Other synonyms are *natajyā*, *nataguṇa*, *prabhā*, etc.
2. *Cf. BrSpSi*, iii. 47; *ŚiDVṛ*, iv. 15; *SiŚe*, iv. 42; *SiŚi*, I, iii. 32. For similar rules see *BrSpSi*, xv. 41-42; *KK* (BC), iii. 8(a-b); *SiŚe*, iv. 43.
3. Another synonym is *nā*.

or according as

$$90^\circ - \phi > \text{or} < 90^\circ - \delta,$$

where δ is the Sun's northern declination and ϕ the latitude of the place.

When the Sun's declination is south, the midday shadow always falls towards the north.

Alternative rule

4. When, (the Sun being in the northern hemisphere), the codeclination together with the local latitude is less than three signs, the (midday) shadow falls towards the south; in the contrary case, towards the north. The difference between three signs and that (i.e., codeclination plus local latitude) gives the (meridian) zenith distance.

5. Or, when the sum of the colatitude and the declination is greater than three signs, the midday shadow falls towards the south; in the contrary case, towards the north. That sum diminished by (or subtracted from) three signs gives the corresponding zenith distance.

That is, when the Sun is in the northern hemisphere, the midday shadow of the gnomon falls towards the south or north according as

$$\phi + (90^\circ - \delta) \lesseqgtr 90^\circ,$$

or according as

$$(90^\circ - \phi) + \delta > \text{or} < 90^\circ.$$

Also, if z denote the meridian zenith distance of the Sun, then

$$z = 90^\circ \sim \{ \phi + (90^\circ - \delta) \}$$

$$= 90^\circ \sim \{ (90^\circ - \phi) + \delta \}.$$

MIDDAY SHADOW AND HYPOTENUSE OF MIDDAY SHADOW

6. The Rsine of the (Sun's) zenith distance multiplied by 12 and divided by the Rsine of the (Sun's) altitude gives (the length of) the shadow cast by the midday Sun.[1] The Rsine of three signs multiplied by 12 and divided by the Rsine of the (Sun's) altitude gives the hypotenuse of the midday shadow.[2]

$$\text{Midday shadow} = \frac{\text{Rsin } z \times 12}{\text{Rsin } a},$$

1. *Cf. BrSpSi*, iii. 28(a-b), 48; *KK* (BC), iii. 8; *ŚiDVr*, iv. 21; *SiŚe*, iv. 45.
2. *Cf. BrSpSi*, iii. 49(a-b); *ŚiDVr*, iv. 17; also iv. 22; *SiŚe*, iv. 45.

$$\text{hypotenuse of midday shadow} = \frac{\text{R} \times 12}{\text{Rsin } a},$$

where a is the Sun's altitude and z the Sun's zenith distance at midday.

DHṚTI AND *ANTYĀ* FOR MIDDAY

7. The Rsine of the (Sun's) codeclination (i.e., the day-radius) diminished or increased by the earthsine, according as the Sun is in the southern or northern hemisphere, gives the *dhṛti* for the middle of the day. Similarly, the radius diminished or increased by the Rsine of the (Sun's) ascensional difference (according as the Sun is in the southern or northern hemisphere) gives the *antyā* for the middle of the day.[1]

$$\text{Midday } dhṛti = \text{Rcos } \delta \mp \text{earthsine}$$

$$\text{and midday } antyā = \text{R} \mp \text{Rsin (asc. diff.)},$$

$-$ or $+$ sign being taken according as the Sun is in the southern or northern hemisphere.

Dhṛti is generally called *hṛti* and *svadhṛti* or *iṣṭadhṛti, iṣṭahṛti.* Some writers call *dhṛti* by the name *cheda* and *antyā* by the name *hāra, hāraka*, etc.

RSINE OF MERIDIAN ALTITUDE FROM *DHṚTI* OR *ANTYĀ*

8. Severally multiply the *dhṛti* by the Rsine of colatitude, the Rsine of declination, the Rsine of the prime vertical altitude, and 12 and divide (the products thus obtained) by the radius, the *agrā*, the *taddhṛti* and the *palakarṇa* ("hypotenuse of the equinoctial midday shadow"), respectively: the results are the Rsines of the altitude.[2]

$$\text{Rsin } a = \frac{dhṛti \times \text{Rcos } \phi}{\text{R}} \tag{1}$$

$$= \frac{dhṛti \times \text{Rsin } \delta}{agrā} \tag{2}$$

$$= \frac{dhṛti \times samaśaṅku}{taddhṛti} \tag{3}$$

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1. *Cf. BrSpSi*, iii. 34; also xv. 52; *ŚiDVṛ*, iv. 18; *MSi*, iv. 13(a-b); *SiŚe*, iv. 46(a-b); *SiŚi*, 1, iii. 34(c-d).
2. *Cf. ŚiDVṛ*, iv. 20; *SiŚe*, iv. 32(d)-34(a-b), 40(a-b). Also see *BrSpSi*, iii. 27(a-b), for (4); and *MSi*, iv. 14(a-b).

$$= \frac{dh\d{r}ti \times 12}{palakar\d{n}a}, \tag{4}$$

where a denotes meridian altitude, δ declination, and ϕ the local latitude.

These are equivalent, because

$$\frac{\text{Rcos } \phi}{\text{R}} = \frac{\text{Rsin } \delta}{agr\bar{a}} = \frac{sama\acute{s}a\dot{n}ku}{taddh\d{r}ti} = \frac{12}{palakar\d{n}a}.$$

Note. It must be remembered that throughout this chapter the *dhṛti* and *antyā*, etc., are those for midday.

9. Severally multiply the product of the day-radius and the *antyā* (called "*ghāta*") by the stated multipliers[1] and divide by the corresponding divisors[2] as multiplied by the radius : then (too) are obtained the Rsines of the altitude.

$$\text{Rsin } a = \frac{(\text{day-radius} \times anty\bar{a}) \times \text{Rcos } \phi}{\text{R} \times \text{R}} \tag{5}$$ [3]

$$= \frac{(\text{day-radius} \times anty\bar{a}) \times \text{Rsin } \delta}{\text{R} \times agr\bar{a}} \tag{6}$$

$$= \frac{(\text{day-radius} \times anty\bar{a}) \times sama\acute{s}a\dot{n}ku}{\text{R} \times taddh\d{r}ti} \tag{7}$$

$$= \frac{(\text{day-radius} \times anty\bar{a}) \times 12}{\text{R} \times palakar\d{n}a}. \tag{8}$$ [4]

Formulae (5) to (8) are equivalent to formulae (1) to (4), because[5]

$$dh\d{r}ti = \frac{\text{day-radius} \times anty\bar{a}}{\text{R}}.$$

The multipliers (viz. Rcos ϕ, Rsin δ, *samaśaṅku*, and 12) and the divisors (viz. R, *agrā*, *taddhṛti*, and *palakarṇa*) of vs. 8 will be referred to as "multipliers" and "divisors". These divisors multiplied by the radius R (i.e., R$\times$R, R $\times$*agrā*, R$\times$*taddhṛti*, and R$\times$*palakarṇa*), which have been used as divisors in vs. 9, will be referred to as "subsequent divisors" (*anantara-hāra*).

1. That is, Rcos ϕ, Rsin δ, *samaśaṅku* and 12. See vs. 8.
2. That is, R, *agrā*, *taddhṛti* and *palakarṇa*. See vs. 8.
3. *Cf. BrSpSi*, iii. 31(c-d); *SiŚe*, iv. 38(c-d).
4. *Cf. BrSpSi*, iii. 32(a-b); *SiŚe*, iv. 39(a-b).
5. See *ŚiDVṛ*, iv. 19.

10. Multiply the *dhṛti* and the *ghāta* (i.e., day-radius × *antyā*) severally by the differences between the divisors and the (corresponding) multipliers and divide by the bare divisors and by the divisors as multiplied by the radius, respectively. The results (thus obtained) subtracted from the *dhṛti* give the Rsines of the altitude.

$$\text{Rsin } a = dhṛti - \frac{dhṛti \times (\text{R} - \text{Rcos } \phi)}{\text{R}} \tag{9}$$

$$= dhṛti - \frac{dhṛti \times (agrā - \text{Rsin } \delta)}{agrā} \tag{10}$$

$$= dhṛti - \frac{dhṛti \times (taddhṛti - samaśaṅku)}{taddhṛti} \tag{11}$$

$$= dhṛti - \frac{dhṛti \times (palakarṇa - 12)}{palakarṇa} \tag{12}$$

and also

$$\text{Rsin } a = dhṛti - \frac{ghāta \times (\text{R} - \text{Rcos } \phi)}{\text{R} \times \text{R}} \tag{13}$$

$$= dhṛti - \frac{ghāta \times (agrā - \text{Rsin } \delta)}{\text{R} \times agrā} \tag{14}$$

$$= dhṛti - \frac{ghāta \times (taddhṛti - samaśaṅku)}{\text{R} \times taddhṛti} \tag{15}$$

$$= dhṛti - \frac{ghāta \times (palakarṇa - 12)}{\text{R} \times palakarṇa}, \tag{16}$$

where *ghāta* = day-radius × *antyā*.

Formulae (9) to (16) are alternative forms of formulae (1) to (8).

11. Multiply the *ghāta* (i.e., day-radius × *antyā*) by the difference between the divisor as multiplied by the radius, and the multiplier, and divide by the divisor as multiplied by the radius : the result subtracted from the *ghāta* gives the Rsine of the altitude.

$$\text{Rsin } a = ghāta - \frac{ghāta \times (\text{R} \times \text{R} - \text{Rcos } \phi)}{\text{R} \times \text{R}} \tag{17}$$

$$= ghāta - \frac{ghāta\,(\text{R} \times agrā - \text{Rsin } \delta)}{\text{R} \times agrā} \tag{18}$$

$$= ghāta - \frac{ghāta \times (\text{R} \times taddhṛti - samaśaṅku)}{\text{R} \times taddhṛti} \tag{19}$$

$$= ghāta - \frac{ghāta \times (\text{R} \times palakarṇa - 12)}{\text{R} \times palakarṇa}, \tag{20}$$

where *ghāta* = day-radius × *antyā*.

Formulae (17) to (20) are alternative forms of formulae (5) to (8).

12(a-b). The Rsines of the altitude are also obtained by dividing day-radius × multiplier × *antyā* by the "subsequent divisor" (i.e., R × divisor).

$$\text{Rsin } a = \frac{\text{day-radius} \times \text{multiplier} \times antyā}{\text{R} \times \text{divisor}},$$

$$\text{i.e., Rsin } a = \frac{\text{day-radius} \times \text{Rcos } \phi \times antyā}{\text{R} \times \text{R}} \tag{21}$$

$$= \frac{\text{day-radius} \times \text{Rsin } \delta \times antyā}{\text{R} \times agrā} \tag{22}$$

$$= \frac{\text{day-radius} \times samaśaṅku \times antyā}{\text{R} \times taddhṛti} \tag{23}$$

$$= \frac{\text{day-radius} \times 12 \times antyā}{\text{R} \times palakarṇa}. \tag{24}$$

These formulae are the same as formulae (5) to (8).

12(c-d)-13(a-b). Or, (severally) multiply the *antyā* by the differences between these divisors and multipliers and divide (the products) by the (corresponding) divisors; then subtract (the quotients obtained) from the *antyā*; and then multiply (the differences) by the day-radius and divide by the radius: the results are the Rsines of the altitude.

$$\text{Rsin } a = \left[antyā - \frac{(\text{divisor} - \text{multiplier}) \times antyā}{\text{divisor}}\right] \times \frac{\text{day-radius}}{\text{R}},$$

$$\text{i.e., Rsin } a = \left[antyā - \frac{(\text{R} - \text{Rcos } \phi) \times antyā}{\text{R}}\right] \times \frac{\text{day-radius}}{\text{R}} \tag{25}$$

$$= \left[antyā - \frac{(agrā - \text{Rsin } \delta) \times antyā}{agrā}\right] \times \frac{\text{day-radius}}{\text{R}} \tag{26}$$

$$= \left[antyā - \frac{(taddhṛti - samaśaṅku) \times antyā}{taddhṛti}\right] \times \frac{\text{day-radius}}{\text{R}} \tag{27}$$

$$= \left[antyā - \frac{(palakarṇa - 12) \times antyā}{palakarṇa}\right] \times \frac{\text{day-radius}}{R}. \quad (28)$$

These formulae are other alternative forms of formulae (5) to (8).

13(c-d)-15. Or, severally multiply the day-radius by the multipliers as multiplied by the *antyā* and divide by the (corresponding) "subsequent divisors" : the results are the Rsines of the altitude.

Or else, severally multiply the day-radius by the differences of the above-mentioned divisors and multipliers and divide by the (corresponding) divisors; then add the quotient to the day-radius if the multiplier is greater than the divisor, or subtract the quotient from the day-radius if the multiplier is less than the divisor; and then multiply by the *antyā* and divide by the radius. Then are obtained the Rsines of the altitude.

$$\text{Rsin } a = \frac{(antyā \times \text{multiplier}) \times \text{day-radius}}{\text{subsequent divisor}},$$

$$\text{i.e., Rsin } a = \frac{(antyā \times \text{Rcos } \phi) \times \text{day-radius}}{R \times R} \quad (29)$$

$$= \frac{(antyā \times \text{Rsin } \delta) \times \text{day-radius}}{R \times agrā} \quad (30)$$

$$= \frac{(antyā \times samaśaṅku) \times \text{day-radius}}{R \times taddhṛti} \quad (31)$$

$$= \frac{(antyā \times 12) \times \text{day-radius}}{R \times palakarṇa}. \quad (32)$$

Or,

$$\text{Rsin } a = \left[\text{day-radius} + \frac{\text{day-radius} \times (\text{multiplier} - \text{divisor})}{\text{divisor}}\right] \times \frac{antyā}{R},$$

if multiplier > divisor, (*A*)

$$\text{or } \left[\text{day-radius} - \frac{\text{day-radius} \times (\text{divisor} - \text{multiplier})}{\text{divisor}}\right] \times \frac{antyā}{R},$$

if multiplier < divisor; (*B*)

i.e.,

$$\left.\begin{aligned} \text{Rsin } a &= \left[\text{day-radius} + \frac{\text{day-radius} \times (\text{Rcos } \phi - R)}{R}\right] \times \frac{antyā}{R} \\ \text{or } &\left[\text{day-radius} - \frac{\text{day-radius} \times (R - \text{Rcos } \phi)}{R}\right] \times \frac{antyā}{R} \end{aligned}\right\} \quad (33)$$

$$\left.\begin{aligned} &= \left[\text{day-radius} + \frac{\text{day-radius} \times (\text{Rsin}\,\delta - \textit{agrā})}{\textit{agrā}}\right] \times \frac{\textit{antyā}}{\text{R}} \\ &\text{or } \left[\text{day-radius} - \frac{\text{day-radius} \times (\textit{agrā} - \text{Rsin}\,\delta)}{\textit{agrā}}\right] \times \frac{\textit{antyā}}{\text{R}} \end{aligned}\right\} \quad (34)$$

$$\left.\begin{aligned} &= \left[\text{day-radius} + \frac{\text{day-radius} \times (\textit{samaśaṅku} - \textit{taddhṛti})}{\textit{taddhṛti}}\right] \times \frac{\textit{antyā}}{\text{R}} \\ &\text{or } \left[\text{day-radius} - \frac{\text{day-radius} \times (\textit{taddhṛti} - \textit{samaśaṅku})}{\textit{taddhṛti}}\right] \times \frac{\textit{antyā}}{\text{R}} \end{aligned}\right\} \quad (35)$$

$$\left.\begin{aligned} &= \left[\text{day-radius} + \frac{\text{day-radius} \times (12 - \textit{palakarṇa})}{\textit{palakarṇa}}\right] \times \frac{\textit{antyā}}{\text{R}} \\ &\text{or } \left[\text{day-radius} - \frac{\text{day-radius} \times (\textit{palakarṇa} - 12)}{\textit{palakarṇa}}\right] \times \frac{\textit{antyā}}{\text{R}} \end{aligned}\right\} \quad (36)$$

These formulae are also equivalent to formulae (5) to (8). Thus Vaṭeśvara gives 36 methods for finding the Rsine of the altitude from *dhṛti* or *antyā*.

Formula (*A*) is irrelevant, because here the multiplier is less than the divisor.

16-17(a-b). (Severally) multiply the previous multipliers by the Rversed-sine of the declination, and add (these products) to the (corresponding) results obtained by multiplying the radius by the difference between the (corresponding) divisors and multipliers. Then are obtained the so called *vivaras*. By these *vivaras* multiply the *antyā* and divide (the resulting products) by the (corresponding) subsequent divisors. The *antyā* (severally) diminished by these results gives the Rsines of the altitude.

$$\text{Rsin}\,a = \textit{antyā} - \frac{\textit{vivara} \times \textit{antyā}}{\text{subsequent divisor}}, \qquad (37)$$

where *vivara* = Rvers δ × multiplier + radius (divisor − multiplier).

This simplifies easily to

$$\text{Rsin}\,a = \frac{\text{day-radius} \times \text{multiplier} \times \textit{antyā}}{\text{subsequent divisor}},$$

the formula stated in verse 13 (c-d).

17(c-d). Or, the *antyā* multiplied by the difference between the subsequent divisor and the *vivara*, and divided by the subsequent divisor also gives the same (Rsine of the altitude).

$$\text{Rsin } a = \frac{(\text{subsequent divisor} - vivara) \times antyā}{\text{subsequent divisor}}, \quad (38)$$

where the *vivara* is the same as defined in vs. 16 above.

This is an alternative form of formula (37).

18-20. The difference between (i) the product of the Rsine of the ascensional difference and the multiplier and (ii) the radius multiplied by the difference between the divisor and the multiplier, gives the so called *bhedas*. The differences or sums of those (*bhedas*) and the (corresponding) subsequent divisors, according as the (Sun's) hemisphere is north or south, give the so called *ghātas*. When the radius multiplied by the difference between the divisor and the multiplier is less than the multiplier multiplied by the Rsine of the ascensional difference, one should take the sum even in the northern hemisphere.

The day-radius multiplied by the *bheda* and divided by the (corresponding) subsequent divisor should be subtracted from or added to the day-radius (according as the Sun's hemisphere is north or south); or, the day-radius should be multiplied by the *ghāta* and divided by the (corresponding) subsequent divisor: then are obtained the Rsines of the Sun's altitude, as before.

The above rule seems to be defective. The correct rule should run as follows:

"The difference or sum of (i) the product of the Rsine of the ascensional difference and the multiplier and (ii) the radius multiplied by the difference between the divisor and the multiplier, according as the (Sun's) hemisphere is north or south, gives the so called *bhedas*. The differences between these (*bhedas*) and the (corresponding) subsequent divisors give the so called *ghātas*. When, in the northern hemisphere, the radius multiplied by the difference between the divisor and the multiplier is less than the multiplier multiplied by the Rsine of the ascensional difference, one should take the sum of the *bheda* and the corresponding subsequent divisor.

The day-radius multiplied by the *bheda* and divided by the (corresponding) subsequent divisor should be subtracted from (but, in the exceptional

case, added to) the day-radius, or the day-radius should be multiplied by the *ghāta* and divided by the (corresponding) subsequent divisor : then are obtained the Rsines of the Sun's (meridian) altitude, as before.

Rationale. Let a denote the Sun's meridian altitude, c the Sun's ascensional difference, M the multiplier and D the corresponding divisor. Then from vs. 12 (a-b), we have

$$\text{Rsin } a = \frac{antyā \times M \times \text{day-radius}}{R \times D}.$$

Case 1. When the Sun is in the northern hemisphere,

$$\text{Rsin } a = \frac{(R + \text{Rsin } c) \times M \times \text{day-radius}}{R \times D}$$

$$= \frac{R \times D - [R \times D - (R + \text{Rsin } c) \times M]}{R \times D} \times \text{day-radius}$$

$$= \frac{\text{subsequent divisor} - [(D-M) \times R - \text{Rsin } c \times M]}{\text{subsequent divisor}} \times \text{day-radius},$$

because R × D = subsequent divisor

$$= \left[\text{day-radius} - \frac{bheda \times \text{day-radius}}{\text{subsequent divisor}}\right] \text{ or } \frac{ghāta \times \text{day-radius}}{\text{subsequent divisor}}. \quad (39)$$

Case 2. When the Sun is in the southern hemisphere,

$$\text{Rsin } a = \frac{(R - \text{Rsin } c) \times M \times \text{day-radius}}{R \times D}$$

$$= \frac{R \times D - [R \times D - (R - \text{Rsin } c) \times M]}{R \times D} \times \text{day-radius}$$

$$= \frac{\text{subsequent divisor} - [(D - M) \times R + \text{Rsin } c \times M)}{\text{subsequent divisor}} \times \text{day-radius}$$

$$= \left[\text{day-radius} - \frac{bheda \times \text{day-radius}}{\text{subsequent divisor}}\right] \text{ or } \frac{ghāta \times \text{day-radius}}{\text{subsequent divisor}}. \quad (40)$$

21. Having subtracted the difference between the square of the radius and the "earlier *ghāta*" (i.e., *antyā* × day-radius) from the square of the radius, and then having multiplied the difference by 12, divide

whatever is obtained by the (corresponding) subsequent divisor: the result is the Rsine of the Sun's altitude as before.

$$\text{Rsin } a = \frac{antyā \times \text{day-radius} \times 12}{\text{R} \times palakarṇa}, \quad \text{[vide formula (8)]}$$

$$= \frac{\text{R}^2 - (\text{R}^2 - antyā \times \text{day-radius})}{\text{R} \times palakarṇa} \times 12$$

$$= \frac{\text{R}^2 - (\text{R}^2 - \text{earlier } ghāta)}{\text{subsequent divisor}} \times 12. \quad (41)$$

22. Diminish the radius by the Rsine of colatitude, multiply that by the radius, subtract that from the square of the radius, divide that by the (corresponding) "subsequent divisor", and by that multiply the *dhṛti*: then is obtained the Rsine of the Sun's altitude at the middle of the day.

$$\text{Rsin } a = \frac{\text{R}^2 - (\text{R} - \text{Rcos } \phi) \times \text{R}}{\text{subsequent divisor}} \times dhṛti. \quad (42)$$

This is equivalent to formula (1).

23. By similar addition and subtraction one may find the Rsines of the altitude for midday in numerous ways.

The Rsines of the zenith distance and altitude should be computed from each other like the Rsines of the *bhuja* and *koṭi*.

RSINE OF MERIDIAN ALTITUDE FROM *ŚAṄKUTALA*

24. Severally multiply the *dhṛti* by the Rsine of latitude, the equinoctial midday shadow, the earthsine and the *agrā* and divide by the radius, the hypotenuse of the equinoctial midday shadow, the *agrā* and the *taddhṛti*, respectively: then is obtained the *śaṅkutala* (in each case).

25. Or, find the *śaṅkutala* from the *dhṛti* or the *antyā* etc. with the help of the multipliers and divisors stated above (in vs. 24), as before. The square root of the square of the *dhṛti* diminished by the square of that (*śaṅkutala*) also gives the Rsine of the altitude.

$$\text{Rsin } a = \sqrt{[(dhṛti)^2 - (śaṅkutala)^2]}, \text{ where} \quad (43)$$

$$śaṅkutala = \frac{dhṛti \times \text{Rsin } \phi}{\text{R}} \text{ or } \frac{antyā \times \text{day-radius} \times \text{Rsin } \phi}{\text{R} \times \text{R}}$$

$$= \frac{dhṛti \times palabhā}{palakarṇa} \text{ or } \frac{antyā \times \text{day-radius} \times palabhā}{\text{R} \times palakarṇa}$$

$$= \frac{dh\dot{r}ti \times \text{earthsine}}{agr\bar{a}} \text{ or } \frac{anty\bar{a} \times \text{day-radius} \times \text{earthsine}}{R \times agr\bar{a}}$$

$$= \frac{dh\dot{r}ti \times agr\bar{a}}{taddh\dot{r}ti} \text{ or } \frac{anty\bar{a} \times \text{day-radius} \times agr\bar{a}}{R \times taddh\dot{r}ti}.$$

26. Or, the Rsine of the altitude (severally) multiplied by the multipliers stated above (in vs. 24) and divided by the Rsine of the colatitude, 12, the Rsine of the declination and the Rsine of the prime vertical altitude, respectively, gives the *śaṅkutalas*. Those (*śaṅkutalas*) multiplied by the above divisors and divided by the (same) multipliers give the Rsines of the altitude.

$$\text{Rsin } a = \frac{śaṅkutala \times \text{divisor}}{\text{multiplier}},$$

$$\text{i.e., Rsin } a = \frac{śaṅkutala \times \text{Rcos } \phi}{\text{Rsin } \phi} \quad (44)$$

$$= \frac{śaṅkutala \times 12}{palabh\bar{a}} \quad (45)$$

$$= \frac{śaṅkutala \times \text{Rsin } \delta}{\text{earthsine}} \quad (46)$$

$$= \frac{śaṅkutala \times samaśaṅku}{agr\bar{a}}, \quad (47)$$

where $śaṅkutala = \frac{\text{Rsin } a \times \text{Rsin } \phi}{\text{Rcos } \phi}$[1]

$$= \frac{\text{Rsin } a \times palabh\bar{a}}{12}$$[2]

$$= \frac{\text{Rsin } a \times \text{earthsine}}{\text{Rsin } \delta}$$

$$= \frac{\text{Rsin } a \times agr\bar{a}}{samaśaṅku}.$$

27. The difference between the squares of the radius and the *dhṛti* divided by the *agrā*, and the quotient (obtained) diminished or increased by the *agrā* and halved, yields respectively the *śaṅkutala* or *dṛgjyā* when the Sun is in the southern hemisphere and the *dṛgjyā* or *śaṅkutala* when the Sun is in the northern hemisphere.

1. *Cf. PSi*, iv. 52; *MBh*, iii. 54; *LBh*, iii, 16; *BrSpSi*, iii. 65; *SiŚe*, iv. 91; *TS*, iii. 47.
2. *Cf. ŚiDVṛ*, iv. 49.

When the Sun is in the southern hemisphere, then at midday we have

$$\tfrac{1}{2}\left[\frac{R^2 - (dh\underset{.}{r}ti)^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}\left[\frac{R^2 - (\acute{s}a\dot{n}ku)^2 - (\acute{s}a\dot{n}kutala)^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}\left[\frac{(R\sin z)^2 - (\acute{s}a\dot{n}kutala)^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}[(R\sin z + \acute{s}a\dot{n}kutala) \mp (R\sin z - \acute{s}a\dot{n}kutala)],$$

since $agr\bar{a} = R\sin z - \acute{s}a\dot{n}kutala$

$$= \acute{s}a\dot{n}kutala \text{ or } R\sin z,$$

according as $-$ or $+$ sign is taken, z being the Sun's zenith distance.

When the Sun is in the northern hemisphere, then at midday (assuming that the Sun is to the south of the zenith) we have

$$\tfrac{1}{2}\left[\frac{(dh\underset{.}{r}ti)^2 - R^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}\left[\frac{(\acute{s}a\dot{n}kutala)^2 + (\acute{s}a\dot{n}ku)^2 - R^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}\left[\frac{(\acute{s}a\dot{n}kutala)^2 - (R\sin z)^2}{agr\bar{a}} \mp agr\bar{a}\right]$$

$$= \tfrac{1}{2}[(\acute{s}a\dot{n}kutala + R\sin z) \mp (\acute{s}a\dot{n}kutala - R\sin z)],$$

since $agr\bar{a} = \acute{s}a\dot{n}kutala - R\sin z$

$$= R\sin z \text{ or } \acute{s}a\dot{n}kutala, \qquad (49)$$

according as $-$ or $+$ sign is taken.

Similarly when the Sun is to the north of the zenith.

28. Multiply the sum of the radius and the *dhṛti* by their difference and divide (the resulting product) by the *agrā*. From the quotient (so obtained) and the *agrā*, obtain the *śaṅkutala* and *dṛgjyā* as before.

That is, when the Sun is in the southern hemisphere, then at midday

$$\left.\begin{aligned} \acute{s}a\dot{n}kutala &= \tfrac{1}{2}\left[\frac{(R + dh\underset{.}{r}ti)(R - dh\underset{.}{r}ti)}{agr\bar{a}} - agr\bar{a}\right] \\ R\sin z &= \tfrac{1}{2}\left[\frac{(R + dh\underset{.}{r}ti)(R - dh\underset{.}{r}ti)}{agr\bar{a}} + agr\bar{a}\right] \end{aligned}\right\} (50)$$

and when the Sun is in the northern hemisphere, then at midday

$$\left.\begin{aligned} \acute{s}a\dot{n}kutala &= \tfrac{1}{2}\left[\frac{(dh\d{r}ti + \mathrm{R})(dh\d{r}ti - \mathrm{R})}{agr\bar{a}} + agr\bar{a}\right] \\ \mathrm{Rsin}\ z &= \tfrac{1}{2}\left[\frac{(dh\d{r}ti + \mathrm{R})\ (dh\d{r}ti - \mathrm{R})}{agr\bar{a}} - agr\bar{a}\right]. \end{aligned}\right\} \quad (51)$$

This rule is the same as the previous one.

MIDDAY SHADOW AND HYPOTENUSE OF MIDDAY SHADOW

29. The product of the square of the radius and the hypotenuse of the equinoctial midday shadow, divided by the product of the day-radius and the *antyā*, is the hypotenuse of the (midday) shadow ;[1] or, the radius multiplied by the hypotenuse of the equinoctial midday shadow and divided by the (Sun's) own *dhṛti* is the hypotenuse of the midday shadow.[2]

$$\text{Hypotenuse of midday shadow} = \frac{\mathrm{R}^2 \times palakar\d{n}a}{\text{day-radius} \times anty\bar{a}} \qquad (52)$$

$$= \frac{\mathrm{R} \times palakar\d{n}a}{dh\d{r}ti}. \qquad (53)$$

Rationale. Let a be the Sun's altitude at midday. Then[3]

$$\text{hypotenuse of midday shadow} = \frac{\mathrm{R} \times 12}{\mathrm{Rsin}\ a}, \qquad \text{(i)}$$

where, by vs. 9,
$$\mathrm{Rsin}\ a = \frac{\text{day-radius} \times anty\bar{a} \times 12}{\mathrm{R} \times palakar\d{n}a} \qquad \text{(ii)}$$

and, by vs. 8,
$$\mathrm{Rsin}\ a = \frac{dh\d{r}ti \times 12}{palakar\d{n}a}. \qquad \text{(iii)}$$

Combining (i) and (ii) we get formula (52), and combining (i) and (iii) we get formula (53).

30. The product of the *agrā* and the radius multiplied by the hypotenuse of the equinoctial midday shadow and divided by the product of the day-radius and the *antyā* gives the *agrā* corresponding to the shadow circle. The *agrâ* multiplied by the hypotenuse of the equinoctial midday shadow and divided by the *dhṛti* is also the same.

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1. *Cf. BrSpSi*, iii. 30(c-d), 31(a-b); *SiŚe*, iv. 38(a-b).
2. *Cf. BrSpSi*, iii. 28(c-d); *MSi*, iv. 17.
3. *Cf. BrSpSi*, iii. 29(a-b).

31. The difference or sum of that and the equinoctial midday shadow according as the Sun is in the northern or southern hemisphere, gives the midday shadow (of the gnomon). The Rsine of the Sun's zenith distance multiplied by the hypotenuse of the equinoctial midday shadow and divided by the *dhṛti*, too, gives the midday shadow (of the gnomon).

$$\text{Midday shadow} = \textit{bhāvṛttīya agrā} \sim \text{or} + \textit{palabhā}, \tag{54}$$

where *bhāvṛttīya agrā* ("*agrā* corresponding to shadow circle")

$$= \frac{\textit{agrā} \times \text{R} \times \textit{palakarṇa}}{\text{day-radius} \times \textit{antyā}} \tag{55}$$

$$= \frac{\textit{agrā} \times \textit{palakarṇa}}{\textit{dhṛti}}, \tag{56}$$

$\sim$ or $+$ sign being taken according as the Sun is in the northern or southern hemisphere.

$$\text{Also} \quad \text{midday shadow} = \frac{\text{Rsin}\, z \times \textit{palakarṇa}}{\textit{dhṛti}}, \tag{57}$$

where z is the Sun's zenith distance (at midday).

Proof. Since

$$\text{hypotenuse of shadow} = \frac{\text{R} \times 12}{\text{Rsin}\, a}$$

and

$$\text{Rsin}\, a = \frac{12 \times \textit{svadhṛti}}{\textit{palakarṇa}},$$

$$\text{therefore} \quad \text{hypotenuse of shadow} = \frac{\text{R} \times \textit{palakarṇa}}{\textit{svadhṛti}}. \tag{i}$$

$$\therefore \quad \textit{bhāvṛttīya agrā} = \frac{\textit{agrā} \times \text{hypotenuse of shadow}}{\text{R}}$$

$$= \frac{\textit{agrā} \times \textit{palakarṇa}}{\textit{svadhṛti}}, \quad \text{using (i)},$$

which gives formula (56).

Again since

$$\textit{dhṛti} = \frac{\text{day-radius} \times \textit{antyā}}{\text{R}},$$

therefore formula (56) becomes

$$bh\bar{a}vrtt\bar{\imath}ya\ agr\bar{a} = \frac{agr\bar{a} \times \text{R} \times palakarna}{\text{day-radius} \times anty\bar{a}},$$

which is formula (55).

Since

$$\text{midday } bhuja = \text{midday } \acute{s}a\dot{n}kutala \sim \text{or} + agr\bar{a},$$

therefore multiplying throughout by

$$\frac{\text{hypotenuse of midday shadow}}{\text{R}},$$

we get

$$\text{midday shadow} = palabh\bar{a} \sim \text{or} + bh\bar{a}vrtt\bar{\imath}ya\ agr\bar{a},$$

$\sim$ or $+$ sign being taken according as the Sun's hemisphere is north or south.

Also, since

$$\text{midday shadow} = \frac{\text{Rsin } z \times \text{hypotenuse of midday shadow}}{\text{R}}$$

and hypotenuse of midday shadow $= \dfrac{\text{R} \times 12}{\text{Rsin } a} = \dfrac{\text{R} \times palakarna}{dhrti}$,

therefore

$$\text{midday shadow} = \frac{\text{Rsin } z \times palakarna}{dhrti}.$$

32. Or, multiply the sum of the hypotenuse of the midday shadow and the hypotenuse of the equinoctial midday shadow by their difference and divide by the difference of the two shadows, and then diminish the quotient (obtained) by the equinoctial midday shadow: the result is the midday shadow.

Let s be the midday shadow and h the hypotenuse of the midday shadow; P the equinoctial midday shadow and H the hypotenuse of the equinoctial midday shadow. Then

$$\text{midday shadow } s = \frac{(h + H)(h \sim H)}{s \sim P} - P. \qquad (58)$$

Proof. Right hand side $= \dfrac{h^2 \sim H^2}{s \sim P} - P$

$$= \frac{s^2 \sim P^2}{s \sim P} - P$$

$$= s, \text{ the midday shadow.}$$

33. Or, when the Sun is in the northern hemisphere, the midday shadow and the *palabhā* are obtained when the reverse of them (i. e., the *palabhā* and the midday shadow) are respectively diminished and increased by the *agrā* for the shadow circle; when the Sun is in the southern hemisphere, the *palabhā* and the midday shadow are obtained when the reverse of them (i.e., the midday shadow and the *palabhā*) are respectively diminished and increased by the *agrā* for the shadow circle.

ALTERNATIVE TRANSLATION

33. Or, when the Sun is in the northern hemisphere, the *palabhā* and the midday shadow (*viparīta*) being respectively diminished and increased by the *agrā* for the shadow circle yield the midday shadow and the *palabhā*; when the Sun is in the southern hemisphere, the midday shadow and the *palabhā* being respectively diminished and increased by the *agrā* for the shadow circle yield the *palabhā* and the midday shadow.

That is, when the Sun is in the northern hemisphere (and the Sun at midday is to the south of the zenith),

midday shadow = *palabhā* — *bhāvṛttāgrā*

palabhā = midday shadow + *bhāvṛttāgrā*;

and when the Sun is in the southern hemisphere,

palabhā = midday shadow — *bhāvṛttāgrā*

midday shadow = *palabhā* + *bhāvṛttāgrā*.

34. When the Sun is in the northern hemisphere, diminish twice the *palabhā* by the *agrā* of the shadow circle and multiply (the difference) by the *agrā* of the shadow circle, then subtract that from the square of the *palakarṇa*, and then take the square-root of that; when the Sun is in the southern hemisphere, increase twice the *palabhā* by the *agrā* of the shadow circle and multiply (the sum) by the *agrā* of the shadow circle, then add that to the square of the *palakarṇa*, and then take the square-root of that: the square-root (in both the cases) gives the hypotenuse of the midday shadow.

That is: When the Sun is in the northern hemisphere, hypotenuse of the midday shadow

$$= \sqrt{(palakar\d{n}a)^2 - (2\,palabh\bar{a} - bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}}\,;$$

and when the Sun is in the southern hemisphere,

hypotenuse of the midday shadow

$$= \sqrt{(palakar\d{n}a)^2 + (2\,palabh\bar{a} + bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}}\,. \quad (60)$$

Rationale. When the Sun is in the northern hemisphere, then

midday shadow $= palabh\bar{a} \sim bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}$.

$\therefore$ Hypotenuse of the midday shadow $= \sqrt{[12^2+(palabh\bar{a} \sim bh\bar{a}v\d{r}tt\bar{a}gr\bar{a})^2]}$

$$= \sqrt{[12^2 + (palabh\bar{a})^2 - (2\,palabh\bar{a} - bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}]}$$

$$= \sqrt{(palakar\d{n}a)^2 - (2\,palabh\bar{a} - bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}}.$$

And when the Sun is in the southern hemisphere, then

midday shadow $= palabh\bar{a} + bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}$.

$\therefore$ Hypotenuse of the midday shadow $= \sqrt{[12^2+(palabh\bar{a}+ bh\bar{a}v\d{r}tt\bar{a}gr\bar{a})^2]}$

$$= \sqrt{[12^2 + (palabh\bar{a})^2 + (2\,palabh\bar{a} + bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}]}$$

$$= \sqrt{(palakar\d{n}a)^2 + (2\,palabh\bar{a} + bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}) \times bh\bar{a}v\d{r}tt\bar{a}gr\bar{a}}.$$

35. The difference between the radius and the *dhṛti*, multiplied by the hypotenuse of the equinoctial midday shadow, when divided by the *dhṛti*, and the resulting quotient subtracted from or added to the hypotenuse of the equinoctial midday shadow, gives the hypotenuse of the midday shadow.

That is: Hypotenuse of midday shadow

$$= palakar\d{n}a \mp \frac{(R \sim dh\d{r}ti) \times palakar\d{n}a}{dh\d{r}ti}, \quad (61)$$

— or + sign being taken according as the *dhṛti* is greater or less than the radius.

This formula is an alternative form of formula (53).

Rationale. When *dhṛti* > R, we can write formula (53) in the form ;

Hypotenuse of midday shadow $= palakarṇa - \frac{(dhṛti - R) \times palakarṇa}{dhṛti}$, (i)

and when R > *dhṛti*, we can write formula (53) in the form :

Hypotenuse of midday shadow $= palakarṇa + \frac{(R - dhṛti) \times palakarṇa}{dhṛti}$. (ii)

Combining (i) and (ii), we get formula (61).

36. The difference between (i) the product of the day-radius and the *antyā* and (ii) the square of the radius, multiplied by the hypotenuse of the equinoctial midday shadow, being divided by the product of the day-radius and the *antyā*, and the resulting quotient subtracted from or added to the hypotenuse of the equinoctial midday shadow, also gives the hypotenuse of the midday shadow.

Hypotenuse of midday shadow = *palakarṇa*

$$\mp \frac{(\text{day-radius} \times antyā \sim R^2) \times palakarṇa}{\text{day-radius} \times antyā}, \qquad (62)$$

− or + sign being taken according as the *dhṛti* is greater or less than the radius.

This formula is an alternative form of formula (52).

Rationale. When *dhṛti* > R and likewise

$$\frac{\text{day-radius}}{R} = \frac{dhṛti}{antyā} > \frac{R}{antyā},$$

$$\text{i. e., day-radius} \times antyā > R^2,$$

we can write formula (52) in the form :

Hypotenuse of midday shadow = *palakarṇa* −

$$-\frac{(\text{day-radius} \times antyā - R^2) \times palakarṇa}{\text{day-radius} \times antyā}; \qquad (i)$$

and when R > *dhṛti*, so that

$$R^2 > \text{day-radius} \times antyā,$$

we can write formula (52) in the form :

Hypotenuse of midday shadow = *palakarṇa* +

$$+\ \frac{(R^2 - \text{day-radius} \times \textit{antyā}) \times \textit{palakarṇa}}{\text{day-radius} \times \textit{antyā}}. \qquad \text{(ii)}$$

Combining (i) and (ii), we get formula (62).

37(a-b). Or, the product of the hypotenuse of the equinoctial midday shadow, the Rsine of the ascensional difference and the radius, divided by the product of the earthsine and the *antyā*, is the hypotenuse of the midday shadow.

Hypotenuse of the midday shadow

$$= \frac{\textit{palakarṇa} \times \text{Rsin (asc. diff.)} \times R}{\text{earthsine} \times \textit{antyā}}. \qquad (63)$$

This is equivalent to formula (52), because

$$\frac{R}{\text{day-radius}} = \frac{\text{Rsin (asc. diff.)}}{\text{earthsine}}.$$

37(c-d)-38. Find the product of the earthsine and the *antyā* and also of the Rsine of the ascensional difference and the radius ; multiply the difference of the two (products) by the hypotenuse of the equinoctial midday shadow and divide that by the first product. The hypotenuse of the equinoctial midday shadow diminished or increased by that quotient is, as before, the hypotenuse of the midday shadow.

Hypotenuse of the midday shadow = *palakarṇa*

$$\mp\ \frac{[\text{earthsine} \times \textit{antyā} \sim \text{Rsin (asc. diff.)} \times R] \times \textit{palakarṇa}}{\text{earthsine} \times \textit{antyā}}, \qquad (64)$$

— or + sign being taken according as the *dhṛti* is greater or less than the radius.

This formula is an alternative form of formula (63).

Rationale. When *dhṛti* > R and likewise

$$\frac{\text{earthsine}}{\text{Rsin (asc. diff.)}} = \frac{\textit{dhṛti}}{\textit{antyā}} > \frac{R}{\textit{antyā}},$$

i. e., earthsine × *antyā* > Rsin (asc. diff.) × R,

we can write formula (63) in the form :

$$\text{hyp. midday shadow} = palakarṇa\ -$$

$$-\ \frac{\text{earthsine} \times antyā - \text{Rsin (asc. diff.)} \times \text{R}}{\text{earthsine} \times antyā} \times palakarṇa\,; \quad \text{(i)}$$

and when *dhṛti* $<$ R and likewise

$$\text{Rsin (asc. diff.)} \times \text{R} > \text{earthsine} \times antyā,$$

we can write formula (63) in the form :

$$\text{hyp. midday shadow} = palakarṇa\ +$$

$$+\ \frac{\text{Rsin (asc. diff.)} \times \text{R} - \text{earthsine} \times antyā}{\text{earthsine} \times antyā} \times palakarṇa. \quad \text{(ii)}$$

Combining (i) and (ii) we get formula (64).

39. The difference between the radius and the *dhṛti* should be multiplied by their sum and (the resulting product should be) multiplied by the hypotenuse of the equinoctial midday shadow and divided by the product of the *agrā* and the *dhṛti*. The difference between the quotient (obtained) and the equinoctial midday shadow is the midday shadow (of the gnomon).

$$\text{Midday shadow} = \frac{(dhṛti \sim \text{R})\,(dhṛti + \text{R}) \times palakarṇa}{agrā \times dhṛti} \sim palabhā. \quad (65)$$

Rationale.

Case 1. When the Sun is in the northern hemisphere and $\delta < \phi$, then at midday

$$(dhṛti)^2 = \text{R}^2 + (agrā)^2 + 2\ agrā \times \text{Rcos}\ a,$$

where a is the Sun's altitude at midday.

$$\therefore\ \frac{[(dhṛti)^2 - \text{R}^2] \times palakarṇa}{agrā \times dhṛti}$$

$$= \frac{agrā \times palakarṇa}{dhṛti} + \frac{2\ \text{Rcos}\ a \times palakarṇa}{dhṛti}$$

$$= \frac{agrā \times 12}{\text{Rsin}\ a} + \frac{2\ \text{Rcos}\ a \times 12}{\text{Rsin}\ a}$$

$$= \frac{agr\bar{a} \times h}{R} + 2\,s,$$

where s denotes the midday shadow of the gnomon and h the hypotenuse of the midday shadow.

$$\therefore \frac{[(dh\d{r}ti)^2 - R^2\,]\,palakar\d{n}a}{agr\bar{a} \times dh\d{r}ti} - palabh\bar{a}$$

$$= 2s - \left[palabh\bar{a} - \frac{agr\bar{a} \times h}{R}\right]$$

$$= 2s - (palabh\bar{a} - ch\bar{a}y\bar{a}kar\d{n}\bar{a}gr\bar{\imath}\ agr\bar{a})$$

$$= 2s - s = s, \text{ i.e., midday shadow.} \qquad \text{(i)}$$

Case 2. When the Sun is in the northern hemisphere and $\delta > \phi$, then at midday

$$(dh\d{r}ti)^2 = R^2 + (agr\bar{a})^2 - 2\ agr\bar{a} \times R\cos a.$$

Therefore, proceeding as above,

$$palabh\bar{a} - \frac{[(dh\d{r}ti)^2 - R^2] \times palakar\d{n}a}{agr\bar{a} \times dh\d{r}ti} = \text{midday shadow.} \qquad \text{(ii)}$$

Case 3. When the Sun is in the southern hemisphere, then at midday

$$(dh\d{r}ti)^2 = R^2 + (agr\bar{a})^2 - 2\ agr\bar{a} \times R\cos a.$$

Hence $$\frac{[R^2 - (dh\d{r}ti)^2] \times palakar\d{n}a}{agr\bar{a} \times dh\d{r}ti} - palabh\bar{a} = \text{midday shadow.} \qquad \text{(iii)}$$

Combining (i), (ii) and (iii), we get formula (65).

Alternative Rationale.

When the Sun is in the northern hemisphere and $\delta < \phi$, then at midday

$$(dh\d{r}ti)^2 = (R\sin a)^2 + (\acute{s}a\dot{n}kutala)^2,$$

$$R^2 = (R\sin a)^2 + (bhuja)^2,$$

and $\acute{s}a\dot{n}kutala - bhuja = agr\bar{a}$.

∴ Right hand side of (65)

$$= \frac{(\acute{s}a\dot{n}kutala + bhuja) \times palakar\d{n}a}{dh\d{r}ti} - palabh\bar{a}$$

$$= \frac{(2\ \textit{śaṅkutala} - \textit{agrā}) \times \textit{palakarṇa}}{\textit{dhṛti}} - \textit{palabhā}$$

$$= \left[2\ \textit{palabhā} - \frac{\textit{agrā} \times 12}{\text{Rsin } a}\right] - \textit{palabhā}$$

$$= \textit{palabhā} - \frac{\textit{agrā} \times (\text{hypotenuse of midday shadow})}{\text{R}}$$

$$= \textit{palabhā} - \textit{chāyākarṇāgrī agrā}$$

$$= \text{midday shadow.}$$

Similarly in the other cases.

40. The hypotenuse of the equinoctial midday shadow multiplied by 286½, and the radius multiplied by that, when divided by *dhṛti* and day-radius × *antyā*, respectively, also give the hypotenuse (of the midday shadow) when 12 is multiplied by (either of) those results.

$$\text{Hypotenuse of the midday shadow} = \frac{286\tfrac{1}{2} \times \textit{palakarṇa}}{\textit{dhṛti}} \times 12 \qquad (66)$$

$$= \frac{286\tfrac{1}{2} \times \textit{palakarṇa} \times \text{R}}{\text{day-radius} \times \textit{antyā}} \times 12. \qquad (67)$$

Rationale. One can easily see that

$$286\tfrac{1}{2} = \frac{\text{R}}{12}.$$

Therefore from formula (53), we have

$$\text{hypotenuse of the midday shadow} = \frac{\textit{palakarṇa} \times \text{R}}{\textit{dhṛti}}$$

$$= \frac{(\text{R}/12) \times \textit{palakarṇa}}{\textit{dhṛti}} \times 12$$

$$= \frac{286\tfrac{1}{2} \times \textit{palakarṇa}}{\textit{dhṛti}} \times 12. \qquad \text{(i)}$$

But since

$$\textit{dhṛti} = \frac{\text{day-radius} \times \textit{antyā}}{\text{R}},$$

therefore (i) reduces to

$$\text{hypotenuse of the midday shadow} = \frac{286\tfrac{1}{2} \times \textit{palakarṇa} \times \text{R}}{\text{day-radius} \times \textit{antyā}} \times 12. \qquad \text{(ii)}$$

DHṚTI, ANTYĀ AND DAY-RADIUS

41. From the square of the radius multiplied by the *palakarṇa* and divided by the hypotenuse of the midday shadow (*dyukhaṇḍakarṇa* or *madhyakarṇa*) multiplied by the radius is obtained the *dhṛti* (for midday); and from the product (of the day-radius and the *antyā*) divided by the radius is obtained the (Sun's) own *dhṛti*.

$$\text{Midday } dhṛti = \frac{palakarṇa \times R^2}{madhyakarṇa \times R} \tag{68}$$

$$Svadhṛti = \frac{\text{day-radius} \times antyā}{R}. \tag{69}$$

Rationale. One can easily see that

$$\text{midday } dhṛti = \frac{madhyaśaṅku \times R}{R\cos\phi}$$

$$= \frac{palakarṇa \times R^2}{madhyakarṇa \times R},$$

because

$$madhyaśaṅku = \frac{12 \times R}{madhyakarṇa}$$

and

$$R\cos\phi = \frac{12 \times R}{palakarṇa}.$$

Hence formula (68). Formula (69) is evident.

42. The product (viz. day-radius × *antyā*) when divided by the day-radius gives the *antyā*, and when divided by the *antyā* gives the day-radius. Also the *antyā* multiplied by the day-radius when divided by the radius gives the *dhṛti*.

$$Antyā = \frac{\text{day-radius} \times antyā}{\text{day-radius}} \tag{70}$$

$$\text{Day-radius} = \frac{\text{day-radius} \times antyā}{antyā} \tag{71}$$

$$Dhṛti = \frac{\text{day-radius} \times antyā}{R}. \qquad (72)[1]$$

These results are trivial.

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1. This is equivalent to the formula given in *SiŚi*, I, iii. 35(a-b).

43. The square of the radius multiplied by the *palakarṇa* when divided by the day-radius multiplied by the hypotenuse (of shadow) gives the *antyā*; and when divided by the product of the hypotenuse (of shadow) and the *antyā*, the quotient obtained is the day-radius.

$$Antyā = \frac{R^2 \times palakarṇa}{\text{day-radius} \times \text{hypotenuse of shadow}} \tag{73}$$

$$\text{Day-radius} = \frac{R^2 \times palakarṇa}{antyā \times \text{hypotenuse of shadow}}. \tag{74}$$

See formula (52).

44. The *antyā* diminished by the result obtained by multiplying the *antyā* by the difference between the radius and the day-radius and dividing by the radius gives the *svadhṛti*; and the *antyā* diminished by the result obtained by multiplying the *antyā* by *carajyā* (i.e., Rsine of the ascensional difference) minus earthsine and dividing by the *carajyā* also gives the (*sva*)*dhṛti*.

$$svadhṛti = antyā - \frac{antyā\,(R - \text{day-radius})}{R} \tag{75}$$

$$= antyā - \frac{antyā\,(carajyā - \text{earthsine})}{carajyā}. \tag{76}$$

Formula (75) simplifies to formula (69); and formula (76) simplifies to

$$svadhṛti = \frac{\text{earthsine} \times antyā}{carajyā},$$

which is equivalent to formula (69), because

$$\frac{\text{earthsine}}{carajyā} = \frac{\text{day-radius}}{\text{radius}}.$$

45(a-b). The Rversed-sine of half the duration of the day, or the radius increased by the Rsine of the excess of the semi-duration of the day over 5400 *asus* is the *antyā* (for the middle of the day).[1]

45(c-d). The *dhṛti* multiplied by the Rsine of the ascensional difference and divided by the earthsine is also the *antyā*.[2]

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1. *Cf. BrSpSi*, xv. 54(a-b); *ŚiDVr*, iv. 50(a-b); *SiŚe*, iv. 93.
2. *Cf. SiŚi*, I, iii. 35(a-b).

That is : If $5400 \pm c$, where c is the Sun's ascensional difference, be the *asus* of half the duration of the day, then

$$antyā = \text{Rvers}\,(5400 \pm c)' \tag{77}$$

$$= \text{R} \pm \text{Rsin}\, c. \tag{78}$$

Also $$antyā = \frac{dhṛti \times \text{Rsin}\, c}{\text{earthsine}}. \tag{79}$$

PARTICULAR CASES

46. When the (Sun's) northern declination amounts to the latitude (of the place), the Rsine of the (Sun's) zenith distance for the middle of the day is non-existent but the Rsine of the (Sun's) altitude is equal to the radius.

When the Sun is on the horizon, the Rsine of the (Sun's) zenith distance is equal to the radius but the Rsine of the (Sun's) altitude is non-existent.

47. When the Rversed-sine of the latitude is zero, the Rsine of the colatitude is equal to the radius ; and then at midday, when the shadow is not cast (due to the Sun being on the equator and at the zenith), the *dhṛti* is stated to be exactly equal to the radius.

48. But here (at Ānandapura) and also at Daśapura, when the Sun is at the summer solstice, the same (*dhṛti*) of the Sun for midday is equal to the radius multiplied by the *palakarṇa* and divided by 12.

Daśapura, according to Monier-Williams (see his Sanskrit-English Dictionary), is "Decapolis", the modern Mandasor (lat. 24.03 N., long. 75.08 E.) in Madhya Pradesh.

From vs. 48 it is evident that Ānandapura and Daśapura were both situated in latitude 24°.

Section 10 : Shadow for the desired time

NATAKĀLA AND *UNNATAKĀLA*

1. The time-interval between the Sun and its position at midday is the *Natakāla* and that between the Sun and its position on the horizon is to be known as the *Unnatakāla*. Half the duration of the day minus the *Unnatakāla* is the *Natakāla*; and half the duration of the day minus the *Natakāla* is the *Unnatakāla*.[1]

By the duration of the day is meant the time-interval from sunrise to sunset. Thus half the duration of the day means the time-interval from sunrise to midday or from midday to sunset.

Thus the *natakāla* is the time to elapse until midday in the first half of the day, and the time elapsed since midday in the second half of the day.

The *unnatakāla* means the time elapsed since sunrise in the first half of the day, and to elapse before sunset in the second half of the day.

Thus the sum of the *natakāla* and the *unnatakāla* is equal to half the duration of the day, so that

unnatakāla = half the duration of day − *natakāla*

and *natakāla* = half the duration of day − *unnatakāla*.

UNNATAJĪVĀ OR *UNNATAJYĀ*

2. The Rsine of the *unnatakāla* diminished or increased by the ascensional difference, according as the Sun is in the northern or southern hemisphere, is the *Unnatajīvā*. The Rsine obtained from the *asus* is the same as the Rsine obtained from the *kalās* or minutes.[2]

That is :

Unnatajyā = Rsin (*unnatakāla* in *asus* $\mp$ asc. diff. in *asus*),

− or + sign being taken according as the Sun is in the northern or southern hemisphere.

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1. *Cf. KK*, I, iii. 10(c-d); *ŚiDVr*, iv. 23; *SiŚe*, iv, 67, 68.
2. *Cf. MSi.* iv. 27(a-b).

SVĀNTYĀ OR *IṢṬĀNTYĀ*

3. That (*unnatajyā*) increased by the Rsine of the ascensional difference in the northern hemisphere and diminished by the same in the southern hemisphere is the *Svāntyā*.[1] The *antyā* (i.e., *svāntyā* for the middle of the day) diminished by the Rversed-sine of the *natakāla* also is the *Svāntyā*.[2]

(1) *Svāntyā* = *unnatajyā* ± Rsin (asc. diff.),

according as the Sun is in the northern or southern hemisphere.

(2) *Svāntyā* = *antyā* − Rvers (*natakāla*).

SVADHṚTI OR *IṢṬADHṚTI*

4. The *svāntyā* should be severally multiplied by the day-radius, the *dhṛti* (i.e., *svadhṛti* for midday) and the earthsine, and divided by the radius, the *antyā* (i.e., *svāntyā* for midday) and the Rsine of the ascensional difference respectively : the resulting quantities each bear the name *Svadhṛti*.

(1) $\textit{Svadhṛti} = \dfrac{\textit{svāntyā} \times \text{day-radius}}{R}$

(2) $\textit{Svadhṛti} = \dfrac{\textit{svāntyā} \times \textit{dhṛti}}{\textit{antyā}}$

(3) $\textit{Svadhṛti} = \dfrac{\textit{svāntyā} \times \text{earthsine}}{\text{Rsin (asc. diff.)}}$.

Formula (1) occurs also in *BrSpSi*, iii. 30 (a-b); *SiŚe*, iv. 37.

5. The results obtained by multiplying the *unnatajīvā* by the (previous) multipliers and dividing by the corresponding divisors when increased or diminished by the earthsine, according as the Sun is in the northern or southern hemisphere, give the *svadhṛti*. From that one may obtain the Rsine of the (Sun's) altitude, etc., as before.

(1) $\textit{Svadhṛti} = \dfrac{\textit{unnatajīvā} \times \text{day-radius}}{R} \pm \text{earthsine}$

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1. *Cf. BrSpSi*, iii. 29(c-d); *KK*, I, iii. 84; *ŚiDVṛ*, iv. 27; *MSi*, iv. 27(c-d); *SiŚe*, iv. 37(a-b).
2. *Cf. ŚiDVṛ*, iv. 28; *SūSi*, iii. 35; *MSi*, iv. 21(a-b), 30(c-d); *SiŚi*, I, iii. 58(a-b).

(2) $Svadhṛti = \dfrac{unnatajīvā \times dhṛti}{antyā} \pm$ earthsine

(3) $Svadhṛti = \dfrac{unnatajīvā \times \text{earthsine}}{\text{Rsin (asc. diff.)}} \pm$ earthsine,

+ or − sign being taken according as the Sun is in the northern or southern hemisphere.

SUN'S ALTITUDE

6. The *svadhṛti* and the *svāntyā* being multiplied by the Rsine of the meridian altitude and divided by the *dhṛti* and the *antyā* respectively, the results are the Rsines of the altitude for the desired time.

(1) $\text{Rsin } a = \dfrac{svadhṛti \times \text{Rsin (mer. alt.)}}{dhṛti}$

(2) $\text{Rsin } a = \dfrac{svāntyā \times \text{Rsin (mer. alt.)}}{antyā}$,

where a is the altitude.

7. The Rsine of the meridian altitude severally multiplied by (i) the *dhṛti* minus *svadhṛti* and (ii) the Rversed-sine of the hour angle (*nata* or *natakāla*), and divided by (i) the *dhṛti* and (ii) the *antyā*, respectively, and then the same (Rsine of the meridian altitude) severally diminished by the results obtained yields the Rsine of the altitude for the desired time.

(1) $\text{Rsin } a = \text{Rsin (mer. alt.)} - \dfrac{(dhṛti - svadhṛti) \times \text{Rsin (mer. alt.)}}{dhṛti}$

(2) $\text{Rsin } a = \text{Rsin (mer. alt.)} - \dfrac{\text{Rvers } H \times \text{Rsin (mer. alt.)}}{antyā}$,

where a is the altitude and H the hour angle (*natakāla*).

These formulae are obviously equivalent to the previous ones, because

$$\text{Rvers } H = antyā - svāntyā.$$

8. The Rsine of the rising point of the ecliptic minus the Sun being multiplied by the Rsine of the altitude of the meridian-ecliptic point and divided by the radius, the result (obtained) is called the Rsine of the (Sun's) altitude by those proficient in Spherics.

$$\text{Rsin } a = \frac{\text{Rsin } (\textit{lagna} - \text{Sun}) \times \text{Rsin } a_m}{\text{R}},$$

where a and a_m are the altitudes of the Sun and the meridian-ecliptic point. Also see *supra*, sec. 8, vs. 14.

SHADOW AND HYPOTENUSE OF SHADOW

9. The *antyā*, *dhṛti* and the Rsine of the meridian altitude being severally multiplied by the hypotenuse of the midday shadow and divided by *svāntyā*, *svadhṛti* and the Rsine of the desired altitude respectively, the results (obtained) are the hypotenuse of the desired shadow.

(1)[1] Hypotenuse of shadow $= \dfrac{\textit{antyā} \times \text{hyp. midday shadow}}{\textit{svāntyā}}$

(2)[2] Hypotenuse of shadow $= \dfrac{\textit{dhṛti} \times \text{hyp. midday shadow}}{\textit{svadhṛti}}$

(3) Hypotenuse of shadow $= \dfrac{\text{Rsin (mer. alt.)} \times \text{hyp. midday shadow}}{\text{Rsin (alt.)}}$.

Rationale. Comparing the similar right-angled triangles (i) and (ii), viz.

	Base	Upright	Hypotenuse
(i)	Rsin (mer. z. d.)	Rsin (mer. alt.)	R
(ii)	midday shadow	12	hyp. midday shadow

we have

$$\text{hyp. midday shadow} = \frac{\text{R} \times 12}{\text{Rsin (mer. alt.)}}; \qquad (A)$$

and comparing the similar right-angled triangles (iii) and (iv), viz.

	Base	Upright	Hypotenuse
(iii)	Rsin (z. d.)	Rsin (alt.)	R
(iv)	shadow	12	hypotenuse of shadow

we have

$$\text{hypotenuse of shadow} = \frac{\text{R} \times 12}{\text{Rsin (alt.)}}. \qquad (B)$$

1. *Cf. BrSpSi*, iii. 35(c-d); *ŚiDVṛ*, iv. 30; *MSi*, iv. 25(c-d); *SiŚe*, iv. 46.
2. *Cf. BrSpSi*, iii. 35(c-d); *KK*, I, iii. 13; *ŚiDVṛ*, iv. 30; *MSi*, iv. 26; *SiŚe*, iv. 46.

Dividing (B) by (A), we get

$$\frac{\text{hypotenuse of shadow}}{\text{hyp. midday shadow}} = \frac{\text{Rsin (mer. alt.)}}{\text{Rsin (alt.)}},$$

whence

$$\text{hypotenuse of shadow} = \frac{\text{Rsin (mer. alt.)} \times \text{hyp. midday shadow}}{\text{Rsin (alt.)}}.$$

This proves (3).

Formulae (1) and (2) are equivalent to formula (3), because

$$\frac{\textit{antyā}}{\textit{svāntyā}} = \frac{\textit{dhṛti}}{\textit{svadhṛti}} = \frac{\text{Rsin (mer. alt.)}}{\text{Rsin (alt.)}}.$$

10. Or, first diminish and then divide the *antyā*, *dhṛti* and the Rsine of the meridian altitude by the *svāntyā*, *svadhṛti* and the Rsine of the desired altitude, (respectively); thereafter add one to each result and multiply by the hypotenuse of midday shadow: then is obtained, in each case, the hypotenuse of shadow for that time.

(1) Hyp. of shadow $= \left[\frac{\textit{antyā} - \textit{svāntyā}}{\textit{svāntyā}} + 1\right] h_m$

(2) Hyp. of shadow $= \left[\frac{\textit{dhṛti} - \textit{svadhṛti}}{\textit{svadhṛti}} + 1\right] h_m$

(3) Hyp. of shadow $= \left[\frac{\text{Rsin (mer. alt.)} - \text{Rsin (alt.)}}{\text{Rsin (alt.)}} + 1\right] h_m$,

where h_m denotes the hypotenuse of midday shadow.

These formulae are equivalent to those stated in vs. 9 above.

11. Having subtracted from the squares of the products of the multipliers *antyā* etc. and the hypotenuse of the midday shadow, the square of 12 times the corresponding divisors, divide the square-roots (of those differences) by the corresponding divisors: the quotient, in each case, is the shadow for that time.

$$\text{Shadow} = \frac{\sqrt{(\text{hyp. midday shadow} \times \text{multiplier})^2 - (12 \times \text{divisor})^2}}{\text{divisor}},$$

where the multipliers are *antyā*, *dhṛti* and Rsine of the meridian altitude and the corresponding divisors are *svāntyā*, *svadhṛti* and Rsine of the altitude, respectively.

This is equivalent to saying that :

$$\text{shadow} = \sqrt{(\text{hyp. of shadow})^2 - 12^2},$$

where the hypotenuse of shadow is given by the formulae of vs. 9 above.

12. Or, multiply the "square roots" (of vs. 11) by *iṣṭaśaṅku* divided by 12 and divide that (severally) by the divisors *svāntyā* etc. : the results (thus obtained) are the Rsines of the zenith distance for that time.

$$\text{Rsin (z. d.)} = \frac{\text{square-root} \times \textit{iṣṭaśaṅku}}{12 \times \text{divisor}},$$

where

$$\text{square root} = \sqrt{(\text{hyp. midday shadow} \times \text{multiplier})^2 - (12 \times \text{divisor})^2}$$

Rationale. We have

$$\text{Rsin (z. d.)} = \frac{\text{shadow} \times \textit{iṣṭaśaṅku}}{12}.$$

Hence using the value of shadow as obtained in vs. 11, we get the desired formula.

13. Divide the product of the Rversed-sine of the declination and 12, by the hypotenuse of the equinoctial midday shadow and add the quotient to the Rversed-sine of the latitude : then is obtained the so called *vivara*. Multiply that (*vivara*) by the *svāntyā* and divide by the radius and subtract the resulting quotient from the *svāntyā* : then is obtained the Rsine of the altitude for that time.

$$\text{Rsin } a = \textit{svāntyā} - \frac{\textit{vivara} \times \textit{svāntyā}}{\text{R}},$$

$$\text{where } \textit{vivara} = \frac{\text{Rvers } \delta \times 12}{\textit{palakarṇa}} + \text{Rvers } \phi,$$

δ being the Sun's declination and ϕ the local latitude.

Rationale. $$\text{Rsin } a = \frac{12 \times \textit{svadhṛti}}{\textit{palakarṇa}}$$

$$= \frac{12}{\textit{palakarṇa}} \times \frac{\text{Rcos } \delta \times \textit{svāntyā}}{\text{R}}$$

$$= \textit{svāntyā} - \left(\text{R} - \frac{\text{Rcos } \delta \times 12}{\textit{palakarṇa}}\right) \times \frac{\textit{svāntyā}}{\text{R}}$$

$$= svāntyā - \left[\frac{(R - R\cos\delta) \times 12}{palakarṇa} + R - R\cos\phi\right] \times \frac{svāntyā}{R}$$

$$= svāntyā - \frac{vivara \times svāntyā}{R}.$$

14. Severally multiply the *antyā* for the middle of the day (i. e., *dyudalāntyā* or simply *antyā*) and the *svāntyā* by the radius diminished by the *vivara* (i. e., by R – *vivara*), and divide (the products) each by the radius : the results are the Rsine of the meridian altitude and the Rsine of the desired altitude, (respectively).

(1) $\text{Rsin (mer. alt.)} = \frac{antyā \times (R - vivara)}{R}$

(2) $\text{Rsin (alt.)} = \frac{svāntyā \times (R - vivara)}{R}.$

Verification.

$$\frac{antyā \times (R - vivara)}{R} = \frac{antyā \times \frac{R\cos\delta \times 12}{palakarṇa}}{R} = \frac{dhṛti \times 12}{palakarṇa}$$
$$= \text{Rsin (mer. alt.)}$$

$$\frac{svāntyā \times (R - vivara)}{R} = \frac{svāntyā \times \frac{R\cos\delta \times 12}{palakarṇa}}{R} = \frac{svadhṛti \times 12}{palakarṇa}$$
$$= \text{Rsin (alt.)}.$$

15. Multiply the *antyā* diminished by the *svāntyā* by the radius diminished by the *vivara* and divide (the resulting product) by the radius : the quotient subtracted from the Rsine of the meridian altitude gives the Rsine of the desired altitude and the quotient added to the Rsine of the given altitude gives the Rsine of the meridian altitude.

(1) $\text{Rsin (alt.)} = \text{Rsin (mer. alt.)} - \frac{(R - vivara) \times (antyā - svāntyā)}{R}$

(2) $\text{Rsin (mer. alt.)} = \text{Rsin (alt.)} + \frac{(R - vivara) \times (antyā - svāntyā)}{R}.$

These results are true, because

$$\frac{(R - vivara) \times (antyā - svāntyā)}{R} = \frac{\frac{R\cos\delta \times 12}{palakarṇa}(antyā - svāntyā)}{R}.$$

$$= \frac{\text{Rcos}\,\delta \times 12 \times \textit{antyā}}{\textit{palakarṇa} \times \text{R}} - \frac{\text{Rcos}\,\delta \times 12 \times \textit{svāntyā}}{\textit{palakarṇa} \times \text{R}}$$

$$= \frac{\textit{dhṛti} \times 12}{\textit{palakarṇa}} - \frac{\textit{svadhṛti} \times 12}{\textit{palakarṇa}}$$

$$= \text{Rsin (mer. alt.)} - \text{Rsin (alt)}.$$

16. Divide the difference between the radius and the Rsine of the *unnata* (i. e., *unnatajyā*) by the *svāntyā* and multiply that by the hypotenuse of the midday shadow; the quotient added to the hypotenuse of the midday shadow gives the desired hypotenuse of shadow and the (same) quotient subtracted from the given hypotenuse of shadow gives the hypotenuse of the midday shadow.

(1) Desired hypotenuse of shadow = hypotenuse of midday shadow

$$+ \frac{(\text{R} - \textit{unnatajyā}) \times \text{hyp. midday shadow}}{\textit{svāntyā}}$$

(2) hypotenuse of midday shadow = given hypotenuse of shadow

$$- \frac{(\text{R} - \textit{unnatajyā}) \times \text{hyp. midday shadow}}{\textit{svāntyā}}.$$

These formulae are equivalent to formula (1) of vs. 10 above, because

$$\textit{antyā} - \textit{svāntyā} = \text{R} - \textit{unnatajyā},$$

where *unnatajyā* = Rsin (*unnatakāla* ± asc. diff.). See vs. 2.

17. The *unnatajyā* multiplied by 15 and divided by the semi-duration of the day (in *ghaṭīs*) gives an approximate value of the *anupātajyā*; the *svāntyā* divided by the *antyā* and multiplied by the radius gives the accurate value (of the *anupātajyā*).

(1) $\textit{anupātajyā} = \frac{15 \times \textit{unnatajyā}}{\text{semi-duration of day in } \textit{ghaṭīs}}$, approx.

(2) $\textit{anupātajyā} = \frac{\textit{svāntyā} \times \text{R}}{\textit{antyā}}$, accurately.

18. The Rsine of the meridian altitude multiplied by the *anupātajyā* and divided by the radius gives the desired Rsine of the altitude; and the hypotenuse of the midday shadow multiplied by the radius and divided by the *anupātajyā* gives the desired hypotenuse of shadow.

(1) $\text{Rsin (alt.)} = \dfrac{anup\bar{a}tajy\bar{a} \times \text{Rsin (mer. alt.)}}{\text{R}}$

(2) $\text{hypotenuse of shadow} = \dfrac{\text{R} \times \text{hyp. midday shadow}}{anup\bar{a}tajy\bar{a}}$.

Replacing *anupātajyā* by its accurate value, these results reduce to formula (2) of vs. 6 and formula (1) of vs. 9, respectively.

The next two verses relate to the case when the Sun is in the northern hemisphere and lies between the celestial horizon and the six o'clock circle.

19. In the northern hemisphere, when the Rsine is obtained from the ascensional difference minus the *unnatakāla*, the Rsine of the ascensional difference diminished by that (Rsine) gives the *svāntyā*.[1] The Rsine of the altitude, etc., in this case, too, should be determined in accordance with the methods stated heretofore.

That is, when

$unnatak\bar{a}la < \text{ascensional difference},$

then

$sv\bar{a}nty\bar{a} = \text{Rsin (asc. diff.)} - \text{Rsin (asc. diff.} - unnatak\bar{a}la).$

20. The same Rsine (i. e., the Rsine of the ascensional difference minus the *unnatakāla*) when multiplied severally by the *dhṛti*, the earthsine, and the day-radius and divided by the *antyā*, the Rsine of the ascensional difference, and the radius (respectively), and the resulting quotients severally subtracted from the earthsine, the result (in each case) is the *svadhṛti*. From that one may find out the Rsine of the altitude as before.

(1) $svadh\d{r}ti = \text{earthsine} - \dfrac{\text{Rsin (asc. diff.} - unnatak\bar{a}la) \times dh\d{r}ti}{anty\bar{a}}$

(2) $svadh\d{r}ti = \text{earthsine} - \dfrac{\text{Rsin (asc. diff.} - unnatak\bar{a}la) \times \text{earthsine}}{\text{Rsin (asc. diff.)}}$

(3) $svadh\d{r}ti = \text{earthsine} - \dfrac{\text{Rsin (asc. diff.} - unnatak\bar{a}la) \times \text{day-radius}}{\text{R}}$.

1. *Cf. BrSpSi*, iii. 33; *MBh*, iii. 25; *LBh*, iii. 11; *ŚiDVṛ*, iv. 29; *SiŚe*, iv. 41 (a-b).

It may be pointed out that

ascensional difference — *unnatakāla*

occurring in the above formulae may be replaced by

natakāla — 15 *ghaṭīs*.

Both the quantities are equal.

21-22(a-b). The hypotenuse of shadow is obtained by dividing the product of the *agrā* corresponding to the shadow circle and the Rsine of latitude by the earthsine, or by dividing the product of the *agrā* corresponding to the shadow circle and the Rsine of the colatitude by the Rsine of declination, or by dividing the product of the *agrā* corresponding to the shadow circle and the radius by the *agrā*.

(1) $$\text{Hypotenuse of shadow} = \frac{\textit{agrā} \text{ of shadow circle} \times \text{Rsin } \phi}{\text{earthsine}}$$

(2) $$\text{Hypotenuse of shadow} = \frac{\textit{agrā} \text{ of shadow circle} \times \text{Rcos } \phi}{\text{Rsin } \delta}$$

(3) $$\text{Hypotenuse of shadow} = \frac{\textit{agrā} \text{ of shadow circle} \times \text{R}}{\textit{agrā}}.$$

Formulae (1), (2), (3) are equivalent, because

(4) $$\frac{\text{Rsin } \phi}{\text{earthsine}} = \frac{\text{Rcos } \phi}{\text{Rsin } \delta} = \frac{\text{R}}{\textit{agrā}}.$$

The right hand side of each of the above three formulae reduces to hypotenuse of shadow by substituting

$$\textit{agrā} \text{ of shadow circle} = \frac{\textit{agrā} \times \text{hyp. of shadow}}{\text{R}}$$

and using (4).

22(c-d). Or, by dividing the product of the radius and the *aṅgulis* of the *bhuja* of the shadow circle by the *bhuja* (of the R-circle).

$$\text{Hypotenuse of shadow} = \frac{\textit{bhuja} \text{ of shadow circle} \times \text{R}}{\textit{bhuja} \text{ of R-circle}}.$$

23. The product of the *agrā* corresponding to the shadow circle and the Rsine of the zenith distance divided by the *agrā* is the shadow.

$$\text{Shadow} = \frac{\textit{agrā} \text{ of shadow circle} \times \text{Rsin (z. d.)}}{\textit{agrā}}.$$

This is true, because

$$\frac{\textit{agrā} \text{ of shadow-circle}}{\textit{agrā}} = \frac{\text{hypotenuse of shadow}}{\text{R}} = \frac{\text{shadow}}{\text{Rsin (z. d.)}}.$$

24. The radius severally multiplied by 12 and the shadow and divided by the hypotenuse of shadow yields the Rsine of the altitude and the Rsine of the zenith distance, (respectively).[1]

The Rsine of the zenith distance multiplied by 12 and divided by the shadow also gives the Rsine of the altitude.

(1) $$\text{Rsin (alt.)} = \frac{\text{R} \times 12}{\text{hyp. of shadow}}$$

(2) $$\text{Rsin (z. d.)} = \frac{\text{R} \times \text{shadow}}{\text{hyp. of shadow}}$$

(3) $$\text{Rsin (alt.)} = \frac{\text{Rsin (z. d.)} \times 12}{\text{shadow}}.$$

DHṚTI, UNNATA AND *UNNATAKĀLA*

25. The Rsine of the altitude severally multiplied by the *taddhṛti*, the *agrā*, the radius and the hypotenuse of the equinoctial midday shadow and divided by the Rsine of the prime vertical altitude, the Rsine of the declination, the Rsine of the colatitude and 12 (respectively), in each case, yields the *dhṛti*.

(1) $$\textit{dhṛti} = \frac{\text{Rsin (alt.)} \times \textit{taddhṛti}}{\text{Rsin (prime vertical alt.)}}$$

(2) $$\textit{dhṛti} = \frac{\text{Rsin (alt.)} \times \textit{agrā}}{\text{Rsin } \delta}$$

(3) $$\textit{dhṛti} = \frac{\text{Rsin (alt.)} \times \text{R}}{\text{Rcos } \phi}$$

(4) $$\textit{dhṛti} = \frac{\text{Rsin (alt.)} \times \textit{palakarṇa}}{12}.$$

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1. See *MSi*, iv. 16 where hypotenuse of shadow and shadow have been obtained by using these formulae.

26(a-b). (The *dhṛti* is obtained also) by multiplying the *śaṅkutala* severally by the abovementioned multipliers and dividing (the products) by the *agrā*, the earthsine, the Rsine of latitude and the equinoctial midday shadow (respectively).

(1) $dhṛti = \dfrac{śaṅkutala \times taddhṛti}{agrā}$

(2) $dhṛti = \dfrac{śaṅkutala \times agrā}{\text{earthsine}}$

(3) $dhṛti = \dfrac{śaṅkutala \times \text{R}}{\text{Rsin } \phi}$

(4) $dhṛti = \dfrac{śaṅkutala \times palakarṇa}{palabhā}$.

26(c-d)-27. The *svadhṛti* diminished or increased by the earthsine, according as the Sun is in the northern or southern hemisphere, is the "multiplicand". This "multiplicand" being severally multiplied by the radius and the Rsine of the ascensional difference and (the resulting products) divided by the day-radius and the earthsine respectively, the arcs corresponding to the Rsines obtained being diminished or increased by the *asus* of the ascensional difference (according as the Sun is in the southern or northern hemisphere), the result (obtained in each case) is the *unnatakāla* (in *asus*).

Let "multiplicand" stand for

$$svadhṛti \mp \text{earthsine},$$

where − or + sign is taken according as the Sun is in the northern or southern hemisphere. Then

(1) $unnatāsu = \text{arc} \left[\dfrac{\text{"multiplicand"} \times \text{R}}{\text{day-radius}}\right] \pm \text{asc. diff. in } asus$

(2) $unnatāsu = \text{arc} \left[\dfrac{\text{"multiplicand"} \times \text{Rsin (asc. diff.)}}{\text{earthsine}}\right] \pm \text{asc. diff. in } asus,$

+ or − sign being taken according as the Sun is in the northern or southern hemisphere.

The above formulae are based on the formula :

unnatāsu (i.e., *unnatakālāsu*) = arc (*unnatajyā*)

$\pm$ asc. diff. in *asus*,

+ or − sign being taken according as the Sun is in the northern or southern hemisphere. See *supra*, vs. 2.

Formulae similar to (1) have been given by Brahmagupta, Lalla and Śrīpati. See *BrSpSi*, iii. 38-39, 41-42; *ŚiDVr*, iv. 31, 32; *SiŚe*, iv. 51-52 (a-b), 53-54(a-b).

28. The *antyā* when multiplied by the hypotenuse of the midday shadow and divided by the given hypotenuse of shadow, and the arc corresponding to (the Rsine equal to) the quotient obtained (*phala*) diminished or increased by the ascensional difference, according as the Sun is in the northern or southern hemisphere, the result is the *unnata*.[1]

$$Unnata = \text{arc}\left[\frac{antyā \times \text{hyp. midday shadow}}{\text{hyp. of shadow}}\right] \mp \text{asc. diff.},$$

i.e., *svāntyā-cāpa* $\mp$ asc. diff.,

− or + sign being taken according as the Sun is in the northern or southern hemisphere.

Proof We have shown (see notes on vs. 9) that

$$\frac{\text{hyp. midday shadow}}{\text{hyp. of shadow}} = \frac{\text{Rsin (alt.)}}{\text{Rsin (mer. alt.)}} = \frac{svāntyā}{antyā}.$$

Hence the above rule.

Unnata is the arc corresponding to *unnatajyā* or *unnatajīvā*, See vs. 2 above.

29. The arc corresponding to the Rversed-sine equal to *antyā* minus the *phala* (of vs. 28) (i.e., *svāntyā*), gives the *natakāla* as measured from midday. When the *antyā* minus the *phala* exceeds the radius, the arc corresponding to (the Rsine equal to) the excess when increased by the minutes corresponding to 3 signs gives the *asus* of the *natakāla*.[2]

1 *Cf. SiŚe*, iv. 56.

2. *Cf. MSi*, iv. 33-34; *SiŚi*, I, iii. 68. Also see *BrSpSi*, iii. 44; *KK*, I, iii. 15; *ŚiDVr*, iv. 33; *SiŚe*, iv 55; *MSi*, iv. 32.

That is : When *antyā* $-$ *svāntyā* $<$ R,

$$\text{Rvers } (natakāla) = antyā - svāntyā;$$

and when *antyā* $-$ *svāntyā* $>$ R,

$$natakāla = 90° + \theta \text{ mins.}$$

$$= 5400 + \theta \; asus,$$

where $\text{Rsin } \theta = (antyā - svāntyā) - \text{R}$.

30. Multiply the *antyā* by the difference between the given hypotenuse of shadow and the hypotenuse of midday shadow and divide that by the given hypotenuse of shadow; and subtract the quotient from the *antyā*. The arc corresponding to (the Rsine equal to) this (difference) increased or diminished by the ascensional difference, as before (see vs. 28), gives the *unnata*.

$$Unnata = \text{arc}\left[antyā - \frac{(\text{hyp. of shadow} - \text{hyp. midday shadow})\; antyā}{\text{hyp. of shadow}} \right] \pm \text{asc. diff.},$$

$+$ or $-$ sign being taken according as the Sun is in the southern or northern hemisphere.

This formula is equivalent to that of vs. 28 above.

31. The Rsine of the Sun's altitude being multiplied by the radius and divided by the Rsine of the altitude of the meridian-ecliptic point, thereafter the arc of (the Rsine equal to) the resulting quotient being multiplied by half the duration of day in terms of *ghaṭīs* and divided by 15, the result is the *unnata-kāla* (in terms of *asus*).

$$Unnatakāla = \frac{(udayalagna - \text{Sun}) \text{ in mins.} \times \dfrac{\text{duration of day in } ghaṭīs}{2}}{15} \; asus,$$

where

$$\text{Rsin } (udayalagna - \text{Sun}) = \frac{\text{Rsin (Sun's alt.)} \times \text{R}}{\text{Rsin (alt. of meridian-ecliptic point)}}.$$

The proportion used is : When (*udayalagna* $-$ Sun) equals 5400 minutes, the *unnatakāla* amounts to

(half the duration of day in *ghaṭīs*) $\times$ 360

asus, how many *asus* will the *unnatakāla* amount to when (*udayalagna* $-$

Sun) has the given value in minutes ? The result is as given by the above formula. (1 *ghaṭī* = 360 *asus*)

This rule is evidently approximate.

32. Severally multiply the (*sva*)*dhṛti* by the radius and the Rsine of the ascensional difference and divide (the products so obtained) by the day-radius and the earthsine (respectively): the result (in each case) is the (*sva*) *antyā*. Diminish or increase that (*svāntyā*) by the Rsine of the ascensional difference (according as the Sun is in the northern or southern hemisphere) : the arc corresponding to (the Rsine equal to) the resulting (difference or sum) is the *unnata*, as before.

(1) $svānty\bar{a} = \dfrac{svadhṛti \times R}{\text{day-radius}}$

(2) $svānty\bar{a} = \dfrac{svadhṛti \times \text{Rsin (asc. diff.)}}{\text{earthsine}}$

(3) $unnata = \text{arc}\,[svānty\bar{a} \mp \text{Rsin (asc. diff.)}]$,

$-$ or $+$ sign being taken according as the Sun is in the northern or southern hemisphere.

33. When the earthsine (being greater than *svadhṛti*) cannot be subtracted from the (*sva*)*dhṛti*, then multiply the difference of the two by the abovementioned multiplier and divide by the corresponding divisor; and by the arc of (the Rsine equal to) the quotient (so obtained) diminish the ascensional difference : the result is the *unnata* (*kāla*), (in terms of *asus*).

That is : When the Sun (being in the northern hemisphere) is above the horizon but below the six o'clock circle, then

$$unnatakāla = \text{asc. diff.} - \text{arc}\left[\frac{(\text{earthsine} - svadhṛti) \times M}{D}\right],$$

where $\dfrac{M}{D} = \dfrac{R}{\text{day-radius}} = \dfrac{\text{Rsin (asc. diff.)}}{\text{earthsine}}$.

This rule is a sequel to the rule given in vss. 26(c-d)-27.

34. When the Rsine of the ascensional difference (being greater than the *svāntyā*) cannot be subtracted from the (*sva*) *antyā*, then the ascensional difference diminished by the arc corresponding to (the Rsine

equal to) the difference of these two gives the *unnatakāla*, (in terms of *asus*), i.e., the time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon.

That is : When the Sun (being in the northern hemisphere) lies between the horizon and the six o'clock circle,

unnatakāla = asc. diff. — arc [Rsin (asc. diff.) — *svāntyā*].

RULES CONCERNING MULTIPLICAND, MULTIPLIER AND DIVISOR

35. When the multiplier is greater than the divisor, one may multiply the multiplicand by the difference between the multiplier and the divisor and divide by the divisor and then add the resulting quotient (to the multiplicand); and when the multiplier is smaller than the divisor, that quotient should be subtracted (from the multiplicand).

That is : When multiplier > divisor, then

$$\frac{\text{multiplicand} \times \text{multiplier}}{\text{divisor}}$$
$$= \text{multiplicand} + \frac{\text{multiplicand} \times (\text{multiplier} - \text{divisor})}{\text{divisor}};$$

and when multiplier < divisor, then

$$\frac{\text{multiplicand} \times \text{multiplier}}{\text{divisor}}$$
$$= \text{multiplicand} - \frac{\text{multiplicand} \times (\text{divisor} - \text{multiplier})}{\text{divisor}}.$$

36. Or, one may subtract the multiplicand from whatever is obtained by multiplying the multiplicand by the sum of the divisor and the multiplier and dividing that by the divisor : even then the final result is alway true.

That is :

$$\frac{\text{multiplicand} \times \text{multiplier}}{\text{divisor}}$$
$$= \frac{\text{multiplicand} \times (\text{divisor} + \text{multiplier})}{\text{divisor}} - \text{multiplicand}.$$

37. (When there are two, three or more multipliers and divisors) the product of (those) two, three or more multipliers is the resultant multiplier (*antya guṇaka*) and similarly the product of the divisors is the (resultant) divisor.

The product of the squares of the multiplier and the multiplicand divided by the square of the divisor, when reduced to its square-root, (gives the same result as the product of the multiplier and the multiplicand divided by the divisor).

That is :

(1) $$\frac{a \times a_1 \times a_2 \times \ldots\ldots \times a_n}{b_1 \times b_2 \times \ldots\ldots \times b_n} = \frac{a \times M}{D},$$

where $M = a_1 \times a_2 \times \ldots\ldots \times a_n$, and $D = b_1 \times b_2 \times \ldots\ldots \times b_n$.

(2) $$\frac{a \times M}{D} = \sqrt{\frac{a^2 \times M^2}{D^2}}.$$

38. The product of the multiplier and the multiplicand when divided by the quotient gives the divisor and when divided by the divisor gives the quotient. The divisor multiplied by the quotient gives the product ; the product when divided by the multiplier gives the multiplicand and when divided by the multiplicand gives the multiplier.[1]

That is : If

$$\frac{\text{multiplicand} \times \text{multiplier}}{\text{divisor}} = \text{quotient},$$

then

(1) $$\frac{\text{multiplicand} \times \text{multiplier}}{\text{quotient}} = \text{divisor}$$

(2) $$\frac{\text{multiplicand} \times \text{multipler}}{\text{divisor}} = \text{quotient}$$

(3) $$\frac{\text{divisor} \times \text{quotient}}{\text{multiplier}} = \text{multiplicand}$$

(4) $$\frac{\text{divisor} \times \text{quotient}}{\text{multiplicand}} = \text{multiplier}.$$

1. *Cf. BrSpSi*, xii. 58.

METHOD OF INVERSION

39. In the method of inversion, starting from the end one should make the multiplier divisor, the divisor multiplier, the additive subtractive, the subtractive additive, the square-root square, and the square square-root.

This rule is found to occur in all the works on Hindu mathematics.

40(a-b). One should know that the multiplier and the multiplicand are like the multiplicand and the multiplier.

That is :

multiplicand × multiplier = multiplier × multiplicand.

CONCLUSION

40(c-d). This is how the shadow can be obtained in a variety of ways (from the given time) and conversely the time (from the given shadow).

Section 11

Sun on the Prime Vertical

SUN'S ALTITUDE, WHEN SUN'S *DIGJYĀ* > SUN'S *AGRĀ*

1-4(a-b). The Rsine of the arc of the horizon which lies between the prime vertical and the (Sun's) vertical circle is called *Digjyā* (i.e., Rsine of the amplitude). Diminish the square of that by the square of the *agrā*, multiply that by 144 and again by the square of the radius : this gives the "first result". Next multiply the product of *agrā*, 12 and *palabhā* by the square of the radius : this gives the "second result". When these "first" and "second" results are divided by the sum of the square of the product of *digjyā* and 12 and the square of the product of *palabhā* and the radius, they become true. Add the square of the "(true) second result" to the "(true) first result" and take the square-root. Increase or diminish the square-root by the "(true) second result" according as the Sun is in the northern or southern hemisphere. The result is the Rsine of the Sun's altitude, provided the Sun is towards the south of the prime vertical.

That is : If

$$\text{first result} = [(\textit{digjyā})^2 - (\textit{agrā})^2] \times 144 \times R^2$$

$$\text{second result} = \textit{agrā} \times 12 \times \textit{palabhā} \times R^2$$

$$\text{true first result} = \frac{\text{first result}}{D}$$

$$\text{true second result} = \frac{\text{second result}}{D},$$

$$\text{where } D = (\textit{digjyā} \times 12)^2 + (\textit{palabhā} \times R)^2,$$

then

$$\text{Rsin (Sun's alt.)} = \sqrt{(\text{true first result}) + (\text{true second result})^2} \pm \text{true second result},$$

+ or – sign being taken according as the Sun is in the northern or southern hemisphere.

Rationale. Let a be the Sun's altitude and z its zenith distance. Then in the present case (when Sun's *digjyā* > Sun's *agrā*), we have

$$bhuja = \acute{s}a\dot{n}kutala \mp agr\bar{a}$$

$$= \frac{palabh\bar{a} \times \text{Rsin}\, a}{12} \mp agr\bar{a}, \qquad \text{(i)}$$

− or + sign being taken according as the Sun is in the northern or southern hemisphere.

$$\text{Also} \quad bhuja = \frac{digjy\bar{a} \times \text{Rsin}\, z}{\text{R}}$$

$$= \frac{digjy\bar{a} \times \sqrt{\text{R}^2 - (\text{Rsin}\, a)^2}}{\text{R}}. \qquad \text{(ii)}$$

Squaring (i) and (ii) and equating, we have

$$\frac{(palabh\bar{a})^2 \times (\text{Rsin}\, a)^2}{144} \mp \frac{2\; palabh\bar{a} \times agr\bar{a} \times \text{Rsin}\, a}{12} + (agr\bar{a})^2$$

$$= (digjy\bar{a})^2 - \frac{(digjy\bar{a})^2 \times (\text{Rsin}\, a)^2}{\text{R}^2}$$

$$\text{or} \left[\frac{(digjy\bar{a})^2}{\text{R}^2} + \frac{(palabh\bar{a})^2}{144}\right](\text{Rsin}\, a)^2 \mp \frac{2.\, palabh\bar{a} \times agr\bar{a} \times \text{Rsin}\, a}{12}$$

$$- [\,(digjy\bar{a})^2 - (agr\bar{a})^2\,] = 0$$

or $[\,(digjy\bar{a} \times 12)^2 + (palabh\bar{a} \times \text{R})^2]\ (\text{Rsin}\, a)^2 \mp 2\, agr\bar{a} \times 12 \times palabh\bar{a} \times \text{R}^2 \times \text{Rsin}\, a - [\,(digjy\bar{a})^2 - (agr\bar{a})^2].\ 144\ \text{R}^2 = 0$

or $D.\ (\text{Rsin}\, a)^2 \mp 2.\ (\text{second result}).\ \text{Rsin}\, a - (\text{first result}) = 0$

or $(\text{Rsin}\, a)^2 \mp 2.(\text{true second result})\ \text{Rsin}\, a - (\text{true first result}) = 0$

$$\therefore \quad \text{Rsin}\, a = \sqrt{(\text{true first result}) + (\text{true second result})^2} \pm \text{true second result},$$

+ or − sign being taken according as the Sun is in the northern or southern hemisphere.

− sign before the radical has been omitted because Rsin a is always positive.

The above rule is applicable when the Sun is in the northern or southern hemisphere and its *digjyā* is greater than *agrā*. The Sun is then necessarily towards the south of the prime vertical.

SUN'S ALTITUDE, WHEN SUN'S *DIGJYĀ* < *AGRĀ*

4(c-d)-6. When the Sun is in the northern hemisphere but the Sun lies towards the north of the prime vertical, the "true second result" should not be added to the square-root, (as taught above). (In this case as also) when the Sun is towards the south of the prime vertical as far as the *agrā* such that the Sun's *digjyā* is less than the *agrā*, one should subtract the square of the *digjyā* from the square of the *agrā* and find out the "(true) first" and "(true) second" results, as before, and then find out the square-root of the square of the "(true) second result" minus the "(true) first result". Whatever square-root is thus obtained should be subtracted from or added to the "(true) second result". Thus is obtained the Rsine of the Sun's altitude (in the two cases), respectively.

That is : When the Sun is in the northern hemisphere and its *digjyā* is less than *agrā*, then

$$\text{Rsin } a = \text{true second result} \mp \sqrt{(\text{true second result})^2 - (\text{true first result})},$$

according as the Sun is towards the north or south of the prime vertical.

In this case,

$$\text{true first result} = \frac{[(agr\bar{a})^2 - (digjy\bar{a})^2] \times 144 \times R^2}{D},$$

the true second result being the same as before.

ALTITUDE OF THE UPPER LIMB

7. The Rsine of the arc of altitude plus the true semi-diameter (i.e., angular semi-diameter), diminished by one-fifteenth part of the mean daily motion of the heavenly body, in terms of minutes, gives the Rsine of the true altitude (of the upper limb of the heavenly body), (above the visible horizon).[1]

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1. *Cf. BrSpSi*, viii. 6; *SiŚe*, x. 32.

That is : If a be the altitude of the centre of a heavenly body and r the angular semi-diameter of the disc of the heavenly body, and m minutes of arc the mean daily motion of the heavenly body, then the Rsine of the altitude of the heavenly body above the visible horizon is equal to :

$$= \text{Rsin}\,(a + r) - \frac{m}{15}.$$

SAMAŚAṄKU OR RSINE OF SUN'S PRIME VERTICAL ALTITUDE

8. From the product of the Rsine of the Sun's *bhuja* and the Rsine of 24°, divided by the Rsine of the (local) latitude, is obtained the Rsine of the Sun's prime vertical altitude (*samaśaṅku*).[1] This exists when the Sun is in the northern hemisphere and the Sun's declination is less than the local latitude.[2]

That is : If λ be the Sun's *bhuja* (tropical), a_p the Sun's prime vertical altitude and ϕ the latitude of the local place, then

$$\text{Rsin}\,a_p \text{ (i.e., } samaśaṅku) = \frac{\text{Rsin}\,\lambda \times \text{Rsin}\,24°}{\text{Rsin}\,\phi}. \quad (1)$$

The Sun's prime vertical altitude exists only when the Sun is in the northern hemisphere and the Sun's declination is less than the latitude of the place. When this condition is not satisfied it does not exist, because then the Sun does not cross the prime vertical.

9. The Rsine of the declination multiplied by the radius and divided by the Rsine of the (local) latitude gives the Rsine of the prime vertical altitude.[3]

The Rsine of the declination multiplied by the hypotenuse of the equinoctial midday shadow and divided by the equinoctial midday shadow also gives the Rsine of the prime vertical altitude.[4]

$$\text{Rsin}\,a_p = \frac{\text{Rsin}\,\delta \times \text{R}}{\text{Rsin}\,\phi} \quad (2)$$

1. *Cf. BrSpSi*, iii. 52 (a-b); *SiŚe*, iv. 58.
2. *Cf. BrSpSi*, iii. 51; *ŚiDVr*, iv. 36 (a-b); *SiŚe*, iv. 57(d).
3. *Cf. SiŚe*, iv. 59.
4. *Cf. BrSpSi*, iii. 51 (a-b); *ŚiDVr*, iv. 6; *SiŚe*, iv. 57 (a-b). Also see *SiŚi*, I, iii. 20.

$$\text{Rsin } a_p = \frac{\text{Rsin } \delta \times \textit{palakarṇa}}{\textit{palabhā}}, \tag{3}$$

where δ is the Sun's declination.

10. Or, the product of the *agrā* and the Rsine of the declination, divided by the earthsine, gives the Rsine of the prime vertical altitude.

Or, the Rsine of the declination multiplied by the *svadhṛti* and divided by the *śaṅkutala* gives the Rsine of the prime vertical altitude.[1]

$$\text{Rsin } a_p = \frac{\text{Rsin } \delta \times \textit{agrā}}{\text{earthsine}} \tag{4}$$

$$\text{Rsin } a_p = \frac{\text{Rsin } \delta \times \textit{svadhṛti}}{\textit{śaṅkutala}}. \tag{5}$$

11. Or, the Rsine of the prime vertical altitude may also be obtained by dividing the product of the Rsine of colatitude and the *agrā* by the Rsine of latitude.

Or, the *agrā* multiplied by 12 and divided by the equinoctial midday shadow gives the Rsine of the prime vertical altitude.[2]

$$\text{Rsin } a_p = \frac{\textit{agrā} \times \text{Rcos } \phi}{\text{Rsin } \phi} \tag{6}$$

$$\text{Rsin } a_p = \frac{\textit{agrā} \times 12}{\textit{palabhā}}. \tag{7}$$

12. Or, the *agrā* multiplied by the Rsine of the desired altitude and divided by the *śaṅkutala* is the Rsine of the prime vertical altitude.

Or, the Rsine of the prime vertical altitude is equal to the square-root of the difference between the squares of *taddhṛti* and *agrā*.[3]

$$\text{Rsin } a_p = \frac{\textit{agrā} \times \text{Rsin } a}{\textit{śaṅkutala}} \tag{8}$$

$$\text{Rsin } a_p = \sqrt{[(\textit{taddhṛti})^2 - (\textit{agrā})^2]}, \tag{9}$$

where a is the desired altitude.

1. See *SiŚi*, I, iii. 22 (a-b).
2. *Cf. BrSpSi*, iii. 52 (c-d).
3. *Cf. SiŚe*, iv. 60 (a-b).

SAMAKARṆA OR HYPOTENUSE OF THE PRIME VERTICAL SHADOW

13. The Rsine of latitude multiplied by 12 and divided by the Rsine of declination gives the hypotenuse of the prime vertical shadow (i.e., the hypotenuse of shadow when the Sun is on the prime vertical).[1]

The Rsine of colatitude multiplied by the equinoctial midday shadow and divided by the Rsine of declination too gives the hypotenuse of the prime vertical shadow.[2]

$$Samakarṇa = \frac{\text{Rsin}\,\phi \times 12}{\text{Rsin}\,\delta} \qquad \text{(i)}$$

$$Samakarṇa = \frac{\text{Rcos}\,\phi \times palabh\bar{a}}{\text{Rsin}\,\delta}. \qquad \text{(ii)}$$

Rationale. *Samakarṇa* $= \frac{\text{R} \times 12}{\text{Rsin}\,a_{\text{p}}}$, and $\text{Rsin}\,a_{\text{p}} = \frac{\text{Rsin}\,\delta \times \text{R}}{\text{Rsin}\,\phi}$. This gives (i). (ii) is obviously equivalent to (i).

14. The radius multiplied by the equinoctial midday shadow and divided by the *agrā* is the hypotenuse of the prime vertical shadow.

The hypotenuse of the prime vertical shadow is also obtained on multiplying the radius by the hypotenuse of the equinoctial midday shadow and dividing by the *taddhṛti*.[3]

$$Samakarṇa = \frac{\text{R} \times palabh\bar{a}}{agr\bar{a}} \qquad \text{(iii)}$$

$$Samakarṇa = \frac{\text{R} \times palakarṇa}{taddhṛti}. \qquad \text{(iv)}$$

These are true, because *samakarṇa*/R, *palabhā*/*agrā*, and *palakarṇa*/*taddhṛti* are each equal to 12/*samaśaṅku*.

15. The product of the radius and the square of the equinoctial midday shadow divided by the product of the hypotenuse of the equinoctial midday shadow and the earthsine gives the hypotenuse of the prime vertical shadow.

1. *Cf. BrSpSi*, iii. 53; *SiŚe*, iv. 61.
2. *Cf. BrSpSi*, iii. 53; *SiŚe*, iv. 61.
3. *Cf. MSi.* iv. 17.

Or, the Rsine of latitude multiplied by the equinoctial midday shadow and divided by the earthsine gives the hypotenuse of the prime vertical shadow.

$$Samakarṇa = \frac{R \times (palabh\bar{a})^2}{palakarṇa \times \text{earthsine}} \qquad \text{(v)}$$

$$Samakarṇa = \frac{\text{Rsin}\,\phi \times palabh\bar{a}}{\text{earthsine}}. \qquad \text{(vi)}$$

SUN'S PRIME VERTICAL ALTITUDE (Continued)

16. The product of the hypotenuse of the equinoctial midday shadow, 12 and the earthsine divided by the square of the equinoctial midday shadow is the Rsine of the prime vertical altitude.

Or, the product of the Rsine of colatitude, the radius and the earthsine divided by the square of the Rsine of latitude is the Rsine of the prime vertical altitude.

$$\text{Rsin}\,a_p = \frac{palakarṇa \times 12 \times \text{earthsine}}{(palabh\bar{a})^2} \qquad (10)$$

$$\text{Rsin}\,a_p = \frac{\text{Rcos}\,\phi \times R \times \text{earthsine}}{(\text{Rsin}\,\phi)^2}. \qquad (11)$$

Proof.

$$\text{Rsin}\,a_p = \frac{R \times \text{Rsin}\,\delta}{\text{Rsin}\,\phi}, \text{ from vs. 9 above}$$

$$= \frac{R}{\text{Rsin}\,\phi} \cdot \frac{\text{Rcos}\,\phi \times \text{earthsine}}{\text{Rsin}\,\phi}, \text{ from sec. 3, vs. 4}$$

$$= \frac{R}{\text{Rsin}\,\phi} \cdot \frac{\text{Rcos}\,\phi}{\text{Rsin}\,\phi} \cdot \text{earthsine} \qquad \text{(i)}$$

$$= \frac{palakarṇa}{palabh\bar{a}} \cdot \frac{12}{palabh\bar{a}} \cdot \text{earthsine}. \qquad \text{(ii)}$$

(ii) gives formula (10) and (i) gives formula (11).

17. Or, the product of the Rsine of colatitude and the hypotenuse of the equinoctial midday shadow, divided by the same hypotenuse (i.e., hypotenuse of the prime vertical shadow), is (the Rsine of the prime vertical altitude).

$$\text{Rsin } a_p = \frac{\text{Rcos } \phi \times palakarṇa}{samakarṇa}. \tag{12}$$

This is true, because

$$\frac{\text{Rsin } a_p}{12} = \frac{\text{R}}{samakarṇa} \text{ and } \frac{palakarṇa}{\text{R}} = \frac{12}{\text{Rcos } \phi}.$$

18. The product of the radius and the Rsine of the desired declination divided by the Rsine of latitude is the Rsine of the prime vertical altitude. It is also equal to the product of the hypotenuse of the equinoctial midday shadow, the Rsine of the desired altitude and the earthsine, divided by the *śaṅkutala* multiplied by the *palabhā*.

$$\text{Rsin } a_p = \frac{\text{R} \times \text{Rsin } \delta}{\text{Rsin } \phi} \tag{13}$$

$$\text{Rsin } a_p = \frac{palakarṇa \times \text{Rsin } a \times \text{earthsine}}{palabhā \times śaṅkutala}, \tag{14}$$

where a is the desired altitude,

Formula (14) is equivalent to formula (10), because

$$\frac{12}{palabhā} = \frac{\text{Rsin } a}{śaṅkutala}.$$

19. Or, the product of the Rsine of the (desired) altitude, the *dhṛti* and the earthsine divided by the square of the *śaṅkutala* is the Rsine of the prime vertical altitude.

Or, the product of *dhṛti*, earthsine and 12 divided by the product of *palabhā* and *śaṅkutala* is the Rsine of the prime vertical altitude.

$$\text{Rsin } a_p = \frac{\text{Rsin } a \times dhṛti \times \text{earthsine}}{(śaṅkutala)^2} \tag{15}$$

$$\text{Rsin } a_p = \frac{dhṛti \times \text{earthsine} \times 12}{palabhā \times śaṅkutala}, \tag{16}$$

where a is the desired altitude.

Formula (15) is equivalent to formula (14), because

$$\frac{palakarṇa}{palabhā} = \frac{dhṛti}{śaṅkutala},$$

and formula (16) is equivalent to formula (15), because

$$\frac{\text{Rsin } a}{\acute{s}a\dot{n}kutala} = \frac{12}{palabh\bar{a}}.$$

20. The product of the Rsine of colatitude, the earthsine and the *dhṛti* divided by the product of the Rsine of latitude and the *śaṅkutala* is the Rsine of the prime vertical altitude.

The Rsine of the prime vertical altitude is also equal to the square-root of the product of the sum and difference of *taddhṛti* and *agrā*.[1]

$$\text{Rsin } a_p = \frac{\text{Rcos } \phi \times \text{earthsine} \times dh\underset{\cdot}{r}ti}{\text{Rsin } \phi \times \acute{s}a\dot{n}kutala} \tag{17}$$

$$\text{Rsin } a_p = \sqrt{(taddh\underset{\cdot}{r}ti + agr\bar{a})\,(taddh\underset{\cdot}{r}ti - agr\bar{a})}. \tag{18}$$

Formula (17) is equivalent to formula (16), and formula (18) equivalent to formula (9).

21. The product of the day-radius and the Rsine of the ascensional difference (*carajyā*), multiplied by the Rsine of colatitude and divided by the square of the Rsine of latitude, is the Rsine of the prime vertical altitude.

The product (of the day-radius and the Rsine of the ascensional difference) multiplied by 12 and divided by the Rsine of latitude as multiplied by the *palabhā*, too, gives the Rsine of the prime vertical altitude.

$$\text{Rsin } a_p = \frac{(\text{day-radius} \times caraj y\bar{a}) \times \text{Rcos } \phi}{(\text{Rsin } \phi)} \tag{19}$$

$$\text{Rsin } a_p = \frac{(\text{day-radius} \times caraj y\bar{a}) \times 12}{\text{Rsin } \phi \times palabh\bar{a}}. \tag{20}$$

Formula (19) may be derived from formula (6) above by substituting

$$agr\bar{a} = \frac{\text{day-radius} \times caraj y\bar{a}}{\text{Rsin } \phi}. \text{ [See formula (24) of sec. 6]}$$

Formula (20) is obviously equivalent to formula (19).

---

1. *Cf.* this second rule with *ŚiDVṛ*, iv. 6 (d).

22-23. The product (of the day-radius and the Rsine of the ascensional difference) multiplied by the Rsine of the declination and divided by the product of the earthsine and the Rsine of latitude (too) gives the Rsine of the prime vertical altitude.

Alternatively, one may obtain the Rsine of the prime vertical altitude from the product of radius, *taddhṛti* and *carajyā* (i.e., Rsine of the ascensional difference) as multiplied by the Rsine of declination. But in this case the division is to be performed by the previous divisor as multiplied by the *antyā*.

$$\text{Rsin } a_p = \frac{(\text{day-radius} \times carajy\bar{a}) \times \text{Rsin } \delta}{\text{Rsin } \phi \times \text{earthsine}} \tag{21}$$

$$\text{Rsin } a_p = \frac{(\text{R} \times taddh\d{r}ti \times carajy\bar{a}) \times \text{Rsin } \delta}{anty\bar{a} \times \text{Rsin } \phi \times \text{earthsine}}. \tag{22}$$

These formulae are obviously equivalent to the previous ones.

LOCUS OF THE SUN'S *ŚAṄKU*

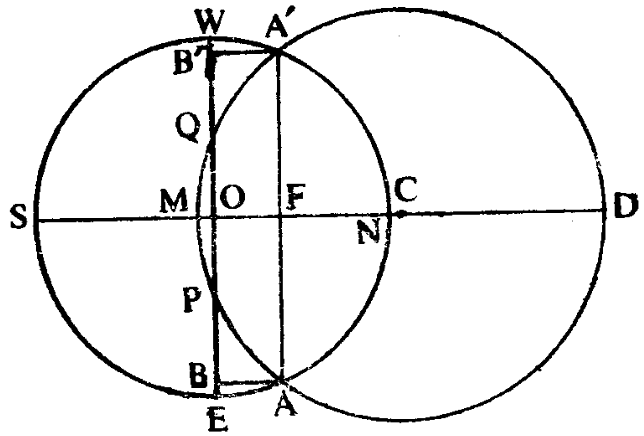
24. Divide the product of the sum and difference of the radius and the (Sun's) *agrā* by the Sun's midday *śaṅkutala* and increase what is thus obtained by the same *śaṅkutala*: this gives the diameter of the circle described by the motion of the Sun's *śaṅku* (i.e., by the Rsine of the Sun's altitude).

That is: Diameter of the circle described by the motion of the Sun's *śaṅku*

$$= \frac{(\text{R} + agr\bar{a})\,(\text{R} - agr\bar{a})}{\text{midday } \acute{s}a\dot{n}kutala} + \text{midday } \acute{s}a\dot{n}kutala.$$

Rationale. In the figure below, the circle ENWS centred at O is the horizon; E, W, N and S are the east, west, north and south cardinal points. A is the point where the Sun rises, A′ the point where the Sun sets, and M the foot of the perpendicular dropped on the plane of the horizon from the Sun at midday. Then AB, the distance of A from the east-west line EW, is the Sun's *agrā*; MO ($=z_m$), the Rsine of the Sun's zenith distance at midday; and MF, the distance of M from the rising-setting line AA′, the Sun's *śaṅkutala* at midday. Evidently, MF = MO+OF = MO+BA = $z_m + agr\bar{a}$.

C is the centre of the circle passing through A, M and A′. This circle has been supposed to be the locus of the Sun's *śaṅku*. Let OC = x. Then



$$MC^2 = AC^2$$

$$\text{or} \quad (MO + OC)^2 = FA^2 + FC^2$$

$$\text{or} \quad (z_m + x)^2 = R^2 - (agr\bar{a})^2 + (x - agr\bar{a})^2, \qquad \text{(i)}$$

where R is the radius of the circle ENWS.

Solving (i) for x, we get

$$2x = \frac{R^2 - (agr\bar{a})^2}{z_m + agr\bar{a}} + agr\bar{a} - z_m$$

$$\therefore\ 2(x + z_m) = \frac{R^2 - (agr\bar{a})^2}{z_m + agr\bar{a}} + z_m + agr\bar{a}$$

$$= \frac{(R + agr\bar{a})(R - agr\bar{a})}{\text{midday } \acute{s}a\dot{n}kutala} + \text{midday } \acute{s}a\dot{n}kutala,$$

because $z_m + agr\bar{a}$ = midday *śaṅkutala*.

This gives the diameter of the circle centred at C, i.e., the diameter of the circle described by the motion of the Sun's *śaṅku* (i.e., Rsine of the Sun's altitude).

LOCUS OF THE GNOMONIC SHADOW

25-26. Find the sum or difference of the instantaneous *bhuja* and the midday shadow according as they are of like or unlike directions. Then subtract the square of the *bhuja* from the square of the (instantaneous) shadow and increase that by the square of that (sum or difference); and then divide whatever is obtained by twice that (sum or) difference. Thus is obtained the radius of the circle described by the (tip of the gnomonic)

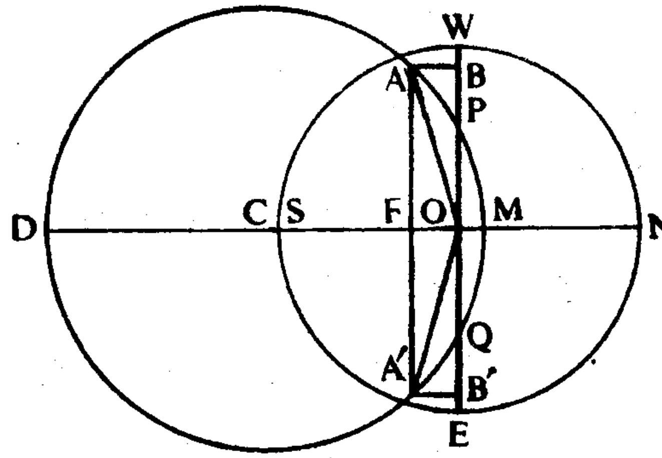
shadow due to the Sun. Twice of that gives the *aṅgulas* of the diameter of the circle described by the motion of the shadow (of the gnomon).

That is : Diameter of the circle described by the tip of the shadow of the gnomon

$$= \frac{(\text{shadow})^2 - (bhuj\bar{a})^2 + (bhuja + \text{ or} \sim \text{midday shadow})^2}{bhuja + \text{ or} \sim \text{midday shadow}}.$$

Rationale. In the adjoining figure ENWS is the circle drawn on level ground and O its centre, E, W, N and S being the east, west, north and south cardinal points. A gnomon of 12 *aṅgulas* is supposed to be set up at O. OA is the instantaneous shadow of the gnomon and AB, the distance of A from the east-west line, the instantaneous *bhuja* (i.e., the *bhuja* of that shadow). OM is the midday shadow.

C is the centre of the circle drawn through A, M and A′, OA′ being the shadow equal to OA, in the other half of the day.



Let OC = x. Then

$$CM^2 = CA^2$$

or $(CO + OM)^2$

$$= AF^2 + CF^2$$

or $(x + \text{midday shadow})^2 = (\text{shadow})^2 - (bhuja)^2 + (x - bhuja)^2$.

$$\therefore\ 2x = \frac{(\text{shadow})^2 - (bhuja)^2}{bhuja + \text{midday shadow}} + (bhuja - \text{midday shadow})$$

$$\therefore\ 2(x + \text{midday shadow}) = \frac{(\text{shadow})^2 - (bhuja)^2}{bhuja + \text{midday shadow}} + (bhuja + \text{midday shadow})$$

$$= \frac{(\text{shadow})^2 - (bhuja)^2 + (bhuja + \text{midday shadow})^2}{bhuja + \text{midday shadow}}.$$

This gives the diameter of the circle centred at C, the locus of the shadow-tip of the gnomon. This is in terms of *aṅgulas* because the quantities involved are measured in *aṅgulas*.

When the point M falls towards the south of O, *bhuja* + midday shadow, in the above formula, becomes *bhuja* $\sim$ midday shadow.

PRIME VERTICAL ZENITH DISTANCE AND PRIME VERTICAL SHADOW FROM THE SHADOW-LOCUS

27. Diminish that (diameter of the circle described by the Sun's *śaṅku*) by the Rsine of the Sun's midday zenith distance and multiply by the same (Rsine of the Sun's midday zenith distance). The square-root thereof is declared as the Rsine of the Sun's prime vertical zenith distance.

Similarly, from the diameter (of the circle described by the shadow-tip of the gnomon), which is in terms of *aṅgulas*, may be obtained the *aṅgulas* of the prime vertical shadow.

That is, if D denote the diameter of the circle described by the Sun's *śaṅku*, z_p the Sun's prime vertical zenith distance and z_m the Sun's meridian zenith distance, then

$$\text{Rsin } z_p = \sqrt{\text{Rsin } z_m \times (D - \text{Rsin } z_m)}.$$

And if d denote the diameter of the circle described by the shadow-tip, and s_p the prime vertical shadow and s_m the meridian shadow, each in terms of *aṅgulas*, then

$$s_p = \sqrt{s_m\ (d - s_m)}.$$

Rationale. From the figure given under vs. 24, we have

$$\text{OP} = \sqrt{\text{MO} \times \text{DO}}.$$

$$\therefore \quad \text{Rsin } z_p = \sqrt{\text{Rsin } z_m \times (D - \text{Rsin } z_m)};$$

and from the figure given under vss. 25 - 26, we have

$$\text{OP} = \sqrt{\text{OM} \times \text{DO}}.$$

$$\therefore \quad s_p = \sqrt{s_m\ (d - s_m)}.$$

Note. The conception of the early Hindu astronomers that the Sun's *śaṅku* or the shadow-tip of the gnomon describes a circular arc is not quite true. Therefore, the results stated in verses 24 to 27 are only approximately correct.

UNNATA-KĀLA OR DAY ELAPSED OR TO ELAPSE

Method 1

28. Divide the product of the radius, the Rsine of the (Sun's) declination and 12 by the product of the equinoctial midday shadow and

the day-radius. To the arc (of the Rsine equal to the resulting quotient) add the (Sun's) ascensional difference: the result gives the measure of the day elapsed (since sunrise in the forenoon) or to elapse (before sunset in the afternoon) when the Sun is on the prime vertical.[1]

That is: When the Sun is on the prime vertical, then

$$\textit{Unnatakāla} = \text{arc}\left(\frac{\text{R} \times \text{Rsin}\,\delta \times 12}{\textit{palabhā} \times \text{day-radius}}\right) + \text{Sun's asc. diff.},$$

which is true, because

$$\frac{\text{R} \times \text{Rsin}\,\delta \times 12}{\textit{palabhā} \times \text{day-radius}} = \frac{\text{Rsin}\,\delta \times 12}{\textit{palabhā}} \cdot \frac{\text{R}}{\text{day-radius}}$$

$$= \frac{\text{Rsin}\,\delta \times \text{Rcos}\,\phi}{\text{Rsin}\,\phi} \cdot \frac{\text{R}}{\text{day-radius}}$$

$$= (\textit{taddhṛti} - \text{earthsine})\frac{\text{R}}{\text{day-radius}}$$

$$= \textit{svāntyā} - \text{Rsin (asc. diff.)}.$$

Proof. Let ZPS be the spherical triangle formed on the celestial sphere by joining Z, the zenith of the place, P, the north celestial pole, and S, the Sun on the prime vertical. Let ϕ be the latitude of the place, $90° - H$ the hour angle and δ the declination of the Sun. Then in the spherical triangle ZPS, $\text{ZP} = 90° - \phi$, $\text{SP} = 90° - \delta$, $\angle\text{ZPS} = 90° - H$, and $\angle\text{SZP} = 90°$. Therefore, using cotangent formula, we get

$$\sin H = \tan\delta \times \cot\phi,$$

giving

$$\text{Rsin}\,H = \frac{\text{R} \times \text{Rsin}\,\delta \times \text{Rcos}\,\phi}{\text{Rcos}\,\delta \times \text{Rsin}\,\phi}$$

$$= \frac{\text{R} \times \text{Rsin}\,\delta \times 12}{\text{Rcos}\,\delta \times \textit{palabhā}}. \qquad \text{(i)}$$

If c be the Sun's ascensional difference, then

unnata-kāla = complement of Sun's hour angle + Sun's ascensional difference

$= H + c,$

where H is given by (i).

1. Same rule is found to occur in *BrSpSi*, xv. 19-20 and *SiŚe*, iv. 96.

Method 2

29. Divide the product of the radius, the *agrā* and the square of 12 by (the product of) the equinoctial midday shadow, the day-radius and the hypotenuse of the equinoctial midday shadow. The arc of (the Rsine equal to) the resulting quotient when increased by the (Sun's) ascensional difference gives the *unnata-kāla* when the Sun is on the prime vertical.

That is : When the Sun is on the prime vertical, then

$$unnata\text{-}k\bar{a}la = \text{arc}\left[\frac{\text{R} \times agr\bar{a} \times 12^2}{palabh\bar{a} \times \text{day-radius} \times palakar\d{n}a}\right] + \text{Sun's asc. diff.}$$

This formula is equivalent to the previous one, for

$$\frac{\text{Rsin}\ \delta}{agr\bar{a}} = \frac{12}{palakar\d{n}a}.$$

Method 3

30. Divide the product of the square of 12, the radius and the earthsine by the product of the square of the equinoctial midday shadow and the day-radius. The arc of (the Rsine equal to) that quotient being added to the (Sun's) ascensional difference gives the *unnata-kāla* when the Sun is on the prime vertical.

That is : When the Sun is on the prime vertical, then

$$unnata\text{-}k\bar{a}la = \text{Sun's asc. diff.} + \text{arc}\left[\frac{12^2 \times \text{R} \times \text{earthsine}}{(palabh\bar{a})^2 \times \text{day-radius}}\right].$$

This formula is equivalent to the previous one, because

$$\frac{\text{earthsine}}{palabh\bar{a}} = \frac{agr\bar{a}}{palakar\d{n}a}.$$

NATA OR HOUR ANGLE

31. Multiply the Rsine of the Sun's prime vertical zenith distance by the radius and divide by the day-radius. The arc of (the Rsine equal to) the quotient obtained briefly gives the *natakālā* ("hour angle") of the Sun on the prime vertical. The *natakāla* ("hour angle") and *unnatakāla* ("complement of hour angle *plus* Sun's ascensional difference") may also be obtained in many ways in the manner stated heretofore.

That is : If z, δ and H be the zenith distance, declination and hour angle of the Sun on the prime vertical, then

$$\text{Rsin}\ H = \frac{\text{Rsin}\ z \times \text{R}}{\text{Rcos}\ \delta}.$$

Section 12

Sun's Altitude in the Corner Directions

CALCULATION OF CORNER ALTITUDE. GENERAL METHOD

1-2. Half of the square of the radius minus the square of the *agrā* when multiplied by the square of 12 gives the "first result". The "other result" is the product of 12, the equinoctial midday shadow and the *agrā*. These results are to be divided by 72 as increased by the square of the equinoctial midday shadow. The square-root of the sum of the "first result" and the square of the "other result" should be increased or diminished by the other result according as the Sun is in the northern or southern hemisphere : the result is the *koṇaśaṅku* (*koṇanā*), i.e., the Rsine of the Sun's altitude when the Sun is in a corner direction.[1] When the Sun is in the northern hemisphere and the "other result" is not smaller than the square-root, even then subtraction should be made (of the square-root from the "other result").

The *koṇaśaṅku* is the Rsine of the Sun's altitude when it is in a corner direction, viz. north-east (Aiśāna, lorded over by Īśāna or Śiva), south-east (Āgneya, lorded over by Agni), south-west (Nairṛtya, lorded over by Nairṛta), or north-west (Vāyavya, lorded over by Vāyu). The *koṇaśaṅku* is also known as *koṇanā*, *koṇanara*, *vidikśaṅku*, *vidiṅnā*, *vidiṅnara*, etc.

Let x be the *koṇaśaṅku* and z the corresponding zenith distance. Then

$$bhuja = \frac{digjy\bar{a} \times \text{Rsin}\, z}{\text{R}} = \frac{(\text{R}/\sqrt{2})\sqrt{(\text{R}^2-x^2)}}{\text{R}} = \sqrt{[(\text{R}^2 - x^2)/2]}. \qquad \text{(i)}$$

Also $$bhuja = \acute{s}a\dot{n}kutala \mp agr\bar{a} = \frac{palabh\bar{a} \times x}{12} \mp agr\bar{a}, \qquad \text{(ii)}$$

according as the Sun is in the northern or southern hemisphere.

From (i) and (ii), by squaring,

$$\left(\frac{palabh\bar{a}}{12}\right)^2 x^2 \mp 2.\frac{palabh\bar{a} \times agr\bar{a}}{12}\, x + (agr\bar{a})^2 = \frac{\text{R}^2 - x^2}{2}$$

---

1. Same rule occurs in *BrSpSi*, iii. 54-56; *SūSi*, iii. 28 (c-d)-32; *SiŚe*, iv. 74-75.

or $\left[\left(\frac{palabh\bar{a}}{12}\right)^2 + \frac{1}{2}\right]x^2 \mp 2.\frac{palabh\bar{a} \times agr\bar{a}}{12}\,x - \left[\frac{R^2}{2} - (agr\bar{a})^2\right] = 0$

or $[(palabh\bar{a})^2 + 72]\; x^2 \mp 2.\; 12.\; palabh\bar{a} \times agr\bar{a}.\, x - \left[\frac{R^2}{2} - (agr\bar{a})^2\right] \times 12^2 = 0$

or $x^2 \mp 2.\; \frac{12\; palabh\bar{a} \times agr\bar{a}}{(palabh\bar{a})^2 + 72}\, x - \frac{[R^2/2 - (agr\bar{a})^2] \times 12^2}{(palubh\bar{a})^2+72} = 0$

or $x^2 \mp 2$ (other result) x — (first result) $= 0$

$\therefore\; x = \pm$ other result $+ \sqrt{[(\text{other result})^2 + \text{first result}]}$,

$+$ or $-$ sign being taken according as the Sun is in the northern or southern hemisphere.

Observations. Hence when the Sun is in the northern hemisphere,

$$x = + \text{ other result} \pm \sqrt{(\text{other result})^2 + \text{first result}} \qquad (1)$$

and when the Sun is in the southern hemisphere,

$$x = - \text{ other result} \pm \sqrt{(\text{other result})^2 + \text{first result}}. \qquad (2)$$

(1) gives one positive value when *agrā* $< R/\sqrt{2}$ and two positive values when *agrā* $> R/\sqrt{2}$. (2) gives one positive value when *agrā* $< R/\sqrt{2}$ and no positive value when *agrā* $> R/\sqrt{2}$.

This means that :

(1) There will be 4 *koṇaśaṅkus* when the Sun is in the northern hemisphere and *agrā* $> R/\sqrt{2}$.

(2) There will be 2 *koṇaśaṅkus* when the Sun is in the northern hemisphere and *agrā* $< R/\sqrt{2}$, and also when the Sun is in the southern hemisphere and *agrā* $< R/\sqrt{2}$

(3) There will be no *koṇaśaṅku* when the Sun is in the southern hemisphere and *agrā* $> R/\sqrt{2}$.

ALTERNATIVE METHOD (PROCESS OF ITERATION)

3-4. When the Sun is in the northern hemisphere, subtract twice the square of the difference between an optional number (chosen for the

Sun's unknown *śaṅkutala*) and the *agrā* from the square of the radius, and find the square-root of the difference (thus obtained). This gives (the first approximation for) the *koṇaśaṅku*. Multiply that by the equinoctial midday shadow and divide by 12 : this gives (a better approximation for) the optional number. Now repeat the process (until the best approximation for the *koṇaśaṅku* is not arrived at).

When the Sun is in the southern hemisphere, the *koṇaśaṅku* is obtained by proceeding in the manner stated above with the sum of the *agrā* and the optional number (instead of their difference).[1]

From that (*koṇaśaṅku*), the Rsine of the Sun's zenith distance, the hypotenuse of shadow and the shadow should be obtained as before.

The above process may be explained more clearly as follows: When the Sun is in the northern hemisphere, one can easily see that

$$koṇaśaṅku = \sqrt{R^2 - 2\,(bhuja)^2}$$

$$= \sqrt{R^2 - 2\,(śaṅkutala \sim agrā)^2} \qquad (1)$$

and when the Sun is in the southern hemisphere

$$koṇaśaṅku = \sqrt{R^2 - 2\,(śaṅkutala + agrā)^2}. \qquad (2)$$

Since the *śaṅkutala* is not known, some optional number (*iṣṭa*) is chosen for it and using the formula (1) or (2), as the case may be, one obtains the first approximation for the *koṇaśaṅku*. From that *koṇaśaṅku* one calculates a better approximation for the *śaṅkutala* by using the formula

$$śaṅkutala = \frac{koṇaśaṅku \times palabhā}{12}.$$

Using this value of the *śaṅkutala* in (1) or (2), as the case may be, one obtains the second approximation for the *koṇaśaṅku*. This process is repeated again and again until the best approximation for the *koṇaśaṅku* is not arrived at.

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1. The same rule occurs in *ŚiDVṛ*, iv. 34-35; *SiŚe*, iv. 72-73; and *SiŚi*, I, iii. 30. To find the approximation for the *koṇaśaṅku*, Bhāskara II takes the optional number to be zero.

ALTERNATIVE FORMS OF "FIRST RESULT" AND "OTHER RESULT"

5-8(a). The square of the radius diminished by twice the square of the *agrā* and then divided by 2 gives the "multiplicand"; the same ("multiplicand") is also equal to half the square of the radius diminished by the square of the *agrā*. This ("multiplicand") multiplied by the square of the Rsine of colatitude is the "first result". The "other result" is the product of *agrā*, the Rsine of latitude and the Rsine of colatitude; or the product of the radius, the Rsine of declination and the Rsine of latitude; or the product of the radius, the Rsine of colatitude and the earthsine; or the Rsine of the prime vertical altitude multiplied by the square of the Rsine of latitude. Their divisor is half the sum of the squares of the Rsine of latitude and the radius, or half the square of the Rsine of colatitude increased by the square of the Rsine of latitude. With the help of these (first and other results) one may calculate the *koṇaśaṅku*, as before.

That is: "first result" $=$ "multiplicand" $\times$ $(\text{Rcos}\ \phi)^2$

and "other result" $=$ *agrā* $\times$ $\text{Rsin}\ \phi \times \text{Rcos}\ \phi$

$= \text{R} \times \text{Rsin}\ \delta \times \text{Rsin}\ \phi$

$= \text{R} \times \text{Rcos}\ \phi \times$ earthsine

$= (\text{Rsin}\ \phi)^2 \times$ *samaśaṅku*,

where multiplicand $= \dfrac{\text{R}^2 - 2\,(agr\bar{a})^2}{2}$ or $\dfrac{\text{R}^2}{2} - (agr\bar{a})^2$

and divisor $= \dfrac{\text{R}^2 + (\text{Rsin}\ \phi)^2}{2}$ or $(\text{Rsin}\ \phi)^2 + \frac{1}{2}\,(\text{Rcos}\ \phi)^2$.

It can be easily seen that the values of the "first result", the "other result" and the "divisor" given here are $(\text{Rcos}\ \phi/12)^2$ times those stated in vs. 1. So the values of (first result)/divisor and (other result)/divisor remain invariant, as it should be.

8(b-d)-9. Or, for another (alternative) calculation, the divisor of the first and other results (stated in vs. 1) may be taken as half the sum of the squares of the equinoctial midday shadow and the hypotenuse of the equinoctial midday shadow, and the "other result" as the product of the Rsine of declination, the hypotenuse of the equinoctial midday shadow and the equinoctial midday shadow, or the earthsine multiplied by the

hypotenuse of the equinoctial midday shadow and 12, or the square of the equinoctial midday shadow multiplied by the Rsine of the prime vertical altitude.

That is : "other result" = Rsin δ × *palakarṇa* × *palabhā*

= earthsine × *palakarṇa* × 12

= (*palabhā*)^2 × *samaśaṅku*,

and divisor = $\frac{(palabh\bar{a})^2 + (palakar\d{n}a)^2}{2}$,

the "first result" remaining the same as stated in vs. 1.

Evidently the values of the "divisor" and the "other result" stated here are equivalent to those stated in vs. 1.

CALCULATION OF *KOṆAŚAṄKU* BY TAKING *PALAKARṆA* FOR THE RADIUS

10. (Severally) multiply the *samaśaṅku*, the Rsine of the declination, the earthsine and the *agrā* by the *palakarṇa* and divide by the radius. From them the *koṇaśaṅku* may be obtained as before (see General Method), in case the *palakarṇa* is taken for the radius.

11. Or, from this *agrā* corresponding to the radius equal to the *palakarṇa*, diminished or increased by an optional number, according as the hemisphere is northern or southern, too, the *koṇaśaṅku* may be obtained by the process of iteration (see Alternative Method).[1]

OTHER FORMS OF "FIRST RESULT" AND "OTHER RESULT"

12-13(a-b). The product of the multiplicand and the square of the Rsine of the declination is the "first result"; the "other result" is the product of the *agrā*, the Rsine of the declination and the earthsine, or the product of the *samaśaṅku*, and the square of the earthsine; the divisor of these products is the square of the earthsine plus half the square of the Rsine of the declination, or half the sum of the squares of the *agrā* and the earthsine.

1. For details of this method, see *SiŚe*, iv. 78.

That is : "first result" $=$ multiplicand $\times$ (Rsin $\delta)^2$,

"other result" $=$ *agrā* $\times$ Rsin δ $\times$ earthsine

$=$ *samaśaṅku* $\times$ (earthsine)^2,

and divisor $=$ (earthsine)^2 $+$ $\frac{1}{2}$(Rsin $\delta)^2$

$$= \frac{(agr\bar{a})^2 + (\text{earthsine})^2}{2}.$$

These values are evidently (Rsin $\delta/12)^2$ times those stated in vs. 1.

CALCULATION OF *KOṆAŚAṄKU* BY TAKING *AGRĀ* FOR THE RADIUS

13(c-d)-14. Multiply the *agrā*, the Rsine of the declination, the earthsine and the *samaśaṅku* by the *agrā* and divide by the radius : the results are the abbreviated quantities. From them may be obtained the *samaśaṅku* as before, in case the *agrā* is taken for the radius.

15. Or, one may, by taking the *agrā* for the radius, obtain the *koṇaśaṅku* from the reduced *agrā* increased by an optional number, in the southern hemisphere, or from the reduced *agrā* diminished by an optional number, in the northern hemisphere, by using the process of iteration, as before.

OTHER FORMS OF "FIRST RESULT" AND "OTHER RESULT"

16-17. The product of the multiplicand and the square of the *samaśaṅku* is the "first result"; the "other result" is the product of the square of the *agrā* and the *samaśaṅku*, or the product of the *taddhṛti*, the *agrā*, and the Rsine of the declination, or the product of the earthsine, the *samaśaṅku* and the *taddhṛti*; and the divisor of these (first and other results) is stated to be one-half of the sum of the squares of the *taddhṛti* and the *agrā*, or the square of the *agrā* increased by half the square of the *samaśaṅku*. From these, one may obtain the *koṇaśaṅku*, as before.

That is : "first result" $=$ multiplicand $\times$ (*samaśaṅku*)^2,

"other result" $=$ (*agrā*)^2 $\times$ *samaśaṅku*

$=$ *taddhṛti* $\times$ *agrā* $\times$ Rsin δ

$=$ earthsine $\times$ *samaśaṅku* $\times$ *taddhṛti*,

$$\text{and divisor} = \frac{(taddhṛti)^2 + (agrā)^2}{2}$$

$$= (agrā)^2 + \frac{(samaśaṅku)^2}{2}.$$

These values are evidently $(samaśaṅku/12)^2$ times those stated in vs. 1.

CALCULATION OF *KOṆAŚAṄKU* BY TAKING *TADDHṚTI* FOR THE RADIUS

18. (Severally) multiply the Rsine of the declination, the earthsine, the *agrā* and the Rsine of the prime vertical altitude by the *taddhṛti* and divide by the radius : the results are their shorter values. With the help of them, one may obtain the *koṇaśaṅku*, as before, in case the *taddhṛti* is taken for the radius.

19. Or, assuming the *taddhṛti* for the radius, one may determine the *koṇaśaṅku*, by the process of iteration, from this (shorter) *agrā* increased or diminished by an optional number, according as the hemisphere is southern or northern.

OTHER FORMS OF "FIRST RESULT" AND "OTHER RESULT"

20-21. The product of the square of the *iṣṭaśaṅku* and the multiplicand is the "first result"; the "other result" is the product of *agrā*, *iṣṭaśaṅku* and *śaṅkutala*, or the product of *dhṛti*, (*śaṅkv*) *agra* (i.e., *śaṅkutala*) and the Rsine of the declination, or the product of *dhṛti*, *śaṅku* and earthsine, or the *samaśaṅku* multiplied by the square of *śaṅkutala*; and their divisor is half the sum of the squares of the *śaṅkutala* and the *dhṛti* or the square of *śaṅkutala* plus half the square of *śaṅku*. From them the other things may be obtained as before.

That is : "first result" = multiplicand × $(iṣṭaśaṅku)^2$,

"other result" = $agrā \times iṣṭaśaṅku \times śaṅkutala$

$= dhṛti \times śaṅkutala \times \text{Rsin } \delta$

$= dhṛti \times śaṅku \times \text{earthsine}$

$= (śaṅkutala)^2 \times samaśaṅku,$

$$\text{and divisor} = \frac{(śaṅkutala)^2 + (dhṛti)^2}{2}$$

$$= (\acute{s}a\dot{n}kutala)^2 + \frac{(\acute{s}a\dot{n}ku)^2}{2}.$$

These values are evidently $(i\d{s}\d{t}a\acute{s}a\dot{n}ku/12)^2$ times those stated in vs. 1.

CALCULATION OF *KOṆAŚAṄKU* BY TAKING *DHṚTI* FOR THE RADIUS

22. The *agrā*, the Rsine of declination, the earthsine and the *sama-śaṅku* (severally) multiplied by the *dhṛti* and divided by the radius are the shorter values (of those elements). With the help of them, one may determine the *koṇaśaṅku*, as before, assuming the *dhṛti* for the radius.

CALCULATION OF *KOṆAŚAṄKU* BY TAKING *IṢṬAKARṆA* FOR THE RADIUS

23. Multiply the *agrā* etc. by the *iṣṭakarṇa* and divide by the radius : the results are the shorter values (of *agrā*, etc.). With the help of them, too, one may determine the *koṇaśaṅku*, assuming the *iṣṭakarṇa* for the radius.

24. Or, one may determine the *koṇaśaṅku* from the shorter *agrā* increased or diminished by an optional number, according as the hemisphere is southern or northern, by applying the process of iteration, and assuming the *iṣṭakarṇa* for the radius.

KOṆAŚAṄKU AND ITS SHADOW

25-26. (The Sun being in the northern hemisphere) if the Sun's *bhuja* (at sunrise) exceeds its *koṭi*, the *koṇaśaṅku* will occur at the four corners of a square (i.e., in the four corner directions). If the Sun's zenith distance is large (and the Sun is in the north-east corner direction), the (corner) shadow of the gnomon will fall towards the south-west; and if the Sun's zenith distance is small (and the Sun is in the south-west corner direction), the (corner) shadow of the gnomon will fall towards the north-east. Or, if the Sun's altitude is small (and the Sun is in the north-west corner direction), the (corner) shadow of the gnomon will fall on the circle of shadow's path towards the south-east; and if the Sun's altitude is large (and the Sun is in the south-east corner direction), the (corner shadow of the gnomon will fall towards the north-west. The *ghaṭikās* (of the hour angle) coresponding to the corner altitude are obtained as before.

What is meant here is this: When the Sun is in the northern hemisphere and the Sun's *bhuja* at sunrise is greater than the Sun's *koṭi*, there will be, in general, 4 *koṇaśaṅkus*, towards north-east, south-east, south-west and north-west. If the Sun's zenith distance is large the *koṇaśaṅku* will lie towards the north-east or north-west and then the shadow of the gnomon will fall towards the south-west or south-east, respectively. If the Sun's zenith distance is small the *koṇaśaṅku* will lie towards the south-east or south-west and then the shadow of the gnomon will fall towards the north-west or north-east, respectively.

Section 13

Sun From Shadow

SUN'S HEMISPHERE

1. The zenith distance of the midday Sun is, as before, the *Khākṣa*[1]. The directions of the Sun's hemisphere are determined with reference to that (*Khākṣa*) or with reference to the *palabhā*.

When the latitude is less than the *Khākṣa*, the Sun's hemisphere is south; when (the latitude is) greater, the Sun's hemisphere is to be known as north.

2. Or, when the *palabhā* is smaller than the midday shadow, the Sun's hemisphere is south; when greater, the Sun's hemisphere is north; when the midday shadow is south, the Sun's hemisphere is always north.

SUN'S AYANA

3(a-b). The midday shadow begins to increase with the Sun's entrance into the sign Cancer and to decrease with the Sun's entrance into the sign Capricorn.

That is to say : When the Sun's midday shadow is on the decrease, the Sun's *ayana* is north; and when the Sun's midday shadow is on the increase, the Sun's *ayana* is south.

The above-mentioned criteria for knowing the Sun's hemisphere and the Sun's *ayana* give us the following two sets of criteria for knowing the Sun's quadrant.

First set of Criteria

The Sun is in the first quadrant if:

The zenith distance of the midday Sun is less than the latitude of the place, and the midday shadow is on the decrease. (In case the zenith distance of the midday Sun is north, the midday shadow is on the increase).

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1. See *supra*, sec. 9, vs. 1(a-b). The term *Khākṣa* is also used in the same sense in *PSi*, iv. 21, but the editors of *PSi* have misspelt it as *Svākṣa*.

The Sun is in the second quadrant if :

The zenith distance of the midday Sun is less than the latitude of the place, and the midday shadow is on the increase. (In case the zenith distance of the midday Sun is north, the midday shadow is on the decrease.)

The Sun is in the third quadrant if:

The zenith distance of the midday Sun is greater than the latitude of the place, and the midday shadow is on the increase.

The Sun is in the fourth quadrant if :

The zenith distance of the midday Sun is greater than the latitude of the place, and the midday shadow is on the decrease.

Second Set of Criteria

The Sun is in the first quadrant if :

The midday shadow is on the decrease and smaller than the *palabhā*. (In case the midday shadow falls towards the south, it is on the increase.)

The Sun is in the second quadrant if :

The midday shadow is on the increase and smaller than the *palabhā*. (In case the midday shadow falls towards the south, it is on the decrease).

The Sun is in the third quadrant if :

The midday shadow is on the increase and greater than the *palabhā*.

The Sun is in the fourth quadrant if :

The midday shadow of the Sun is on the decrease but greater than the *palabhā*.

The second set of criteria was later given by Śrīpati also.[1]

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1. See *SiŚe*, iv. 70-71. Also see *SiTVi*, iii. 192-193.

SUN'S DECLINATION

3(c-d)-4(a-b). The difference or the sum of the *Khākṣa* ("Sun's meridian zenith distance") and the latitude, according as they are of like or unlike directions, gives the Sun's declination.

The *Khākṣa* being zero, the Sun's declination is equal to the latitude (of the place), because the midday shadow is then non-existent.

It should be noted that the latitude is always of south direction and that the direction of the *Khākṣa* ("Sun's meridian zenith distance") is north or south according as the Sun is to the north or south of the zenith.

SUN'S *BHUJA*

4(c-d). The Rsine of the Sun's declination multiplied by the radius and divided by the Rsine of 24° gives the Rsine of the Sun's *bhuja*.[1]

Let λ be the *bhuja* of the Sun's tropical longitude and δ the Sun's declination. Then

$$\text{Rsin}\ \lambda = \frac{\text{R} \times \text{Rsin}\ \delta}{\text{Rsin}\ 24°}. \tag{1}$$

5. The product of the Rsine of the codeclination, the Rsine of the ascensional difference and the *śaṅkutala*, multiplied by the *samaśaṅku* into the Rsine of the colatitude, and divided by the product of the Rsine of the latitude, the *agrā*, the *śaṅku* ("Rsine of the Sun's altitude"), and the Rsine of 24° is the Rsine of the (Sun's) *bhuja*.

$$\text{Rsin}\ \lambda = \frac{\text{Rcos}\ \delta \times \text{Rsin (asc. diff.)} \times \textit{śaṅkutala} \times \textit{samaśaṅku} \times \text{Rcos}\ \phi}{\text{Rsin}\ \phi \times \textit{agrā} \times \textit{śaṅku} \times \text{Rsin}\ 24°} \tag{2}$$

Rationale. This easily reduces to formula (1). For,

$$\text{R. H. S.} = \frac{\text{R} \times \text{earthsine} \times \textit{śaṅkutala} \times \textit{samaśaṅku} \times \text{Rcos}\ \phi}{\text{Rsin}\ \phi \times \textit{agrā} \times \textit{śaṅku} \times \text{Rsin}\ 24°},$$

$$\text{because}\ \frac{\text{Rsin (asc. diff.)}}{\text{earthsine}} = \frac{\text{R}}{\text{Rcos}\ \delta}$$

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1. *Cf. BrSpSi*, iii. 61(a-b); *SiŚe*, iv. 63.

$$= \frac{\text{Rsin } \phi \times samaśaṅku}{\text{Rsin } 24°}, \quad \text{because } \frac{\text{R} \times \text{earthsine}}{\text{Rsin } \phi} = agrā,$$

$$\text{and } \frac{śaṅkutala}{śaṅku} = \frac{\text{Rsin } \phi}{\text{Rcos } \phi}$$

$$= \frac{\text{R} \times \text{Rsin } \delta}{\text{Rsin } 24°} = \text{Rsin } \lambda.$$

6. Or, the Rsine of the (Sun's) prime vertical altitude multiplied by the Rsine of the latitude and divided by the Rsine of 24° gives the Rsine of the Sun's *bhuja*.[1] Or, the *taddhṛti* multiplied by the Rsine of the latitude and divided by the *agrā* corresponding to the end of Gemini also yields the same.

$$\text{Rsin } \lambda = \frac{samaśaṅku \times \text{Rsin } \phi}{\text{Rsin } 24°} \tag{3}$$

$$\text{Rsin } \lambda = \frac{taddhṛti \times \text{Rsin } \phi}{agrā \text{ for end of Gemini}}. \tag{4}$$

Formula (3) follows from formula (1) by using formula (14) of sec. 3. Formula (4) reduces to formula (3), for

$$taddhṛti = \frac{samaśaṅku \times \text{R}}{\text{Rcos } \phi} \text{ and } agrā \text{ for end of Gemini } = \frac{\text{R} \times \text{Rsin } 24°}{\text{Rcos } \phi}.$$

7. The Rsine of the Sun's *bhuja* may also be obtained by dividing the product of the Rsine of the colatitude and the *taddhṛti* by the *samaśaṅku* corresponding to (the Sun's position at) the end of Gemini; or, by multiplying the product of the *taddhṛti* and the Rsine of the latitude by 12 and dividing that by the product of the *palakarṇa* and (the Rsine of) 24°.

$$\text{Rsin } \lambda = \frac{\text{Rcos } \phi \times taddhṛti}{samaśaṅku \text{ for end of Gemini}} \tag{5}$$

$$\text{Rsin } \lambda = \frac{taddhṛti \times \text{Rsin } \phi \times 12}{palakarṇa \times \text{Rsin } 24°}. \tag{6}$$

1. *Cf BrSpSi*, xv. 26; *SiŚe*, iv. 101.

Formula (5) is equivalent to formula (4), because

$$\frac{\textit{agrā} \text{ for end of Gemini}}{\textit{samaśaṅku} \text{ for end of Gemini}} = \frac{\text{Rsin } \phi}{\text{Rcos } \phi};$$

and formula (6) is equivalent to formula (3), because

$$\frac{\textit{taddhṛti}}{\textit{samaśaṅku}} = \frac{\textit{palakarṇa}}{12}.$$

8. Multiply the product of the radius and the Rsine of the prime vertical altitude severally by the Rsine of the latitude, the earthsine and the *iṣṭaśaṅkutala*, and divide by the Rsine of 24° multiplied respectively by the radius, the *agrā* and the *dhṛti* : the result (in each case) is the Rsine of the Sun's *bhuja*.

$$\text{Rsin } \lambda = \frac{(\text{R} \times \textit{samaśaṅku}) \times \text{Rsin } \phi}{\text{R} \times \text{Rsin } 24^\circ}. \tag{7}$$

$$\text{Rsin } \lambda = \frac{(\text{R} \times \textit{samaśaṅku}) \times \text{earthsine}}{\textit{agrā} \times \text{Rsin } 24^\circ} \tag{8}$$

$$\text{Rsin } \lambda = \frac{(\text{R} \times \textit{samaśaṅku}) \times \textit{iṣṭaśaṅkutala}}{\textit{iṣṭadhṛti} \times \text{Rsin } 24^\circ}. \tag{9}$$

Rationale. From formula (1) above,

$$\text{Rsin } \lambda = \frac{\text{R} \times \text{Rsin } \delta}{\text{Rsin } 24^\circ}. \tag{i}$$

But (*vide supra*, sec. 3, vs. 6)

$$\text{Rsin } \delta = \frac{\textit{samaśaṅku} \times \text{Rsin } \phi}{\text{R}} \tag{ii}$$

$$= \frac{\textit{samaśaṅku} \times \text{earthsine}}{\textit{agrā}} \tag{iii}$$

$$= \frac{\textit{samaśaṅku} \times \textit{iṣṭaśaṅkutala}}{\textit{iṣṭadhṛti}}. \tag{iv}$$

Substitution of (ii), (iii) and (iv) in (i) gives (7), (8) and (9).

9-10(a-b). The product of the radius, the *agrā* and the *śaṅku* (i.e., Rsine of the altitude) divided by the product of the (*iṣṭa*)*dhṛti* and the Rsine of 24° gives the Rsine of the Sun's *bhuja*. The product of the *agrā*

of the shadow circle, the radius and the Rsine of the colatitude being divided by the hypotenuse of shadow and the Rsine of the (Sun's) greatest declination, the result is also the Rsine of the Sun's *bhuja*.

$$\text{Rsin}\ \lambda = \frac{\text{R} \times agr\bar{a} \times \acute{s}a\dot{n}ku}{i\d{s}\d{t}adh\d{r}ti \times \text{Rsin}\ 24^\circ} \tag{10}$$

$$= \frac{bh\bar{a}v\d{r}tt\bar{a}gr\bar{a} \times \text{R} \times \text{Rcos}\ \phi}{\text{hyp. of shadow} \times \text{Rsin}\ 24^\circ}. \tag{11}$$

Rationale. Substituting formula (9) of sec. 3, viz.

$$\text{Rsin}\ \delta = \frac{agr\bar{a} \times \acute{s}a\dot{n}ku}{i\d{s}\d{t}adh\d{r}ti}$$

in formula (1) above, we get (10), and substituting

$$agr\bar{a} = \frac{bh\bar{a}v\d{r}tt\bar{a}gr\bar{a} \times \text{R}}{\text{hyp. of shadow}} \text{ and } \frac{\acute{s}a\dot{n}ku}{i\d{s}\d{t}adh\d{r}ti} = \frac{\text{Rcos}\ \phi}{\text{R}}$$

in (10), we get (11).

10(c-d)-11. The radius multiplied by the Rsine of the Sun's prime vertical zenith distance and divided by the Rsine of the corresponding hour angle gives the Rcosine of the Sun's declination. From the Rcosine of the (Sun's) declination obtain the Rsine of the (Sun's) declination and from that determine the Rsine of the Sun's *bhuja* as before.

Or, with the help of the tabulated Rsine-differences, obtain the Sun's declination and therefrom find the Rsine of the Sun's *bhuja*.

$$\text{Rsin}\ \lambda = \frac{\text{R} \times \text{Rsin}\ \delta}{\text{Rsin}\ 24^\circ}, \text{ [See formula (1)]} \tag{12}$$

where

$$\text{Rcos}\ \delta = \frac{\text{R} \times \text{Rsin}\ z}{\text{Rsin}\ H},$$

z and H being the Sun's prime vertical zenith distance and hour angle, respectively.

Brahmagupta (*BrSpSi*, xv. 57-58), Lalla (*ŚiDVṛ*, iv. 40) and Śrīpati (*SiŚe*, iv. 64) give the following formula for finding Rsin λ from the corner shadow :

$$\text{Rsin}\ \lambda = \frac{\text{R} \times \text{Rsin}\ \delta}{\text{Rsin}\ 24^\circ},$$

where

$$\text{Rsin } \delta = \left[\sqrt{\frac{(\text{corner shadow})^2}{2}} + \text{ or } \sim \textit{palabhā}\right] \frac{\text{Rcos } \phi}{\text{hyp. of corner shadow}},$$

+ or ~ sign being taken according as the shadow-tip falls towards the north or south of the east-west line.[1]

PERPETUAL DAYLIGHT

12. When the Sun's declination is equal to (or greater than) the colatitude of the place, the rising and setting of the Sun do not take place. In that case, the minutes of the ecliptic traversed by the Sun in the sky (while the Sun's rising and setting do not take place) divided by the mean daily motion of the Sun gives the days (during which the Sun's rising and setting do not take place).[2]

That is, the Sun does not rise or set when

$$\delta \geqslant 90° - \phi$$

and this happens for

$$\frac{2(5400-\lambda)}{\text{Sun's mean daily motion}} \text{ days},$$

where Rsin $\lambda = (\text{R} \times \text{Rcos } \phi) / \text{Rsin } 24°$, λ being the minutes of the Sun's (tropical) longitude when $\delta = 90° - \phi$.

13. (The Sun does not rise or set even) when the earthsine is equal to the Rsine of latitude, or when the Rsine of latitude is equal to the Rsine of the (Sun's) codeclination, or when the *agrā* or the Rsine of the Sun's ascensional difference is equal to the radius.

That is, the Sun does not rise or set, when

(1) earthsine = Rsin ϕ

(2) Rsin ϕ = Rcos δ

(3) *agrā* = R

(4) Rsin (asc. diff.) = R.

This happens when the Sun's diurnal circle just touches the horizon.

1. For methods based on iteration, see *BrSpSi*, xv. 21-23; *SiŚe*, iv. 97-98; *SiŚi*, I, iii, 82-83.

2. *Cf. BrSpSi*, xv. 55-56; *SiŚe*, iv. 118; *SiŚi*, II, *tripraśnavāsanā*, 6 (c-d)-7.

UNKNOWN PLANET FROM A KNOWN ONE

14. Find the *asus* of oblique ascension intervening between the known planet and the planet to be known. With the help of the known planet and these *asus* calculate the longitude of the rising point of the ecliptic, as in the case of the Sun. This (longitude of the rising point of the ecliptic) is the longitude of the planet to be known.

TITHIS ELAPSED

15. The time in *ghaṭīs* (measured since sunset) at which the Moon sets or rises in the night in the light half or the dark half of the month, respectively, when reduced to half, is said to give the *tithis* elapsed (since the beginning of the light half or the dark half of the month, respectively).

In the light half: when the first *tithi* begins the Sun and Moon are together so the Sun and Moon set together; hence no *tithi* is elapsed. When the second *tithi* begins the Moon is 12° in advance of the Sun, so the Moon sets 2 *ghaṭīs* after sunset; hence the number of *tithis* elapsed is 2/2 or 1. When the third *tithi* begins the Moon is 24° in advance of the Sun, so the Moon sets 4 *ghaṭīs* after sunset; hence the number of *tithis* elapsed is 4/2 or 2. And so on.

In the dark half: when the first tithi begins the Moon is 180 degrees in advance of the Sun, so the Moon rises when the Sun sets; hence no *tithi* is elapsed. When the second *tithi* begins the Moon is 180 + 12 degrees in advance of the Sun, so the Moon rises 2 *ghaṭīs* after sunset; hence the number of *tithis* elapsed is 2/2 or 1. When the third *tithi* begins the Moon is 180 + 24 degrees in advance of the Sun, so the Moon rises 4 *ghaṭīs* after sunset; hence the number of *tithis* elapsed is 4/2 or 2. And so on.

SUN'S LONGITUDE FROM SUN'S *BHUJA*

16. Reduce the Rsine of the Sun's *bhuja* to the corresponding arc. When the Sun is in the first quadrant, that arc itself is the Sun's longitude; when the Sun is in the second quadrant, that arc subtracted from 6 signs (lit. the number of signs in a circle) gives the Sun's longitude; when the Sun is in the next quadrant, that arc increased by 6 signs gives the Sun's longitude; and when the Sun is in the last quadrant, that arc subtracted from 12 signs gives the Sun's longitude.[1]

1. *Cf. BrSpSi*, iii. 61-62(a-b); *ŚiDVṛ*, iv. 38(c-d); *SiŚe*, iv. 63(c-d).

The above rule gives the Sun's true longitude. To obtain the Sun's mean longitude, the corrections applicable to the Sun should be applied reversely and the process should be iterated. See *supra* ch. 2, sec. 3, vs. 21.[1]

SEASONS

17. Since from the characteristic features of the seasons, the various quadrants (of the Sun's orbit) are clearly recognized, so I shall briefly state some of those characteristic features of the seasons.

VASANTA OR SPRING

18. In the spring season, which is distinguished by the breeze laden with the fragrance of the full-blown flowers of the various flowering trees, by the musical humming of swarms of black bees, by the sweet notes of the melodious cuckoo, by the lotuses growing on the Malaya mountain, which looks fair and beautiful by the creepers of garlands of stars radiating the lustre of (tiny pieces of) ice, and which is marked by the presence of self-respecting people being bathed with water poured by the hands of accomplished amorous ladies,

19. the forest looks bright and lovely by the distinctly visible forest-wealth provided by the blooming flowers of Karṇikāra,[2] excellent Aśoka,[3] beautiful Campaka,[4] the mango blossoms and the flowers of Bakula[5]

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1. Also see *BrSpSi*, iii. 62.

2. Karṇikāra, commonly known as Amalatāsa, is *Cassia fistula*. It has excellent bright yellow colour but no fragrance.

3. Aśoka (*Sita Asoka*) is *Saraca indica*. It is a medium-sized ever-green tree with ornamental foliage, leaves like the Litchi and scarlet-coloured fragrant flowers in large bunches. For detailed botanical description, see Biswas, T. K. and Debnath, P. K., "*Aśoka* (*Saraca Indica*—Linn)—A Cultural and Scientific Evaluation," *IJHS*, Vol. 7, No. 2, p. 105.

4. Campaka (or Campā) is *Michelia campaka*. The Campaka flowers are cream-coloured and bell-shaped and bear strong but pleasing fragrance.

5. Bakula is *Mimusops elengi*. It is commonly known as Maulaśrī. For the botanical description of Bakula, see Roma Mitra, "*Bakula*—A Reputed Drug of Ayurveda, its history, uses in Indian Medicine", *IJHS*, Vol. 16, No. 2, p. 171.

and Palāsa[1] as well as by the trees of Kurabaka[2] and Pāribhadra[3] and well-blossoming Koṇī[4] and is greatly agitated by swarms of black bees and bears the glory of a nicely painted wall.

GRĪṢMA OR SUMMER

20. In the months of Śukra and Śuci (which define the summer season), the wind becomes noisy and scorching due to the condition of the Sun and immensely afflicts the body, the earth (gets heated up and) appears as if it were covered with the powder of chaff-fire, the quarters become defiled by clouds of smoke caused by unending forest-conflagration and the sky gets obscured by volumes of enormous dust.

VARṢĀ OR RAINY SEASON

21. In the rainy season, when the surface of the Earth is made tawny-coloured by its association with the mango fruit, the stars and planets are thrown out of sight by the trees which have borne new sprouts, tender twigs and flowers, the clouds get attended by cool breeze due to their contact with the river of nectar set in motion by the gods, and the air becomes perfumed by the fragrance of the Nava-mallikā[5],

22. (there are dark clouds and rain with occasional flashes of rainbow and lightning and it appears as if) a mighty hunter holding the bow of Indra and the pointed arrows of (rain and) lightning, bearing the beauty of a herd of buffaloes, and endowed with sweet musical voice, on account of enmity with the echoing deer on the Moon, at the advent of the rainy

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1. Palāsa is *Butea frondosa.*
2. Kurabaka is *Barleria dichotoma.* It is a small plant growing under large trees and bears bell-shaped pink flowers in large profusion. Kurabaka is that variety of Kaṭasaraiyā or Piyāvāsā which bears pink flowers. शोणे कुरबकस्तथा पीते कुरण्टकः (अमरकोष:) Kurabaka is also identified with *Amaranthus cruentus* or the scarlet variety of Amaranthus.

   In case Kurabaka is used here in the sense of a tree as appears from the context, then it might be a celestial tree like Mandāra, Pārijāta-kalpavṛkṣa, etc., as suggested to me by Shri R. S. Singh (Professor of Rasa-Shastra, B. H. U., Varanasi).
3. Pāribhadra is *Erythrina indica.* It is a huge tree with three-leaflets to a leaf like *Butea frondosa* and tiny thorns on its branches. It bears coral-red flowers in large bunches. Pāribhadra is commonly known as Pharahada.
4. Koṇī could not be identified. If it is colloquialised form of Koṭi (Koṭi > Koḍi > Koṇi), then it is *Melilotus alba* (White Sweet Clover).
5. Navamallikā is *Jasminum arborescens.* It is a large shrub or woody climber with leaves like the mango and flowers white, fragrant and in large bunches.

season when the Moon gets out of sight, kills the deer-eyed lady, suffering separation from her husband, sitting against the wall like a deer.

23. The wind (in this season) is accompanied by soft, low but distinct humming of the she black bees who are rejoicing under the (delightful) spell of intoxication caused by breeding in the decoction of (putrefied) flowers of the Priyaka,[1] Śilīndhra,[2] Nīla,[3] Kuṭaja,[4] Arjuna,[5] and Ketaka[6] trees, it is lovely on account of the dances of the rejoicing peacocks and the screams of the delightful *Cakora* birds (the Greek partridges), and the forests look glorified by the beauty of green grass, swarms of (red velvety) insects and the rainbow.

ŚARADA OR AUTUMN

24. The autumn season, in which the beauty of the Moon is restored, is like a sportive young lady, whose (lotus-like) face is worshipped by the humming (lit. prayer) of the black bees (*pada* or *ṣaṭpada*) whose lovely eyes are the newly opened lotuses, whose voice is the sweet sound of the swans, who is adorned with the necklaces of pearls in the form of the vast multitude of twinkling lovely stars and who is endowed with proficiency in all arts.

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1. Priyaka is either Kadamba (*Anthocephalus cadamba*) or Vijayasāra (*Pterocarpus marsupium* or Indian Kino tree). Kadamba is well known. Vijayasāra (also known as Asana Raktacandana) is a large tree with yellowish white flowers. In the present context Priyaka means Kadamba and not Vijayasāra, because Kadamba flowers in the rainy season (Varṣā) whereas Vijayasāra flowers in the autumn season (Śarada).
2. Śilīndhra means (i) Plantain or (ii) the flower of the plantain tree. It also means a mushroom or fungus, but this meaning does not seem to be intended here.
3. Nīla is *Indigofera tinctoria*, flowering in Aug.-Jan. It may also be identified with Nīlavṛkṣa or *Cryptocarya wightiana*, which is a large tree with stout branches, leaves like the mango but large and broader, flowers yellowish and in large bunches, met with in the western ghats from Kanara southwards and in Ceylon. For details concerning this tree and its uses see *The Wealth of India* (A Dictionary of Indian Raw Materials and Industrial Products), Raw Materials vol. II, Chief Editor—B. N. Sastri, C. S. I. R., Delhi 1950, p. 385, and *Flowering Plants of Travancore* by M. Rama Rao (Govt. Press, Trivandrum), 1914.

   In case the correct reading is *Nīpa* instead of Nīla then it is a variety of Kadamba (*Anthocephalus*).
4. Kuṭaja is *Holarrhena antidysenterica*. It is also called Girimallikā, Kuraiyā, Kauraiyā, or Kuḍā The Kuṭaja tree bears long leaves and white flowers. Its seeds are called Indrayava.
5. Arjuna is *Terminalia arjuna*. It is a large tree, 60 to 80 feet in height with stem ashy white in colour. It bears tiny flowers of white colour with green sprinkles in bunches.
6. Ketaka is *Pandanus odoratissimus*. It is commonly known as Ketakī of Kevarā.

HEMANTA OR WINTER

25(a-b). In the winter season the Sun is covered with frost, the Priyaṅgu[1] forest is in the distressed state of mind (i. e., in the withering or ruinous condition)[2], at places there are troubles caused by snowfall, and the (burning) fire appears like a blaze of light.

ŚIŚIRA OR COLD SEASON

25(c-d). In the cold season the Sun is dull-rayed, the wind is chilly and due to abundance of frost the sky is dim and impenetrable to the eye-sight, but there is pleasure in the sugarcane juice.[3]

EQUINOXES AND SOLSTICES

26. The equinoxes (*viṣuva*) occur (when the Sun happens to be) at the beginnings of the signs Aries and Libra; the Sun's northerly course (*uttarāyaṇa*) occurs when the Sun is in the six signs beginning with Capricorn; and the Sun's southerly course (*dakṣiṇāyana*) occurs when the Sun is in the six signs beginning with Cancer.[4]

When the Sun is in the beginning of Aries, the equinox is called the vernal equinox (*vasanta-viṣuva*); and when the Sun is in the beginning of Libra, the equinox is called the autumnal equinox (*śarada-viṣuva*).

SAṄKRĀNTIS

27. When the Sun arrives at the beginning of a fixed sign, it is called Viṣṇupada; and when the Sun comes into contact with a sign called Dvitanu, it is called Ṣaḍvandya (or Ṣaḍaśītimukha).[5]

The signs Taurus, Leo, Scorpio and Aquarius are called fixed signs (*sthira-rāśi*) and the signs Gemini, Virgo, Sagittarius and Pisces are called Dvitanu.

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1. Priyaṅgu is *Callicarpa macrophylla*. It is a shrub with bunches of small fragrant flowers. Priyaṅgu is commonly known as Phūla-priyaṅgu (फूल प्रियङ्गु) or Gul-phiranga (गुलफिरंग).
2. प्रिये प्रियङ्गुः प्रियविप्रयुक्ता विपाण्डुतां याति विलासिनीव । (ऋतुसंहारः)
3. प्रचुरगुडविपाकः स्वादुशालीक्षुरम्यः । (ऋतुसंहारः)
4. *Cf. PSi*, iii. 23(a-b), 25; *MSi*, iii. 37.
5. षडशीत्याननं चापनृयुक्कन्याझषौ भवेत् ।
तुलादौ विषुवं विष्णुपदं सिंहालिगोघटे ॥
बृहज्ज्योतिःसारः, पृ. ३६०

The above passage purports to say that the time when the Sun is at the beginning of a fixed sign, is called Viṣṇupadī Saṅkrānti (*saṅkrānti* = Sun's transit into a sign); and the time when the Sun is at the beginning of a Dvi-tanu sign, is called Ṣaḍvandya or Ṣaḍaśītimukha Saṅkrānti.

The statements made in verses 27 and 28 will be clearly understood from the following table.

Table of Saṅkrāntis

Sun's entrance into	is called	Sun's entrance into	is called
1. Aries	Vasanta viṣuva (vernal equinox)	7. Libra	Śarada viṣuva (autumnal equinox)
2. Taurus	Viṣṇupada	8. Scorpio	Viṣṇupada
3. Gemini	Ṣaḍaśītimukha	9. Sagittarius	Ṣaḍaśītimukha
4. Cancer	Dakṣiṇāyana (summer solstice)	10. Capricorn	uttarāyaṇa (winter solstice)
5. Leo	Viṣṇupada	11. Aquarius	Viṣṇupada
6. Virgo	Ṣaḍaśītimukha	12. Pisces	Ṣaḍaśītimukha

SEASONS DEFINED AND NAMED

28. The seasons beginning with Vasanta (Spring) are defined by the Sun's motion through the successive pairs of signs beginning with Aries. The names of the seasons (occurring after Vasanta) are Grīṣma (Summer), Varṣā (Rainy), Śarada (Autumn), Hemanta (Winter), and Śiśira (Cold) respectively, there being six seasons in all.[1]

VEDIC NAMES OF MONTHS

29. The months Caitra etc. are called, (according to the Vedas), Madhu, Mādhava, Śukra, Śuci, Nabhas, Nabhasya, Iśa, Ūrja, Sahas, Sahasya, Tapas and Tapasya, respectively.[2] **The names of the seasons have come down to us since the time of the Vedas.**

1. *Cf. BrSpSi*, xxiii. 7; *SiŚe*, i. 52.
2. *Cf. Jyotiṣa-ratnamālā*, i. 20.

The correspondence of the months Caitra etc. with the Vedic ones will be clear from the following table.

Table of Months and Seasons

Months	Vedic Months	Seasons
1. Caitra	1. Madhu	1. Vasanta
2. Vaiśākha	2. Mādhava	
3. Jyeṣṭha	3. Śukra	2. Grīṣma
4. Āṣāḍha	4. Śuci	
5. Śrāvaṇa	5. Nabhas	3. Varṣā
6. Bhādrapada	6. Nabhasya	
7. Āśvina	7. Iṣa	4. Śarada
8. Kārtika	8. Ūrja	
9. Āgrahāyaṇa	9. Sahas	5. Hemanta
10. Pauṣa	10. Sahasya	
11. Māgha	11. Tapas	6. Śiśira
12. Phālguna	12. Tapasya	

Section 14

Graphical Representation of Shadow

AGRĀ, BHUJA AND *ŚAṄKUTALA* FOR THE SHADOW-CIRCLE

1. As before,[1] the sum or difference of the *śaṅkutala* and the *agrā* (according as they are of like or unlike directions) is the *bhuja*. Multiply that by (the length of) the shadow (of the gnomon) and divide by the corresponding Rsine of the Sun's zenith distance : the result is the *bhuja* (for the shadow-circle) in terms of *aṅgulas*. The sum or difference of that and the *palabhā* is the *agrā* for the shadow-circle.

2. This *agrā* is in terms of *aṅgulas*. The so called *śaṅkutala* for the shadow-circle is the same as *palabhā*. The length of the rising-setting line for the shadow-circle, in terms of *aṅgulas*, should be determined by the usual method.

The circle drawn with radius equal to the shadow of the gnomon is called the shadow-circle.

ALTERNATIVE METHOD

3. Subtract the square of the *agrā* (from the square of the radius) and the square of the *bhuja* from the square of the Rsine of the Sun's zenith distance, and extract the square-root of the two results. The first square-root multiplied by two gives the length of the rising-setting line; the other square-root is the *koṭi* corresponding to the *bhuja*.

4. Multiply the rising-setting line, the *agrā*, the *bhuja* and the *koṭi* by 12 and divide by the Rsine of the Sun's altitude; or else, multiply by the hypotenuse of the shadow and divide by the radius. Then those quantities are reduced to *aṅgulas* (and correspond to the circle of radius equal to the shadow or to the sphere of radius equal to the hypotenuse of shadow).

One can easily see that :

$$\frac{\text{shadow of gnomon}}{\text{Rsin (Sun's z. d.)}} = \frac{12}{\text{Rsin (Sun's alt.)}} = \frac{\text{hypotenuse of shadow}}{\text{R}}.$$

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1. See *supra*, sec. 1, vs. 18.

CONSTRUCTION OF PATH OF SHADOW AND PATH OF GNOMON

5. On the ground, levelled by means of water, draw a circle with radius equal to the length of one's own desired shadow (and therein draw the east-west and north-south lines). Along the north-south line, in its own direction, lay off the midday shadow, from the centre.

6-8. (Then, from the centre, along the east-west line, lay off the *koṭi* of the shadow for the desired time, towards the west as well as towards the east; and from the two points thus obtained lay off the *bhuja* of the same shadow, in the direction contrary to that of the *bhuja*). With the help of (the end points of) the two *bhujas* (thus) laid off in the contrary direction and the tip of the midday shadow draw two fish-figures. This process is to be adopted whether the Sun be in the southern hemisphere or in the northern hemisphere. Then (stretching and) tying two threads one passing through the head and tail of one fish-figure and the other passing through the head and tail of the other fish-figure, keep the leg of the compass at the junction of those threads and with the mouth (of the compass holding chalk or pencil) distinctly draw the circle (representing the path) of the shadow, passing through the three points (two at the ends of the *bhujas* and the third at the tip of the midday shadow). (The tip of) the shadow (of the gnomon) does not leave this circle in the same way as a lady born in a noble family does not leave the customs and traditions of the family.[1] With the remaining points (obtained by laying off the *bhujas* in their own directions and the midday shadow in the reverse direction), one should, in the same way, draw the circle representing the path of the gnomon (which moves in such a way that the tip of the shadow cast by it always falls at the centre of the circle).[2]

The same process has been described by Mallikārjuna Sūri in his commentary on *ŚiDVṛ*, iv. 44-46.

CLARIFICATION

9. When, the Sun being in the northern hemisphere, the *bhuja* is of the southern direction, then the circle representing the path of the shadow should be drawn by laying off the *bhujas* towards the north and the midday shadow too towards the north. The path of the gnomon should be drawn by laying them off towards the south.

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1. *Cf. ŚiDVṛ*, iv. 42-45(a-b), 46(a-b); *SiŚe*, iv. 81, 82; *TS*, iii. 42(c-d)-47.
2. *Cf. ŚiDVṛ*, iv. 45(c-d), 46(c-d); *SiŚe*, iv. 83.

10. In case the midday shadow of the gnomon is of the southern direction, then the circle representing the path of the shadow should be drawn through the points lying at the tip of the midday shadow (laid off in its own direction) and at the ends of the *bhujas* laid off in the directions contrary to their own. The path of the gnomon should be drawn through the points obtained by laying off the midday shadow in the contrary direction and the *bhujas* in their own directions.

11. When the Sun is in the southern hemisphere, the circle representing the path of shadow should be drawn through the points lying at the tip of the midday shadow falling northwards and at the ends of the *bhujas* laid off northwards. The path of the gnomon should be drawn through the points obtained by laying off the midday shadow as well as the *bhujas* towards the south.

SHADOWS IN VARIOUS DIRECTIONS

12-13(a-b). Where the direction-lines are seen to meet the circle representing the path of shadow, between that point and the gnomon lie the shadows (of those directions). When the Sun is on the prime vertical, the shadow is called the east-west shadow; when the Sun is in a corner direction, the shadow is called the corner shadow; and when the Sun is on the meridian, the shadow is called the midday shadow.

ABSENCE OF EAST-WEST SHADOW, CORNER SHADOW AND MIDDAY SHADOW

13(c-d). When the Sun is in the six signs beginning with the sign Libra, the Sun does not cross the prime vertical; so is also the case when the Sun's meridian zenith distance is north.

14. When the *agrā* of south direction exceeds the Rsine of one and a half signs, the Sun does not enter a corner direction (i. e., a corner vertical circle).

When the Sun is in the northern hemisphere and the Sun's meridian zenith distance (*khākṣa*) is equal to 90 (degrees), the Rsine of the Sun's zenith distance at midday equals the radius.

MOTION OF GNOMON AND SHADOW-TIP AND THAT OF THE SUN'S *ŚAṄKU*

15. The motion of the (moving) gnomon, which keeps its shadow-tip permanently at the centre, resembles that of the Sun. The shadow-tip

of the gnomon, set up at the centre, has its motion opposite to that of the Sun.

OBSERVATION OF SUN THROUGH AN APERTURE IN THE ROOF, OR IN OIL, MIRROR OR WATER, OR THROUGH A HOLLOW TUBE.

16. By means of *Parilekha* ("diagram") inside a house one may, by breaking open the roof of the house in the direction of the hypotenuse of shadow and keeping one's eye at the confluence of the direction-lines, see the Sun or the desired planet.

17. One may, even during the day, see the revolving Sun, in oil, mirror or water placed on the path of the gnomon, by keeping one's eye at the (upper) extremity of the gnomon (set up at the centre).

18. Or, one may see the heavenly body moving along the path of the gnomon by keeping one's eye at the (lower) extremity of the bamboo directed towards the centre (and held along the hypotenuse of shadow). Or, one may, through the hole inside a (hollow) gnomon, see the Sun as if clinging to (the other extremity of) it.

The observations contemplated in the above stanzas have been described by Brahmagupta (*BrSpSi*, vii. 15-17; x. 57-61), Lalla (*ŚiDVr*, iv. 47-48), Śrīpati (*SiŚe*, iv. 84-86) and Bhāskara II (*SiŚi*, I, iii. 105-8), and in *Sūrya-siddhānta* (vii. 16-17). But they have been more fully described in the commentaries.

OBSERVATION OF PLANETS AND STARS

19. Similarly, one should accomplish the observation of the planets and the zodiacal asterisms, of the hunter-star (i. e., dog-star or Sirius), Agastya (i. e., Canopus) and the Saptarṣi (i. e., Great Bear) in oil etc. with the help of their true declinations.

EQUINOCTIAL MIDDAY SHADOW AND ITS HYPOTENUSE IN THE SHADOW-CIRCLE

20-21(a-b). One should lay off the *agrā* (for the shadow-circle) like the *bhuja* (i. e., along the perpendicular to the east-west line), (towards the east as well as the west); the line which joins the two points (thus obtained) is the rising-setting line. Between this line and (the moving gnomon on the circumference of) the shadow-circle lie the *aṅgulas* of the equinoc-

tial midday shadow, and between that line and the upper extremity of the gnomon lies the hypotenuse of the equinoctial midday shadow.[1]

CONSTRUCTION OF THE SHADOW CIRCLE

21(b)-23. Or, Set down a point for the centre (of the circle). Whatever point is thus chosen as the centre, taking that as the centre draw a neat circle beariug the marks of the cardinal points and the divisions of the *ghaṭikās* (on its circumference). The lines drawn (from those points) by chalk should be made to reach the centre. Now draw the locus of the (moving) gnomon as also the locus of (the tip of) the (moving) shadow. Since in this neat circle the gnomon and the shadow-tip trace out their loci accurately, so the declination etc. computed from (the position of) the Sun for that time give their true values.

ASTRONOMICAL PARAMETERS BY OBSERVATION

24. Carefully determine the positions of the planets (for any desired day) and also for the day next to that. Their differences, reduced to minutes of arc, multiplied by the number of (civil) days in a *yuga* and divided by 21600 give the revolutions of the planets in a *yuga*.[2]

Let d be the difference, in minutes of arc, between the positions of a planet for two consecutive days, and c the number of civil days in a *yuga*; then the number of revolutions of that planet in a *yuga* $= d \times c / 21600$.

This follows from the fact that the motion of a planet in one civil day in terms of minutes, viz. a, is equal to

$$\frac{\text{revolutions of the planet in a } yuga \times 21600}{c}.$$

25. The Sun is determined from the conjunction of the Sun and the Earth; the Moon from the conjunction of the Moon and the Sun; all the planets from the conjunction of the Moon and the planets; the polar latitudes and the polar longitudes of the stars from the conjunction of the planets and the stars.[3]

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1. *Cf. SiŚe*, iv. 87.
2. *Cf. BrSpSi*, xix. 12.
3. *Cf. Ā*, iv. 48; *MSi*, xi. 11(a-b).

26. Since the true motion of the planets cannot be truly ascertained without the help of the apogees and the ascending nodes of the planets, therefore their *kṣepas* (for the beginning of Kaliyuga) as well as their revolutions in his own life have been stated by Brahmā.

The revolution-numbers of the apogees and the ascending nodes of the planets during the life of Brahmā have been stated in chap. I, sec. 1, vss. 16-17, and their *kṣepas* for the beginning of Kaliyuga in chap. I, sec. 4, vss. 56-62.

Section 15
Examples on Chapter III

1. It is by no means possible for the astronomers to know the numerous questions that can be asked on "Three Problems" (*Tripraśna*), so I shall set forth here a small chapter on examples based on the methods pertaining to the shadow of the gnomon, the time in *nāḍīs* and the (various) Rsines, in clear words with clear meanings, hearing which, in the royal courts, the ignorant astronomers, strangers to spherics, become depressed and dejected due to the violent upheaval of the heap of mental dirt.[1]

2. One who tells the cardinal points from the entrance (into a circle in the forenoon) and the exit (out of the circle in the afternoon) of the shadow (of the gnomon set up at the centre of the circle) or with the help of three shadows (of the gnomon); or, one who tells the locus of (the tip of) the shadow (of a gnomon) without the help of the Sun's longitude, the (Sun's) declination and the latitude (of the local place) is an astronomer.[2]

3. One who knows the directions with the help of the (Sun's) declination, the tip of the (gnomonic) shadow and the degrees of the (local) latitude;[3] one who knows the midday shadow from the locus of (the tip of) the (gnomonic) shadow; and one who finds out the equinoctial midday shadow for his own station by knowing the Sun from the midday shadow is a *Gaṇaka*.

4. One who by observing the Sun at its rising knows (the longitude of) the Sun, or, by the *Yaṣṭi*-process knows the entire measure of the Sun's altitude; or, one who from the knowledge of the Rsine of the latitude knows the hypotenuse of the equinoctial midday shadow as well as the equinoctial midday shadow is proficient in the shadow-processes.

5. One who, knowing the equinoctial midday shadow, finds out in various ways the Rsines of the latitude and the colatitude; or one who,

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1. A similar statement has been made by Bhāskara II. See *SiŚi*, II, *praśnādhyāya*, 1.

2. *Cf. BrSpSi*, xv. 1, 2.

3. *Cf. BrSpSi*, xv. 3(a-b).

knowing the midday shadow (of the gnomon) and the Sun, tells the degrees of the local latitude is versed in astronomy.[1]

6. One who, from the Rsine of the codeclination, the Rsine of the declination, the Rsine of the ascensional difference, the *agrā*, and the earthsine, etc., can mutually determine them in numerous ways; or, one who determines in a variety of ways the shadow (of the gnomon) due to the midday Sun is the foremost amongst the astronomers.

7. He who knows (how to find) the *samaśaṅku*, *yāmyottaraśaṅku*, *abhīṣṭaśaṅku*, and *vidikśaṅku* for his local place, each even by a single method differing from those stated by me, is an astronomer.[2]

8. One who determines in numerous ways the desired shadow, the prime vertical shadow, or the corner shadow (of the gnomon), or, with the help of them, finds in many ways the position of the Sun (at that time) or the corresponding time, is versed in astronomy.

9. One who, knowing the (Sun's) ascensional difference and the degrees of the (local) latitude, obtains the position of the Sun; or, from the given ascensional difference (of the Sun), finds the equinoctial midday shadow; or, from the equinoctial midday shadow for the local place determines the (Sun's) ascensional difference, has true knowledge of what is taught in the chapter on "Three Problems" (*Tripraśna*).[3]

10. One who, knowing the Rsine of the Sun's altitude and the Rsine of the altitude of the meridian ecliptic point, obtains the longitude of the rising point of the ecliptic by a method differing from that given in the *Tantra* ("astronomical work") written by me is the foremost amongst the astronomers here.

11. One who, without knowing the times of rising of the signs, finds out the *nāḍīs* of oblique ascension intervening between the rising point of the ecliptic and the Sun, or determines the Rsine of the (Sun's) altitude or zenith distance at midday with the help of the earthsine, or obtains the longitude of the Sun from the shadow of the gnomon is versed in astronomy.

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1. Same example occurs in *BrSpSi*, xv. 4.
2. *Samaśaṅku* = Rsine of the Sun's prime vertical altitude, *yāmyottaraśaṅku* = Rsine of the Sun's meridian altitude, *abhīṣṭaśaṅku* = Rsine of the Sun's altitude for the desired time, *vidikśaṅku* = Rsine of the Sun's corner altitude.
3. Same example occurs in *BrSpSi*, xv. 14.

12. The person who, in the circle constructed with the given shadow as the radius, determines the path traced out by the shadow with the help of the (instantaneous) *bhuja* and *koṭi* and the *aṅgulas* of the midday shadow; finds the *ghaṭīs* (elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon) during the day (corresponding to the given shadow); and also finds the shadow (corresponding to the given *ghaṭīs* elapsed or to elapse) is an astronomer with clear mind (lit. free from mental pollution).

13. One who, breaking open (a hole in) the roof of the house shows the Sun through it, or shows the Sun in oil, mirror or water, or at the extremity of a bamboo is versed in astronomy.

14. The Sun having arrived at the end of the sign Gemini, the duration of the night at a (certain) place is (found to be) 25 *ghaṭīs*. Tell me, O good friend, the measure of the latitude there, if you have put in labour in the study of the *Vaṭeśvara-tantra*.

This example is practically the same as set in *SiŚe*, xx. 10. The only difference is that in place of the latitude, the equinoctial midday shadow is asked there.

15. (At a certain place) the time of rising of Aries is 2 *ghaṭīs*. Say what the people (there) mention as the length of day when the Sun reaches the fag end of Gemini. Also say the degrees of latitude (of that place).

16. (At a certain place) where the degrees of the latitude are 20, the *aṅgulas* of the shadow (of the gnomon) cast by the Sun at midday are 9. (Tell me), O friend, what is the position there of the Sun, the store-house of brilliant heat and light and competent to destroy thickset darkness, at the middle of the day

17 The Sun having arrived at the middle of the sign Libra, the midday shadow (of the gnomon) measures 10 *aṅgulas*. Tell me quickly what the measure of the latitude is there.

18. At a certain place the degrees of the latitude amount to 36; say quickly the measure of the shadow (of the gnomon) there when the Sun has reached the end of Gemini and 2 *ghaṭīs* of the day have elapsed.

19. Quickly tell me after full deliberation, if you have studied mathematics and spherics at the teacher's house, how much is the latitude at the place where the Sun, having traversed one-third of Libra, rises towards the south-east (point of the horizon).

20. The degrees of the latitude (of the place) being 36, say how much of the zodiac (lit. the circle of asterisms) has been traversed by the Sun on the day when the Sun's *agrā* for sunrise amounts to 24 degrees north.

21. The Sun having reached the highest point (i. e , the *ucca*), say how much does the shadow extend on the (shadow) board towards the south-east direction at a place where the latitude amounts to 30 degrees.

22. The degrees of the latitude being equal to 27, say how long does the Sun, who has traversed one-third of Scorpio, cast its shadow when it happens to be towards the south-east and the south directions.

23. The Sun having gone to the north (of the equator), the measure of the shadow (of the gnomon) is 16 (*aṅgulas*), the measure of the *bhuja* (of the shadow) is 12 plus 1/3 (*aṅgulas*); and the *ghaṭīs* of the hour angle are 6. One who quickly calculates (from this data) the position of the Sun and the degrees of the local latitude is indeed one who, in this world, correctly and clearly knows mathematics as well as spherics.

24. The Sun having gone towards the south-east direction, the *ghaṭīs* of the hour angle amount to 6 and (the length of) the shadow (of the gnomon) is equal to 16 (*aṅgulas*). One who, applying the methods of spherical astronomy, tells (from this data) the measure of the latitude (of the place) and the *bhuja* of the Sun is free from intellectual impurities (i. e., ignorance).

This example occurs in the *Siddhānta-śekhara* of Śrīpati also. See *SiŚe*, xx. 13.

25. Say the latitude (of the place) where the Sun, resembling liquid-gold and lying at the end of Aries, rises towards the north-east of the horizon; also calculate (the same) if he be situated at the end of Taurus or Gemini. Also say, along with the rationale based on spherics, what latitude of that place is where the Sun, lying at the end of Libra, Scorpio or Sagittarius, rises towards the south-east point of the horizon.

A similar example occurs in the *Siddhānta-śekhara* of Śrīpati also. See *SiŚe*, xx. 7.

26. I calmly bow down to the feet of that more intelligent person who, observing on the horizon the Sun's setting disc, resembling a golden pitcher being submerged into the worldly ocean, and also observing the Pole-star, tells the position of the Sun and the part of the (current) *yuga* that has elapsed.

A similar example occurs in the *Siddhānta-śekhara* of Śrīpati. See *SiŚe*, xx. 9.

27. The Sun having gone to the south-east quarter, the *bhuja* and the *koṭi* (of the shadow) are each 12 *aṅgulas* in length, and the degrees of the latitude (of the local place) are 27. Quickly say, if you know, the position of the Sun at that place.

28. Quickly say, if you are versed in the ocean of the *tantra*, the position of the Sun at the place where the latitude is 36 degrees and the Sun's prime vertical shadow, 25 (*aṅgulas*).

29. Say in succession the latitude and the position of the Sun at the place where the *asus* of the hour angle of the Sun, situated on the prime vertical, are 2400 and the *aṅgulas* of the (prime vertical) shadow, 9.

This example occurs in the *Siddhānta-śekhara* of Śrīpati also. See *SiŚe*, xx. 16. It may be mentioned that 7 minus 1/3 *naḍīs* of Śrīpati's example are equivalent to 2400 *asus* of Vaṭeśvara's example above.

30. Quickly say how much is stated by the people to be the Rsine of latitude of the place where the Sun, bearing the reddish glow of saffron, having arrived at the end of Gemini, stops setting (below the horizon).

This example too occurs in the *Siddhānta-śekhara* of Śrīpati. See *SiŚe*, xx. 17(a-b).

31. Tell me, O learned (astronomer), the measure of the Rsine of the altitude of the Sun, lying midway between the south-east and the east directions, at the place where the degrees of the latitude are 34, the Sun being situated at the end of Gemini.

32. The latitude of a place is 70 degrees. Think over (and say) when the Sun rises at that place (so as to make a perpetual day) and after how much time thereafter it goes to set, and also what the longitude of the Sun then is.

33. If the Sun having once risen (above the horizon) remains (continuously) visible for 150 days, tell us quickly what is the Rsine of the latitude (at that place) and what is the longitude traversed by the Sun at that time (of rising).

This example with 150 days replaced by 100 days is found to occur in the *Siddhānta-śekhara* of Śrīpati. See *SiŚe*, xx. 18. The reading *sakalaṁ śataṁ* is also possible. In that case both the examples will become identical.

34. I bow down to the feet of one with excellent intellect who, not making use of the *Siddhānta* composed by me, computes the various elements of the lunar and solar eclipses, without using the degrees of the declination and the latitude.

This problem is solved in chap. V, sec. 6, below.

35-36. He who, observing (the gnomonic) shadow cast by the Sun, finds the longitudes of the Moon and the Sun and computes *karaṇa*, *vāra*, *nakṣatra* and *tithi*, one after another, and, without knowing the longitudes of the Moon and the Sun, determines the omitted lunar days and the intercalary months as well as the longitudes of Mars etc. stands pre-eminent on account of the wealth of his excellent fame spread around.

Chapter IV
LUNAR ECLIPSE

INTRODUCTION

1. In the world people in general have belief in the eclipses of the Moon and the Sun. So I (first) proceed to give out everything pertaining to the eclipse of the Moon, its commencement etc., lucidly and briefly.

DISTANCE OF A PLANET

2. Multiply the orbit of a planet by 10000 and divide the resulting product by 62832 : then are obtained the *yojanas* of the planet's distance.

Or, the radius of a planet's own orbit may be obtained by multiplying the circumference by 625 and dividing by 3927.[1]

(1) $$\text{Planet's distance} = \frac{\text{planet's orbit} \times 10000}{62832}$$

(2) $$\text{Planet's distance} = \frac{\text{planet's orbit} \times 625}{3927}.$$

In formulating these rules, Vaṭeśvara has adopted Āryabhaṭa I's value of π, viz.[2]

$$\pi = \frac{62832}{20000} = \frac{3927}{1250} = 3{\cdot}1416.$$

DISTANCES OF SUN AND MOON

3. Or, divide the distance of the asterisms by 60 : the result is the distance of the Sun. And multiply the radius (in minutes, i. e., 3737·7) by 10 : the result is the distance of the Moon.

4. Multiply the distance of the Sun and the Moon by the orbits of one another and divide by their own orbits : the results are the distances of the Moon and the Sun, respectively.

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1. *Cf. BrSpSi*, xxi. 31 (a-b); *ŚiDVr*, v. 3(c-d). Also see *SiŚe*, v. 4 (a-b), where $\sqrt{10}$ has been taken as the value of π.
2. See *Ā*, ii. 10.

5. 459585 (*yojanas*) is the distance of the Sun, and 34377 (*yojanas*) the distance of the Moon. This is the distance between the Earth and the planet (Sun or Moon).

$$\text{Sun's distance} = \frac{\text{distance of asterisms}}{60} = 459585 \textit{ yojanas}$$

$$\text{Moon's distance} = 3437{\cdot}7 \times 10 \textit{ yojanas} = 34377 \textit{ yojanas}.$$

Also

$$\text{Sun's distance} = \frac{\text{Moon's distance} \times \text{Sun's orbit}}{\text{Moon's orbit}}$$

$$\text{Moon's distance} = \frac{\text{Sun's distance} \times \text{Moon's orbit}}{\text{Sun's orbit}}.$$

The above-mentioned values of the Sun's and the Moon's distances are the same as those given by Āryabhaṭa I and his followers.[1]

TRUE DISTANCES OF SUN AND MOON

6-7(a-b). That (distance between the Earth and the planet) multiplied by the *aviśeṣakalākarṇa* (i.e., distance in minutes obtained by iteration) and divided by the radius gives the true distance (in *yojanas*).[2] Or, the same distance (of the Sun or Moon) multiplied by the mean daily motion (thereof) and divided by its own true daily motion is declared by the learned as the true distance (in *yojanas*) of the Sun or Moon.[3]

$$\text{Sun's true distance} = \frac{\text{Sun's mean distance} \times \text{Sun's } \textit{aviśeṣakalākarṇa}}{R}$$

$$\text{Moon's true distance} = \frac{\text{Moon's mean distance} \times \text{Moon's } \textit{aviśeṣakalākarṇa}}{R}$$

or

$$\text{Sun's true distance} = \frac{\text{Sun's mean distance} \times \text{Sun's mean daily motion}}{\text{Sun's true daily motion}}$$

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1. See *MBh*, v. 2; *LBh*, iv. 2; *ŚiDVr*, v. 4. For similar values, see *SiŚe*, v. 7.
2. *Cf. BrSpSi*, xxi. 31 (c-d); *ŚiDVr*, v. 5 (a-b); *SiŚe*, v. 4 (c-d); *SiŚi*, I, v. 5 (a-b).
3. *Cf. ŚiDVr*, v. 5 (c-d).

$$\text{Moon's true distance} = \frac{\text{Moon's mean distance} \times \text{Moon's mean daily motion}}{\text{Moon's true daily motion}}.$$

It can be easily seen that these two sets of formulae are equivalent.

DIAMETERS OF SUN AND MOON

7(c-d). **4412 (*yojanas*) is the diameter of the Sun; 330 (*yojanas*) that of the Moon.**

According to Āryabhaṭa I and his followers,[1]

Sun's diameter = 4410 *yojanas*

Moon's diameter = 315 *yojanas*.

According to Śrīpati[2] and Bhāskara II,[3]

Sun's diameter = 6522 *yojanas*

Moon's diameter = 480 *yojanas*,

Śrīpati and Bhāskara II's *yojana* being 2/3 times that of Āryabhaṭa I.

According to the *Sūrya-siddhānta*[4]

Sun's diameter = 6500 *yojanas*

Moon's diameter = 480 *yojanas*.

According to Vaṭeśvara, as also according to Āryabhaṭa I and his followers, 1′ of the Sun's orbit corresponds to 133·688 *yojanas* and 1′ of the Moon's orbit corresponds to 10 *yojanas*. Therefore, according to Vaṭeśvara,

Sun's diameter = 33′ approx.

Moon's diameter = 33′.

According to Brahmagupta, Śrīpati, Bhāskara II as also according to the author of the *Sūrya-siddhānta*, 1′ of the Sun's orbit corresponds to 200·5

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1. See *Ā*, i. 7; *MBh*, v. 4; *ŚiDVr*, v. 6 (a-b); *TS*, iv. 10 (a-b).
2. See *SiŚe*, v. 3.
3. See *SiŚi*, I, v. 5 (c-d).
4. iv. 1.

yojanas and 1′ of the Moon's obit corresponds to 15 *yojanas*. Thus according to these astronomers, the mean angular diameters of the Sun and the Moon are as shown in the following table.

Mean angular Diameters of Sun and Moon

	Āryabhaṭa I	Brahmagupta, Śrīpati and Bhāskara II	*Sūrya-siddhānta*	Modern
Sun's diameter	33′ approx.	32′ 31″ approx.	32′ 25″ approx.	32′ 2″
Moon's diameter	31′30″	32′	32′	31′8″

The above table shows that the Hindu values are in each case greater than the modern values.

DIAMETER OF EARTH'S SHADOW

8. The Sun's distance multiplied by the Earth's diameter and divided by the Sun's diameter minus the Earth's diameter, gives the length of the Earth's shadow. That (length of the Earth's shadow) diminished by the Moon's distance, when multiplied by the Earth's diameter and divided by the length of the Earth's shadow, certainly gives the diameter of the Shadow (at the Moon's distance).[1]

9-10(a-b). Or, divide the difference between the diameters of the Sun and the Earth by the Sun's distance and multiply by the Moon's distance, and subtract the result from the Earth's diameter;[2] **or, multiply the Earth's diameter by the Moon's distance and divide by the length of the Earth's shadow, and subtract the resulting quotient from the Earth's diameter : the remainder obtained (in each case) is the diameter of Shadow (at the Moon's distance).**

(1) $$\text{Length of Earth's shadow} = \frac{\text{Sun's distance} \times \text{Earth's diameter}}{\text{Sun's diameter} - \text{Earth's diameter}}$$

(2) Diameter of Shadow

$$= \frac{\text{Earth's diameter} \times (\text{length of Earth's shadow} - \text{Moon's distance})}{\text{length of Earth's shadow}}$$

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1. *Cf. BrSpSi*, xxiii. 8-9; *ŚiDVr*, v. 7.
2. *Cf. BrSpSi*, xxi. 33; *SiŚe*, v. 5; *SiŚi*, I, v. 6.

(3) Diameter of Shadow = Earth's diameter

$$-\frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times \text{Moon's distance}}{\text{Sun's distance}}$$

(4) Diameter of Shadow = Earth's diameter

$$-\frac{\text{Earth's diameter} \times \text{Moon's distance}}{\text{length of Earth's shadow}}.$$

For the rationales of (1) and (2), the reader is referred to my notes on *Ā*, iv. 39-40 or *MBh*, v. 71-73. The rationales of (3) and (4) may be easily obtained from the figures given there.

TRUE ANGULAR DIAMETERS OF SUN, MOON AND SHADOW

Method 1

11. Severally multiply the diameters of the Sun, the Moon, and the Shadow by the radius and divide the resulting products by the true distances of the Sun, the Moon and the Moon, respectively ; the results are their diameters in terms of minutes.[1]

(1) Sun's true diameter $= \dfrac{\text{Sun's diameter in } yojanas \times R}{\text{Sun's true distance in } yojanas}$ mins.

(2) Moon's true diameter $= \dfrac{\text{Moon's diameter in } yojanas \times R}{\text{Moon's true distance in } yojanas}$ mins.

(3) True diameter of Shadow $= \dfrac{\text{Diameter of Shadow in } yojanas \times R}{\text{Moon's true distance in } yojanas}$ mins.

Method 2

12. Divide the Earth's diameter and the difference between the diameters of the Earth and the Sun by the distances of the Moon and the Sun (respectively) and multiply (both) by the radius. The difference of the results (obtained) gives the measure (i.e., diameter) of the Shadow in terms of minutes.[2]

$$\text{Diameter of Shadow in terms of minutes} = \frac{\text{Earth's diameter} \times R}{\text{Moon's distance in } yojanas} - \frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times R}{\text{Sun's distance in } yojanas}.$$

1. *Cf. BrSpSi*, xxi. 34; also xxiii. 9(c-d); *ŚiDVr*, v. 8; *SiŚe*, v. 6; *SiŚi*, I, v. 7.
2. *Cf. BrSpSi*, xxiii. 10.

Rationale. Multiplying formula (4), stated above under vss. 8-10(a-b), by R and dividing by the Moon's distance in *yojanas,* we get

Diameter of Shadow in terms of minutes

$$= \frac{\text{Earth's diameter} \times \text{R}}{\text{Moon's distance in } yojanas} - \frac{\text{Earth's diameter} \times \text{R}}{\text{length of Earth's shadow}}$$

$$= \frac{\text{Earth's diameter} \times \text{R}}{\text{Moon's distance in } yojanas} - \frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times \text{R}}{\text{Sun's distance in } yojanas}$$

on substituting the value of "length of Earth's shadow".

Note. This formula may also be stated as :

Diameter of Shadow in terms of minutes

$$= \frac{\text{Earth's diameter} \times \text{R}}{\text{Moon's distance in } yojanas}$$

$$- \frac{\text{Earth's semi-diameter} \times \text{R} \times (\text{Sun's diameter} - \text{Earth's diameter})}{\text{Sun's distance in } yojanas \times \text{Earth's semi-diameter}}$$

$$= \frac{\text{Moon's daily motion} \times 2}{15}$$

$$- \frac{\text{Sun's daily motion} \times (\text{Sun's diameter} - \text{Earth's diameter})}{15 \times \text{Earth's semi-diameter}},$$

because

$$\text{Moon's horizontal parallax} = \frac{\text{Moon's daily motion}}{15}$$

and

$$\text{Sun's horizontal parallax} = \frac{\text{Sun's daily motion}}{15}.$$

These forms for the diameter of Shadow in terms of minutes are due to Brahmagupta.[1]

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1. See *BrSpSi,* xxiii. 10, 11.

Method 3

13. Or, divide the motion-correction by the own mean daily motion and multiply by 33; add that to or subtract that from 33 as in the case of daily motion. The results thus obtained are stated to be the measures (diameters) of the Sun and the Moon, in terms of minutes.

(1) $$\text{Sun's diameter} = 33 \pm \frac{33 \times \text{Sun's motion-correction}}{\text{Sun's mean daily motion}} \text{ mins.}$$

(2) $$\text{Moon's diameter} = 33 \pm \frac{33 \times \text{Moon's motion-correction}}{\text{Moon's mean daily motion}} \text{ mins.,}$$

$+$ or $-$ sign being taken according as the Sun or Moon is in the half anomalistic orbit beginning with the sign Cancer or in that beginning with Capricorn.

Since, in the case of the Sun and Moon,

true daily motion = mean daily motion $\pm$ motion-correction,

therefore the above formulae may be written as:

$$\text{Sun's diameter} = \frac{33 \times \text{Sun's true daily motion}}{\text{Sun's mean daily motion}} \text{ mins.}$$

$$\text{Moon's diameter} = \frac{33 \times \text{Moon's true daily motion}}{\text{Moon's mean daily motion}} \text{ mins.}$$

These formulae are stated in the next verse.

Method 4

14. Or, multiply the true daily motion by 33 and divide by the mean daily motion: then is obtained the measure (diameter) (of the Sun or Moon). Also, the true daily motions (of the Sun and the Moon) divided by 20 and 24 (respectively), and in the case of the Sun multiplied by 11, give the true values of their diameters.

(1) $$\text{Sun's diameter} = \frac{33 \times \text{Sun's true daily motion}}{\text{Sun's mean daily motion}} \text{ mins.}$$

(2) $$\text{Moon's diameter} = \frac{33 \times \text{Moon's true daily motion}}{\text{Moon's mean daily motion}} \text{ mins.}$$

or,

(3) $\text{Sun's diameter} = \dfrac{11 \times \text{Sun's true daily motion}}{20}$ mins.

(4) $\text{Moon's diameter} = \dfrac{\text{Moon's true daily motion}}{24}$ mins.

According to Brahmagupta,[1] Āryabhaṭa II[2] and Śrīpati[3] :

(5) $\text{Sun's diameter} = \dfrac{11 \times \text{Sun's true daily motion}}{20}$ mins.

(6) $\text{Moon's diameter} = \dfrac{10 \times \text{Moon's true daily motion}}{247}$ mins.

According to Lalla[4]:

(7) $\text{Sun's diameter} = \dfrac{11 \times \text{Sun's true daily motion}}{20}$ mins.

$= \dfrac{4 \times \text{Sun's true daily motion}}{17}$ *aṅgulas*

(8) $\text{Moon's diameter} = \dfrac{11 \times \text{Moon's true daily motion}}{272}$ mins.

$= \dfrac{10 \times \text{Moon's true daily motion}}{577}$ *aṅgulas*,

where 1 *aṅgula* = $2\frac{1}{3}$ mins.

According to Bhāskara II[5] :

(9) $\text{Sun's diameter} = \dfrac{11 \times \text{Sun's true daily motion}}{20}$ mins.

(10) $\text{Moon's diameter} = \dfrac{3 \times \text{Moon's true daily motion}}{74}$ mins.

Rationale. We know that

$$\text{Sun's true diameter} = \frac{\text{Sun's mean diameter} \times R}{\text{Sun's true distance}} \text{ mins.}$$

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1. See *BrSpSi*, iv. 6 (a-b); *KK*, I, iv. 2 (a-b).
2. See *MSi*, v. 5 (c-d).
3. See *SiŚe*, v. 9 (a-b).
4. See *ŚiDVr*, v. 9 (a-b); vii. 3.
5. See *SiŚi*, I, v. 8 (a-b).

and also that

$$\frac{R}{\text{Sun's true distance in mins.}} = \frac{\text{Sun's true daily motion}}{\text{Sun's mean daily motion}}.$$

Therefore,

$$\text{Sun's true diameter} = \frac{\text{Sun's mean diameter in mins.} \times \text{Sun's true daily motion}}{\text{Sun's mean daily motion}} \text{ mins.}$$

$$= \frac{33 \times \text{Sun's true daily motion}}{\text{Sun's mean daily motion}} \text{ mins.},$$

because Sun's mean diameter in minutes $= \frac{4412}{133 \cdot 7} = 33$ minutes approx.

(1′ of Sun's orbit $= 133 \cdot 7$ *yojanas*.)

In the case of the Moon, 1′ of Moon's orbit = 10 *yojanas* and likewise Moon's mean diameter in minutes $= \frac{330}{10} = 33$ minutes. Hence proceeding as above, we have

$$\text{Moon's true diameter} = \frac{33 \times \text{Moon's true daily motion}}{\text{Moon's mean daily motion}} \text{ mins.}$$

Method 5

15. Multiply the Moon's daily motion by 2 and divide (the resulting product) by 15; and multiply the Sun's daily motion by 41 and divide (the resulting product) by 113. The difference of the two results obtained also gives the value of the diameter of Shadow in terms of minutes.[1]

$$\text{Diameter of Shadow} = \frac{2 \times \text{Moon's daily motion}}{15} - \frac{41 \times \text{Sun's daily motion}}{113} \text{ mins.}$$

According to the *Karaṇasāra* of Vaṭeśvara, says Al-Bīrūnī[2]:

$$\text{Diameter of Shadow} = \frac{4 \times \text{Moon's daily motion} - 13 \times \text{Sun's daily motion}}{30}.$$

1. Similar rules are found in *BrSpSi*, iv. 6 (c-d); *KK*, I, iv. 2 (c-d); *ŚiDVr*, v. 9 (c-d); *MSi*, v. 6; *KP*, iv. 2 (c-d); *SiŚe*, v. 9 (c-d); *SiŚi*, I, v. 9.
2. See *India*, II, p 79.

According to Brahmagupta[1], Śrīpati[2] and Āryabhaṭa II[3],

$$\text{Diameter of Shadow} = \frac{2 \times \text{Moon's daily motion}}{15} - \frac{5 \times \text{Sun's daily motion}}{12}.$$

The same formula is given by Lalla[4] but the value given by him is in *aṅgulas*, obtained by dividing the value in minutes by $2\frac{1}{3}$.

Brahmagupta[5] gives also the following general formula :

Diameter of Shadow in minutes

$$\doteq \frac{[\text{Moon's daily motion} \times \text{Earth's diameter} - \text{Sun's daily motion (Sun's diameter} - \text{Earth's diameter)}]}{15 \times \text{Earth's semi-diameter}}.$$

Method 6

16. Multiply 25′7″ by the radius and divide by the Sun's *manda-karṇa*; and multiply 105′24″ by the radius and divide by the Moon's *mandakarṇa*. The difference of the results thus obtained is the measure of the diameter of Shadow (in terms of minutes etc.).

$$\text{Diameter of Shadow} = \frac{105'\,24''\,R}{\text{Moon's } \textit{mandakarṇa}} - \frac{25'\,7''\,R}{\text{Sun's } \textit{mandakarṇa}}.$$

Rationale. We have (vide vs. 12 above)

$$\text{Diameter of Shadow} = \frac{\text{Earth's diameter} \times R}{\text{Moon's true distance}} - \frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times R}{\text{Sun's true distance}}$$

$$= \left(\frac{\text{Earth's diameter} \times R}{\text{Moon's mean distance}}\right)\frac{\text{Moon's mean distance}}{\text{Moon's true distance}}$$

$$- \left\{\frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times R}{\text{Sun's mean distance}}\right\}\frac{\text{Sun's mean distance}}{\text{Sun's true distance}}$$

1. See *BrSpSi*, iv. 6(c-d); *KK*, I, iv. 2 (c-d).
2. See *SiŚe*, v. 9 (c-d).
3. See *MSi*, v. 6.
4. See *ŚiDVr*, vii. 4.
5. See *BrSpSi*, xxiii. 11.

$$= \left(\frac{\text{Earth's diameter} \times R}{\text{Moon's mean distance}}\right)\frac{R}{\text{Moon's } mandakarṇa}$$

$$- \left\{\frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times R}{\text{Sun's mean distance}}\right\}\frac{R}{\text{Sun's } mandakarṇa}$$

$$= \frac{105'\,24''\,R}{\text{Moon's } mandakarṇa} - \frac{25'\,7''\,R}{\text{Sun's } mandakarṇa},$$

because, according to Vaṭeśvara,

$$\frac{\text{Earth's diameter} \times R}{\text{Moon's mean distance}} = \frac{1054 \times 3437{\cdot}7}{34377} \text{ mins.} = 105'\,24''$$

and

$$\frac{(\text{Sun's diameter} - \text{Earth's diameter}) \times R}{\text{Sun's mean distance}} = \frac{(4412-1054) \times 3438}{459585} \text{ mins}$$
$$= 25'\,7''.$$

Methods 7 and 8

17. Or, multiply (the same numbers, 105′24″ and 25′7″) by the own motion-corrections (of the Moon and the Sun) and divide by their own mean daily motions, and apply them to the same (numbers) as a positive or negative correction, as in the case of motion-correction : the difference of the two results (thus obtained) is the measure of (the diameter of) the Shadow.

18(a-b). Or, multiply (the same numbers) by the true daily motions and divide by the mean daily motions (of the Moon and the Sun respectively) : the difference (of the two results) is the diameter of the Shadow.

(1) Diameter of Shadow $= \left[105'\,24'' \pm \frac{105'\,24'' \times \text{Moon's motion-correction}}{\text{Moon's mean daily motion}}\right]$

$$- \left[25'\,7'' \pm \frac{25'\,7'' \times \text{Sun's motion-correction}}{\text{Sun's mean daily motion}}\right]$$

(2) Diameter of Shadow $= \frac{105'\,24'' \times \text{Moon's true daily motion}}{\text{Moon's mean daily motion}}$

$$- \frac{25'\,7'' \times \text{Sun's true daily motion}}{\text{Sun's mean daily motion}}.$$

Rationale. (2) follows from the formula of vs. 16 by substituting

$$\text{Sun's } \textit{Mandakarṇa} = \frac{\text{Sun's mean daily motion} \times R}{\text{Sun's true daily motion}}$$

and $$\text{Moon's } \textit{Mandakarṇa} = \frac{\text{Moon's mean daily motion} \times R}{\text{Moon's true daily motion}}.$$

(1) is obtained as follows: Since

true daily motion = mean daily motion ± motion-correction,

therefore from (2) we have

$$\text{Diameter of Shadow} = \frac{105'\,24''}{\text{Moon's mean daily motion}}\left[\text{Moon's mean daily motion} \pm \text{Moon's motion-correction}\right]$$

$$- \frac{25'\,7''}{\text{Sun's mean daily motion}}\left[\text{Sun's mean daily motion} \pm \text{Sun's motion-correction}\right]$$

$$= \left[105'\,24'' \pm \frac{105'24'' \times \text{Moon's motion-correction}}{\text{Moon's mean daily motion}}\right]$$

$$- \left[25'\,7'' \pm \frac{25'\,7'' \times \text{Sun's motion-correction}}{\text{Sun's mean daily motion}}\right]$$

Method 9

18(c-d). Or, the radius multiplied by 33 and divided by the *manda-karṇas* of the Sun and the Moon (separately) gives the measures (of their diameters).

(1) $$\text{Sun's diameter} = \frac{33\,R}{\text{Sun's } \textit{mandakarṇa}} \text{ mins.}$$

(2) $$\text{Moon's diameter} = \frac{33\,R}{\text{Moon's } \textit{mandakarṇa}} \text{ mins.}$$

These formulae are equivalent to formulae (1) and (2) given under verse 14, because

$$\frac{\text{true daily motion}}{\text{mean daily motion}} = \frac{R}{\textit{mandakarṇa}}.$$

MOON'S LATITUDE

19. Increase the Moon's longitude for the time of geocentric conjunction of the Moon and the Shadow by the longitude of the Moon's ascending node for the same time. Multiply the Rsine thereof by 270 and divide by the radius. Then is obtained, in minutes etc., the Moon's celestial latitude, which is stated to be north when the longitude of the Moon as increased by the longitude of the Moon's ascending node is in the half-orbit beginning with Aries and south when in the half-orbit beginning with Libra.[1]

That is: If M, N denote the longitudes of the Moon and the Moon's ascending node (the latter being measured westwards), and β the Moon's latitude, then

$$\beta = \frac{\text{Rsin}\,(M+N) \times 270}{\text{R}} \text{ mins., approx.} \tag{1}$$

The accurate formula is

$$\text{Rsin}\,\beta = \frac{\text{Rsin}\,(M+N) \times 270}{\text{R}} \text{ mins.} \tag{2}$$

But since, in the case of an eclipse, β is very small, therefore β and Rsin β are practically the same. 270′ is the Rsine of the Moon's greatest latitude.

20. Or, the Rsine derived from the Moon's longitude increased by that of the Moon's ascending node, divided by one-fourth of the diameter and multiplied by 135; or multiplied by 100 and divided by 1273; or multiplied by 4 and divided by 51, gives the Moon's latitude.[2]

This is the simplified version of the previous rule, and has been obtained by taking $135/(\text{R}/2)$, $100/1273$, and $4/51$ in place of $270/\text{R}$. One can easily see that

$$\frac{270}{\text{R}} \text{ or } \frac{270}{3438} = \frac{135}{3438/2} \approx \frac{100}{1273} \approx \frac{4}{51}.$$

1. *Cf. BrSpSi,* iv. 5; *SiŚi,* I, v. 10.

2. For similar rules, see *KK,* I, iv. 1 (c-d); *SiŚe,* v. 10 (a-b). The ratio 4:51 has been adopted in Nārāyaṇa's *Uparāgakriyākrama* (1563 A. D.) (ii. 1 a) and in Śaṅkaravarman's *Sadratnamālā* (A.D. 1820) (v. 24a).

THE ECLIPSER AND MEASURE OF ECLIPSE

21. The Shadow is the eclipser of the Moon and the Moon that of the Sun. Hence, half the sum of (the diameters of) the Shadow and the Moon diminished by the Moon's latitude gives the measure of the lunar eclipse.[1]

Measure of the eclipsed portion of the Moon's diameter

= ½ (diameter of Shadow + diameter of Moon)
— Moon's latitude.

TOTAL OR PARTIAL ECLIPSE

22. When this is (equal to or) greater than the diameter of the eclipsed body (i.e., the Moon), the (lunar) eclipse is total; when less, the (lunar) eclipse is partial.[2] **When the Moon's latitude exceeds half the sum of (the diameters of) the eclipsing and the eclipsed bodies, then there is no eclipse. This is what the sages have said.**

In the case of a total lunar eclipse, the amount by which the measure of eclipse exceeds the Moon's diameter is called *khagrāsa.*

MEASURE OF UNECLIPSED PORTION OF MOON

23. As much as remains after diminishing the Moon's latitude by the difference of the semi-diameters of the Shadow and the Moon, so much of the Moon's diameter is seen visible in the sky. When the remainder is nil, the eclipse is total.[3]

Uneclipsed portion of the Moon's diameter

= Moon's diameter — eclipsed portion of Moon's diameter

= Moon's diameter — [(semi-diameter of Shadow + semi-diameter of Moon) — Moon's latitude]

= Moon's latitude — (semi-diameter of Shadow — semi-diameter of Moon).

1. *Cf. BrSpSi,* iv. 7 (a-b); *KK,* I, iv. 3 (a-b); *KR,* iii. 18; *ŚiDVr,* v. 12 (a-b); *MSi,* v. 7; *SiŚe,* v. 10 (c-d); *SiŚi,* I, v. 11 (a-b); *SuSi,* I, iv. 5 (d).

2. *Cf. BrSpSi,* iv. 7 (c-d); *KK,* I, iv. 3 (c-d); *ŚiDVr,* v. 12 (c-d); *SiŚi,* I, v. 11 (c-d).

3. *Cf. Ā,* iv. 43; *LBh,* iv. 9; *ŚiDVr,* v. 13; *MSi,* v. 7 (c-d); *SiŚe,* v. 11.

STHITYARDHA AND *VIMARDĀRDHA*

24. Severally increase and diminish the semi-diameter of the eclipsing body by the semi-diameter of the eclipsed body; then find their squares; then diminish them by the square of the Moon's latitude; and then find their square-roots : then are obtained the minutes of the *sthityardha* (i.e., half the duration of the eclipse) and the *vimardārdha* (i.e., half the duration of totality of the eclipse).[1]

25. Or, severally increase and decrease half the sum (or difference) of the diameters of the eclipsed and eclipsing bodies, by the Moon's latitude; multiply the two results (thus obtained) one by the other; and take the square-root (of the product) : then is obtained the *sthityardha* (or *vimardārdha*) (in terms of minutes).

That is, if S, M denote the diameters of the eclipsing and eclipsed bodies, and β the Moon's latitude, then

(1) *Sthityardha* $= \sqrt{\left(\frac{S+M}{2}\right)^2 - \beta^2}$

(2) *Vimardārdha* $= \sqrt{\left(\frac{S-M}{2}\right)^2 - \beta^2}$

or,

(3) *Sthityardha* $= \sqrt{\left(\frac{S+M}{2} + \beta\right)\left(\frac{S+M}{2} - \beta\right)}$

(4) *Vimardārdha* $= \sqrt{\left(\frac{S-M}{2} + \beta\right)\left(\frac{S-M}{2} - \beta\right)}$.

BEGINNING AND END OF ECLIPSE

26. They (i.e., *sthityardha* and *vimardārdha* in minutes) when divided by the motion-difference of the Sun and Moon, in terms of degrees, give the corresponding *ghaṭīs*.[2] **The daily motions (of the Moon etc.) multiplied by (the *ghaṭīs* of) the *sthityardha* and divided by 60 when subtracted from or added to the longitudes of the planets for the time of opposition (of the Sun and Moon) give the longitudes of the planets for the beginning and end of the eclipse. From the corresponding latitudes**

1. *Cf. BrSpSi*, iv. 8; *KK*, I, iv. 4; *ŚiDVr*, v. 14; *MSi*, v. 8; *SiŚe*, v. 12; *SiŚi*, I, v. 12.
2. *Cf. SuSi*, I, iv. 5 (a-b).

(of the Moon) and the *sthityardha* obtain the true values of the *spārśika* and *maukṣika sthityardhas* by the process of iteration.[1]

27. Or, multiply the daily motions of the planets (concerned) by the minutes of the *sthityardha* and divide by the degrees of the daily motion-difference of the Sun and Moon. The results should be subtracted from or added to the longitudes of the (respective) planets (for the time of opposition) and the process of iteration should be applied to obtain the true values of the *spārśika* and *maukṣika sthityardhas.*

28. The time of opposition (of the Sun and Moon) when (severally) diminished by the *nāḍīs* of the *spārśika sthityardha* and increased by the *nāḍīs* of the *maukṣika sthityardha* gives the time of the beginning of the eclipse and the time of the end of the eclipse (i. e., the time of entrance of the Moon into the Shadow and the time of exit of the Moon out of the Shadow), (respectively).[2]

In order to obtain the times of immersion and emersion, one should obtain the (*spārśika* and *maukṣika*) *vimardārdhas* in the same way.[3]

29. The eclipse starts as many *ghaṭīs* before the time of opposition as there are in the *spārśika sthityardha* and ends as many *ghaṭīs* after the time of opposition as there are in the *maukṣika sthityardha.* The immersion of the Moon occurs as many *ghaṭīs* prior to the time of opposition as there are in the *spārśika vimardārdha* and the emersion takes place as many *ghaṭīs* after the time of opposition as there are in the *maukṣika vimardārdha.*[4]

30. The middle of the eclipse takes place at the true time of opposition.[5] The sum of the *spārśika* and *maukṣika sthityardhas* gives the total duration of the eclipse; and the sum of the true *spārśika* and *maukṣika vimardārdhas* gives the *nāḍīs* of the totality of the eclipse.[6]

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1. *Cf. BrSpSi,* iv. 9; *ŚiDVṛ,* v. 14-16; *KK,* I, iv. 5; *SiŚe,* v. 13; *SiŚi,* 1, v. 13.
2. *Cf. ŚiDVṛ,* v. 17(a-b); *MSi,* v. 9; *SuSi,* I, iv. 5(c).
3. *Cf. MSi,* v. 9.
4. Brahmagupta (*BrSpSi,* iv. 10) says : "The immersion occurs as much time after the (first) contact as is obtained by subtracting the (*spārśika*) *mardārdha* from the (*spārśika*) *sthityardha*; the emersion occurs as much time prior to separation." Also see *ŚiDVṛ,* v. 18(c-d).
5. *Cf. BrSpSi,* iv. 15(a); *ŚiDVṛ,* v. 18(a); *SiŚe,* v. 16(c).
6. *Cf. ŚiDVṛ,* v. 17(c-d); *SiŚe,* v. 17(c-d).

IṢṬAGRĀSA OR ECLIPSE FOR THE GIVEN TIME

31-32. The difference between the true daily motions of the Sun and Moon in terms of degrees, multiplied by the *nāḍīs* which are to elapse (at the given time) before the time of opposition or which have elapsed since the time of opposition, gives the Base; the Moon's latitude for the given time gives the Upright; and the square-root of the sum of the Base and the Upright gives the Hypotenuse. This Hypotenuse being subtracted from half the sum of the own diameters of the eclipsed and the eclipsing bodies, the residue gives the *iṣṭagrāsa* or the measure of the eclipsed portion (of the Moon) for the given time.[1]

33. Whatever results after subtracting half the sum of the diameters of the eclipsed and eclipsing bodies from the sum of the diameter of the eclipsed body and the Hypotenuse (for the given time) gives the measure of the uneclipsed or bright portion (of the Moon) at the given time.

That is, if M, S denote the diameters of the eclipsed and eclipsing bodies and H the Hypotenuse (i. e., the distance of the Moon from the centre of the Shadow at the given time), then

(1) *iṣṭagrāsa* $= \frac{1}{2}(S+M) - H$

(2) Uneclipsed diameter of the Moon $= (M+H) - \frac{1}{2}(S+M)$.

GRĀSA FOR IMMERSION AND EMERSION

34. The values of the Hypotenuse for the times of immersion and emersion are equal to half the difference between the diameters of the eclipsing and eclipsed bodies; the values of the Base are equal to the minutes of the (*spārśika* and *maukṣika*) *vimardārdhas* (respectively); and the values of the Upright are equal to the Moon's latitudes for those times.[2]

TIME AT THE GIVEN *GRĀSA*

35. Subtract the *iṣṭagrāsa* ("given eclipsed portion") from half the sum of the diameters of the eclipsed and eclipsing bodies; then diminish

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1. *Cf. BrSpSi*, iv. 11-12; *KK*. I, iv. 6; *ŚiDVṛ*, v. 19, 20(d); *MSi*, v. 14; *SiŚe*, v. 14; *SiŚi*, I, v. 15-17(a-b).
2. *Cf. ŚiDVṛ*, v. 20.

the square of that by the square of the Moon's latitude (for the time of opposition); the square-root thereof is the Base. That divided by the degrees of difference between the (true daily) motions of the Sun and the Moon, gives the *nāḍīs* (corresponding to the Base, i. e., the *nāḍīs* intervening between the time of opposition and the desired time). From the Moon's latitude for that time (i.e., for the time obtained by subtracting those *nāḍīs* from or adding them to the time of opposition), obtain the true value of the *nāḍīs* of the Base, by applying the process of iteration. The true *ghaṭīs* which are thus fixed by iteration give the desired time which is so many *ghaṭīs* before or after the time of opposition.[1]

AKṢA-DIGVALANA OR *AKṢAVALANA*

36. Multiply the Rversed-sine of the *asus* of hour angle (of the eclipsed body) for the beginning, middle or end of the eclipse (as the case may be) by the equinoctial midday shadow for the local place and divide by the hypotenuse of the equinoctial midday shadow for the local place. The minutes in the arc corresponding to (the Rsine equal to) the result obtained give the *akṣavalana* whose direction is north or south (according as the eclipsed body is) in the eastern or western hemisphere, respectively.[2]

When the hour angle exceeds three signs, the Rsine of the excess should be added to the radius (and the result treated as the Rversed-sine of the hour angle).

37. Or else, the Rversed-sine of the hour angle (severally) multiplied by the *agrā*, the *akṣajyā* (i e., the Rsine of the latitude), the *śaṅkutala* and the earthsine and divided by the *taddhṛti*, the radius, the *svadhṛti* and the *agrā*, respectively, is (in each case) declared as the *digjyā* (i e., the Rsine of the *digvalana* or *akṣavalana*).

Let H be the hour angle of the eclipsed body. Then

$$(1)\quad \text{Rsin}\ (ak\d{s}avalana) = \frac{\text{Rvers}\ H \times palabh\bar{a}}{palakar\d{n}a}$$

$$(2)\quad \text{Rsin}\ (ak\d{s}avalana) = \frac{\text{Rvers}\ H \times agr\bar{a}}{taddh\d{r}ti}$$

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1. *Cf. BrSpSi*, iv. 13-14; *ŚiDVṛ*, v. 21-22; *MSi*, v. 15; *SiŚe*, v. 15; *SiŚi*, I, v. 17(c d)-18.
2. *Cf. ŚiDVṛ*, v. 23; *SiŚe*, v. 18-19. Also see *MBh*, v. 42-44; *LBh*, iv. 15-16.

(3) $\text{Rsin}\,(ak\d{s}avalana) = \dfrac{\text{Rvers}\,H \times \text{Rsin}\,\phi}{\text{R}}$

(4) $\text{Rsin}\,(ak\d{s}avalana) = \dfrac{\text{Rvers}\,H \times \textit{śaṅkutala}}{\textit{svadhṛti}}$

(5) $\text{Rsin}\,(ak\d{s}avalana) = \dfrac{\text{Rvers}\,H \times \text{earthsine}}{\textit{agrā}}.$

According to Brahmagupta[1] and Āryabhaṭa II[2],

$$\text{Rsin}\,(ak\d{s}avalana) = \frac{\text{Rsin}\,H \times \text{Rsin}\,\phi}{\text{R}}.$$

AYANA-DIGVALANA OR *AYANAVALANA*

38. Find the Rsine of the declination from the Rversed-sine of (the *bhuja* of) the longitude of the eclipsed body as increased by three signs; and treating it as direct Rsine obtain the minutes of its arc. These are to be taken as the minutes of the *ayanavalana* for the eclipsed body. Its direction is said to accord to the hemisphere (north or south) of the point which is three signs in advance of the eclipsed body.[3]

That is, if λ be the (tropical) longitude of the eclipsed body and B the *bhuja* of $(90° + \lambda)$, then

$$\text{Rsin}\,(ayanavalana) = \frac{\text{Rvers}\,B \times \text{Rsin}\,24°}{\text{R}}.$$

According to Brahmagupta[4] and Āryabhaṭa II[5],

$$\text{Rsin}\,(ayanavalana) = \frac{\text{Rsin}\,B \times \text{Rsin}\,24°}{\text{R}}.$$

According to Bhāskara II,[6]

$$\text{Rsin}\,(ak\d{s}avalana) = \frac{\text{Rsin}\left[\dfrac{\text{hour angle in } ghaṭīs \times 90}{\text{semi-duration of night}^{7} \text{ in } ghaṭīs}\right] \times \text{Rsin}\,\phi}{\text{Rcos}\,\delta}$$

1. See *BrSpSi*, iv. 16 and *KK*, I, iv. 7(a-b).
2. See *MSi*, v. 16.
3. *Cf. ŚiDVṛ*, v. 25; *SiŚe*, v. 20(a-b). Also see *MBh*, v. 45; *LBh*, iv. 17.
4. See *BrSpSi*, iv. 18 and *KK*, I, iv. 7(c-d).
5. See *MSi*, v. 17.
6. See *SiŚi*, I, v. 20-21(a-b), 21(c-d)-22(a-b).
7. In the case of a lunar eclipse. If the eclipse is solar, one should take semi-duration of day.

$$\text{Rsin }(ayanavalana) = \frac{\text{Rcos } \lambda \times 1397}{\text{Rcos } \delta}.$$

Bhāskara II has criticised the astronomers who have prescribed the use of the Rversed-sine in finding the *valana*.[1] Writes he :

"Those who have prescribed the use of the Rversed-sine in finding this *valana*, are not well-versed in the motion of the Celestial Sphere."

Criticising Lalla for the same reason, he writes :

"When the Sun is in the zenith and the ecliptic looks like a vertical circle, then the *valana* obviously looks on the horizon like the *agrā* corresponding to the Sun's longitude increased by three signs. If you, O friend, proficient in spherics, can find out the same from the Rversed-sine (of the Sun's longitude increased by three signs), then indeed I must admit the flawlessness of the formula for the *valana* as stated in the *Śiṣya-dhī-vṛddhida* etc.

When, at a place in latitude 66°, the contact of the rising Sun, situated in Aries or Taurus, Aquarius or Pisces, takes place towards the south, the ecliptic coincides with the horizon (and the Rsine of the *valana* is equal to the radius). Say, how will (the Rsine of) the *valana* be equal to the radius by using the Rversed sine."[2]

Note. It may be mentioned that the directions of the *akṣa* and *ayana valanas* prescribed in the text are for the eastern side of the disc of the eclipsed body (i.e., in relation to the east point of the eclipsed body). Those for the western side are just the reverse.

RESULTANT *DIGVALANA* OR *DIGVALANA* OR *VALANA*

39. Take the sum of the *akṣavalana* (*digjyācāpa*) and the *ayanavalana* (*krāntivikṣepa*) when they are of like directions, and the difference when they are of unlike directions; and find the Rsine of that (sum or difference).[3] This corresponds to the circle of signs (i. e., the circle of radius R = 3438′), which is marked with the cardinal points. That for the desired circle should be determined by proportion.

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1. See *SiŚi*, I, v. 23(c-d).
2. *SiŚi*, I, v. 38-39.
3. *Cf. BrSpSi*, iv. 18; *SiŚe*, v. 20(c-d).

RELATION BETWEEN MINUTES AND *AṄGULAS*

40. Divide the *nāḍīs* of the *Unnatakāla* (i. e., the time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon) by the true semi-duration of the day (daylight) and increase the result by 3 : then are obtained the minutes in an *aṅgula*, which is defined as the measure of the central width of eight barley corns with their husk pealed off.[1]

$$1 \textit{ aṅgula} = 3 + \frac{G}{D} \text{ minutes,}$$

where D denotes the *ghaṭīs* of the semi-duration of daylight and G the *ghaṭīs* elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon.

This relation is based on the assumption that on the horizon

$$1 \textit{ aṅgula} = 3 \text{ minutes}$$

and on the meridian

$$1 \textit{ aṅgula} = 4 \text{ minutes.}$$

According to Lalla[2], Śrīpati[3] and Bhāskara II[4],

$$1 \textit{ aṅgula} = 2\tfrac{1}{2} + \frac{G}{D} \text{ minutes.}^5$$

CONVERSION OF MINUTES INTO *AṄGULAS*

41(a-b). This (value of an *aṅgula* in terms of minutes) is prescribed to be the divisor of (the minutes of) the Moon's latitude, the measures of the diameters of the eclipsing and eclipsed bodies, and also of the *iṣṭagrāsa* (i. e., the eclipsed part of the eclipsed body at the given time) (for converting them into *aṅgulas*).[6]

SIDE ECLIPSED FIRST

41(c-d). The eclipse of the Moon's limb takes place towards its eastern side, the direction of its latitude being reversed (in representing the eclipse graphically); that of the Sun's, towards the western side.[7]

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1. Same rule occurs in *SūSi*, iv. 26. For similar rules, see *BrSpSi*, xvi. 11, 12; according to Brahmagupta 1 *aṅgula* = 6 barley corns with husk.
2. See *ŚiDVr*, v. 27(c-d). 3. *SiŚe*, v. 23. 4. See *SiŚi*, I, v. 24(c-d).
5. Lalla has sometimes deviated from this rule. In *ŚiDVr*, xi. 7(b) he prescribes $2\frac{1}{2}$ mins. as equivalent to an *aṅgula* and in *ŚiDVr*, vii. 3(a-b) he takes $2\frac{1}{3}$ mins. as equivalent to an *aṅgula*.
6. *Cf. BrSpSi*, xvi. 13(c-d); *ŚiDVr*, v. 28; *SiŚe*, v. 24.
7. *Cf. SiŚe*, v. 26(a-b); *SiŚi*, II, viii. 1, 4, 6.

Chapter V

SOLAR ECLIPSE

Section 1 : *Lambana* or Parallax in Longitude

INTRODUCTION

1. Since great confusion prevails even amongst the astronomers who are versed in *Gaṇita* and *Gola* in the case of a solar eclipse, so here is being stated by me an excellent (process of) computation (of the solar eclipse) which will be immensely astonishing to the intelligent.

2. This (process) should not be imparted to any one who is envious or to one who is a scholar of any other (astronomical) *tantra*, even though he takes oath (to keep it secret). If anyone imparts it (to such a person), he shall lose his good deeds and longevity. It should be imparted to one who is devoted and obedient and to one's own son as well.

EXISTENCE OF *LAMBANA* AND *AVANATI*

3. When the Sun's longitude is equal to that of the central ecliptic point, the *lambana* is non-existent. When the Sun's longitude is greater or less (than that), the *lambana* exists and causes defect or excess in the time of apparent conjunction, no matter whether it is obtained by the process of iteration or directly.[1]

4(a-b). When (the degrees of) the local latitude are not equal to the degrees of the (northern) declination of the central ecliptic point, then and then only does the *avanati* ("parallax in latitude") exist.[2]

The next $10\frac{1}{2}$ verses relate to the determination of the five well known Rsines, viz. *Dṛkkṣepajyā*, *Dṛgjyā*, *Madhyajyā*, *Udayajyā* and *Dṛggatijyā*.

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1. For similar rules see, *BrSpSi*, v. 2(a-b); *ŚiDVṛ*, vi. 14 (c); *SūSi*, v. 1(a-b); *SiŚe*, vi. 1(a-b); *SiŚi*, I, vi. 2.

2. *Cf*. *BrSpSi*, v. 2(c-d); *SūSi*, v. 1(c-d); *SiŚe*, vi. 1(c-d).

DṚKKṢEPAJYĀ, *DṚGJYĀ*, *MADHYAJYĀ* AND *UDAYAJYĀ*

4(c-d)-5. In that case (viz. when the *lambana* and *avanati* exist), one should find out the Rsine of altitude (of the central ecliptic point) with the help of the day elapsed since sunrise, the *asus* of its own ascensional difference, the Rsine of the local latitude and the own day-radius.[1] The square-root of the difference between the squares of that and the radius is the *dṛkkṣepa* (or *dṛkkṣepajyā*). In the same way, one should find out the Rsines of the Sun's altitude and the Sun's zenith distance (*dṛgjyā*), the *madhyajīvā* ("the Rsine of the zenith distance of the meridian ecliptic point") and the *udayajyā* (lit. the *agrā* for the rising point of the ecliptic) as in the case of the Sun.

The *dṛkkṣepa* or *dṛkkṣepajyā* is the Rsine of the zenith distance of the central ecliptic point. It is said to be of the northern or southern direction according as the central ecliptic point is towards the north or south of the zenith.[2]

MADHYALAGNA OR MERIDIAN ECLIPTIC POINT

6. (The signs etc. corresponding to) the *asus* of the right ascension lying between midday and *tithyanta* ("time of conjunction of Sun and Moon") should be subtracted from the Sun's longitude at *tithyanta* or added to it according as the *tithyanta* falls in the first half or the second half of the day. The difference or sum, thus obtained, gives the *madhyalagna* ("the longitude of the meridian ecliptic point"), in terms of signs etc.[3]

ALTERNATIVE METHOD

7. Or, (in case the *tithyanta* falls in the afternoon), increase the *parva* (i. e., *tithyanta*, which is measured in this case by the time to elapse before sunset) by half the measure of the night and subtract half a circle from the Sun's longitude (at the *tithyanta*). (Treating the resulting quantities as the time to elapse from *tithyanta* up to midday and the Sun's longitude for that time respectively) subtract the signs corresponding to the *asus* of right ascension lying from the *tithyanta* up to midday, in

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1. *Cf. BrSpSi*, v. 3(b-d); *SiŚe*, vi. 1(c-d)-2.
2. *Cf. BrSpSi*, v. 8; *MSi*, vi. 10; *SiŚi*, I, vi. 10(c-d).
3. This rule is similar to that found to occur in *ŚiDVṛ*, vi. 2(c-d).

the reverse order, (from the Sun's longitude) : what is thus obtained is called the *madhyalagna* ("the longitude of the meridian ecliptic point") by the learned.[1]

MADHYAJYĀ

8. Take the sum of its declination and the local latitude, when the meridian ecliptic point is in the half orbit beginning with Libra; and their difference, when the meridian ecliptic point is in the half orbit beginning with Aries. The Rsine of this (sum or) difference is called *madhyajyā*. This is the base (of a right-angled triangle); the Rsine of the altitude of the meridian ecliptic point is the upright of that (triangle).

DṚGGATI AND *LAGHU DṚGGATI*

9. The square-root of the product of the difference and sum of *dṛkkṣepa* and *dṛgjyā* is the so called *dṛggati*.

10. The square-root of the product of the sum and difference of *dṛkkṣepa* and *madhyajyā* is called *dṛggati* for the middle of the day. It is said to be equal to the square-root of the difference of the Rsines of the altitudes of the central and meridian ecliptic points. The same is also said to be equal to the square-root of the product of the difference and sum of the Rsines of those altitudes.

In order to distinguish between *dṛggati* and *dṛggati* for the middle of the day, the former is called larger *dṛggati*. Thus we have

(1) *dṛggati* or larger *dṛggati* $= \sqrt{(dṛgjyā + dṛkkṣepa)\,(dṛgjyā - dṛkkṣepa)}$

(2) *dṛggati* for midday or smaller *dṛggati*

$$= \sqrt{(madhyajyā + dṛkkṣepa)\,(madhyajyā - dṛkkṣepa)}$$

$$= \sqrt{(vitribhaśaṅku)^2 - (madhyalagnaśaṅku)^2}$$

$$= \sqrt{(vitribhaśaṅku + madhyalagnaśaṅku)\,(vitribhaśaṅku - madhyalagnaśaṅku)}.$$

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1. For a similar rule applicable when the *tithyanta* falls in the forenoon, see *ŚiDVṛ*, vi. 2(a-b).

ANOTHER FORM FOR LARGER *DṚGGATI*. SUM AND DIFFERENCE OF *DṚGGATIS*.

11. The sum of the *dṛgjyā* ("Rsine of the Sun's zenith distance") and the *madhyajyā* ("Rsine of the zenith distance of the meridian ecliptic point") multiplied by their difference should be added to the square of the smaller *dṛggatijyā* : the square-root of that is the larger *dṛggati*. The sum of that (larger *dṛggati*) and the smaller *dṛggati* is the "Earth". (The sum of the *dṛgjyā* and the *madhyajyā* multiplied by their difference) divided by that (Earth) is the "*Antara*" or "Difference" (of the larger and smaller *dṛggatis*).

12. The Earth (severally) increased and decreased by the *Antara* ("*dṛggati*-difference") and divided by 2 gives the larger and smaller *dṛggatis*, (respectively). The squares of the *dṛgjyā* and the *madhyajyā* being (severally) divided by the *Antara* ("*dṛggati*-difference"), the difference between the (resulting) quotients is also the Earth.

(1) $$\text{Larger } dṛggati = \sqrt{(dṛgjyā + madhyajyā)\,(dṛgjyā - madhyajyā) + (\text{smaller } dṛggati)^2}$$

(2) $$\text{Earth} = \text{larger } dṛggati + \text{smaller } dṛggati$$

(3) $$Antara \text{ (or } dṛggati\text{-difference)} = \text{larger } dṛggati - \text{smaller } dṛggati$$
$$= \frac{(dṛgjyā)^2 - (madhyajyā)^2}{\text{Earth}}$$

(4) $$\text{Larger } dṛggati = \tfrac{1}{2}\,(\text{Earth} + Antara)$$
$$\text{Smaller } dṛggati = \tfrac{1}{2}\,(\text{Earth} - Antara)$$

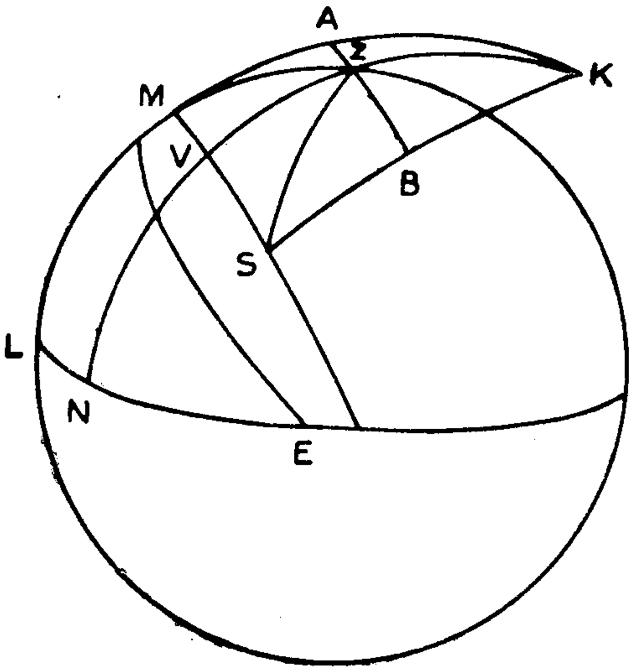
(5) $$\text{Earth} = \frac{(dṛgjyā)^2 - (madhyajyā)^2}{Antara} = \frac{(dṛgjyā)^2}{Antara} - \frac{(madhyajyā)^2}{Antara}.$$

ANOTHER FORM FOR SMALLER *DṚGGATI*

13. Multiply the product of the *madhyajyā* and the *udayajyā* by the Rsine of altitude of the central ecliptic point (*dṛkkṣepavṛttodbhavaśaṅkumaurvī* or *dṛkkṣepaśaṅku* or *vitribhaśaṅku*) and divide by the square of the radius: whatever is obtained as the result of that is the *laghu-dṛggatijyā* (smaller *dṛggati*).

$$\text{Smaller } d\d{r}ggati = \frac{madhyajy\bar{a} \times udayajy\bar{a} \times vitribha\acute{s}a\dot{n}ku}{R^2}.$$

Rationale. The adjoining figure represents the Celestial Sphere in which LNE is the horizon and Z the zenith. MVS is the ecliptic and K its pole. M is the meridian ecliptic point, V the *vitribha-lagna* or the central ecliptic point and S the Sun. ZA is perpendicular to MK and ZB perpendicular to SK, so that Rsin ZA is the smaller *dṛggati* and Rsin ZB the larger *dṛggati*.



Then we have

$$\text{Rsin MV} = \frac{\text{Rsin LN} \times \text{Rsin ZM}}{R}$$

$$\text{Rsin ZA} = \frac{\text{Rsin MV} \times \text{Rsin KZ}}{R}$$

$$= \frac{\text{Rsin MV} \times \text{Rsin VN}}{R}$$

$$\therefore \text{Rsin ZA} = \frac{\text{Rsin ZM} \times \text{Rsin LN} \times \text{Rsin VN}}{R^2}$$

$$\text{or, smaller } d\d{r}ggati = \frac{madhyajy\bar{a} \times udayajy\bar{a} \times vitribha\acute{s}a\dot{n}ku}{R^2}.$$

OTHER RESULTS

14. Severally increase the square of the *dṛkkṣepa* ("Rsine of the zenith distance of the central ecliptic point") by the squares of the larger and smaller *dṛggatis* (lit. by the squares of the larger and smaller segments of the "Earth"); and severally diminish the square of the *vitribha-śaṅku* ("Rsine of altitude of the central ecliptic point") by the same (i.e., by the squares of the larger and smaller *dṛggatis*). The square-roots of the results obtained are the Rsine of the Sun's zenith distance (*dṛgjyā*), the Rsine of zenith distance of the meridian ecliptic point (*madhyajīvā*), the Rsine of the Sun's altitude (*nara* or *śaṅku*), and the Rsine of altitude of the meridian ecliptic point (*madhyaśaṅku*), respectively.

(1) *Dṛgjyā* or Rsin $z = \sqrt{(\textit{dṛkkṣepa})^2 + (\text{larger } \textit{dṛggati})^2}$

(2) *Madhyajīvā* $= \sqrt{(\textit{dṛkkṣepa})^2 + (\text{smaller } \textit{dṛggati})^2}$

(3) *Śaṅku* or Rsin $a = \sqrt{(\textit{vitribhaśaṅku})^2 - (\text{larger } \textit{dṛggati})^2}$

(4) *Madhyaśaṅku* $= \sqrt{(\textit{vitribhaśaṅku})^2 - (\text{smaller } \textit{dṛggati})^2}$,

where a is the Sun's altitude and z the Sun's zenith distance.

These results follow easily from the following formulae :

(5) Larger *dṛggati* $= \sqrt{(\text{Rsin } z)^2 - (\textit{dṛkṣepa})^2}$

$$= \sqrt{(\textit{vitribhaśaṅku})^2 - (\text{Rsin } a)^2}$$

(6) Smaller *dṛggati* $= \sqrt{(\textit{madhyajyā})^2 - (\textit{dṛkkṣepajyā})^2}$

$$= \sqrt{(\textit{vitribhaśaṅku})^2 - (\textit{madhyaśaṅku})^2}.$$

LAMBANA OR *LAMBANĀNTARA-LAMBANA* (MOON'S *LAMBANA* MINUS SUN'S *LAMBANA*)

Method 1

15. Multiply the *dṛggati* by the Earth's semi-diameter and set down the result in two places. In one place divide it by the Sun's true distance in *yojanas* and in the other place divide it by the Moon's true distance in *yojanas*. The difference of the two results, which is in terms of minutes etc., gives the *lambana* for the time of conjunction of the Sun and Moon, in terms of minutes etc.[1]

$$\textit{Lambana} = \left[\frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Moon's true distance in } \textit{yojanas}} - \frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Sun's true distance in } \textit{yojanas}}\right] \text{minutes.}$$

The *lambana*, in minutes, when multiplied by 60 and divided by the difference between the true daily motions of the Sun and the Moon is reduced to *ghaṭīs*. Thus

1. *Cf. MBh*, v. 24-27; *ŚiDVṛ*, vi. 6-7.

lambana in terms of *ghaṭīs* $=\dfrac{\textit{lambana}\text{ in minutes}\times 60}{\text{(Moon's true daily motion} - \text{Sun's true daily motion)}}$.

Rationale. Consider Fig. 1. The circle centred at O denotes the Earth, L being the local place on its surface. S, M are the Sun and the Moon at the

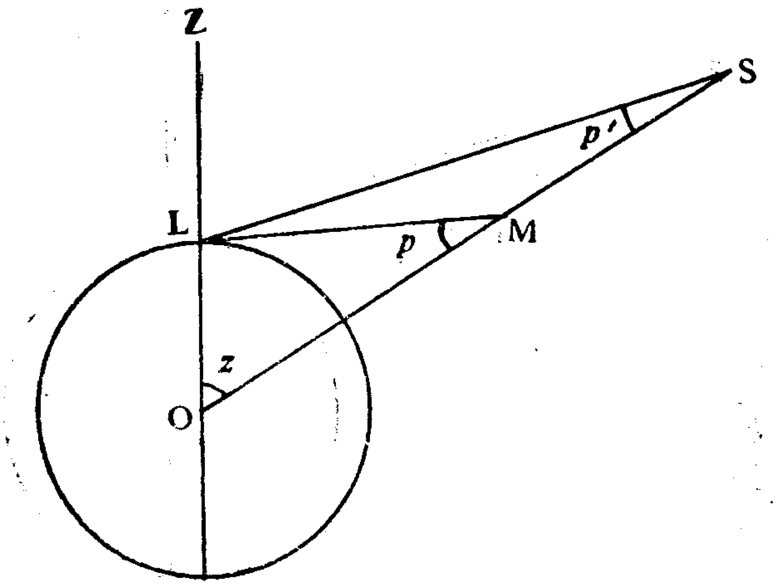


Fig. 1

time of their geocentric conjunction, z being their common geocentric zenith distance. Then angle LMO ($=p$) is the Moon's parallax in zenith distance and angle LSO ($=p'$) is the Sun's parallax in zenith distance.

From triangle LMO, we have

$$\frac{\text{Rsin } p}{\text{Rsin } z}=\frac{\text{LO}}{\text{LM}}\approx\frac{\text{LO}}{\text{MO}}=\frac{\text{Earth's semi-diameter (in } \textit{yojanas})}{\text{Moon's true distance in } \textit{yojanas}}. \tag{1}$$

Similarly from triangle SLO, we have

$$\frac{\text{Rsin } p'}{\text{Rsin } z}=\frac{\text{Earth's semi-diameter (in } \textit{yojanas})}{\text{Sun's true distance in } \textit{yojanas}}. \tag{2}$$

Now consider Fig. 2. VM is the ecliptic and K its pole. V is the *vitribhalagna* or central ecliptic point, Z the zenith and M the common position of the Moon and the Sun at the time of their geocentric conjunction (in longitude). M′ is the apparent position of the Moon and S′ the apparent position of the Sun. Thus MM′ $=p$ and MS′ $=p'$. M′D and S′D′ are perpendiculars on the ecliptic and M′B and S′B′ perpendiculars on KM produced. Then MD denotes Moon's parallax in longitude and MD′ Sun's

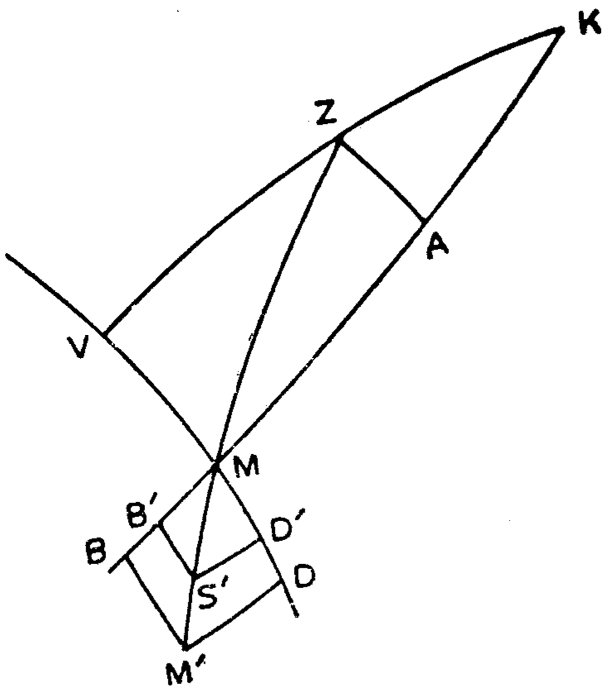


Fig. 2

parallax in longitude. Their difference D′D is the *lambana* for the time of geocentric conjunction of the Sun and Moon.

Let ZA be the perpendicular from Z to KM. Then comparing the triangles MBM′ and ZAM, we have

$$\text{Rsin}\,(\text{BM}') = \frac{\text{Rsin ZA} \times \text{Rsin MM}'}{\text{Rsin ZM}}.$$

But Rsin BM′ = BM′ or MD approx. Therefore,

$$\text{MD} = \frac{\text{Rsin ZA} \times \text{Rsin MM}'}{\text{Rsin ZM}}$$

$$= \frac{\textit{dṛggati} \times \text{Rsin}\ p}{\text{Rsin}\ z}$$

$$= \frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Moon's true distance in } \textit{yojanas}}, \text{ using (1).}$$

Similarly,

$$\text{MD}' = \frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Sun's true distance in } \textit{yojanas}}, \text{ using (2).}$$

Hence, on subtraction,

$$\textit{lambana}\ \mathrm{D'D} = \frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Moon's true distance in } \textit{yojanas}} - \frac{\textit{dṛggati} \times \text{Earth's semi-diameter}}{\text{Sun's true distance in } \textit{yojanas}}.$$

This is *lambana* for the time of geocentric conjunction of the Sun and Moon. But really we need *lambana* for the time of apparent conjunction of the Sun and Moon, because

time of apparent conjunction = time of geocentric conjunction

± *lambana* in time for the time of apparent conjunction,

+ or − being taken according as the Sun and the Moon at the time of apparent conjunction lie to the west or east of the central ecliptic point.

The *lambana* for the time of apparent conjunction depends on the time of apparent conjunction itself. But as the time of apparent conjunction is unknown, the corresponding *lambana* cannot be obtained directly and recourse is taken to the method of iteration.

Method 2

16. Alternatively, the *dṛggati* multiplied by 4 and then divided by the radius gives the *lambana* in terms of *ghaṭīs*.[1] It should be subtracted from the time of (geocentric) conjunction or added to that in the manner stated before (i. e., according as the geocentric conjunction occurs to the east or west of the central ecliptic point). By iterating this process one may obtain the true *lambana* (i. e., *lambana* for the time of apparent conjunction).

$$\textit{Lambana} = \frac{4 \times \textit{dṛggati}}{\mathrm{R}}\ \textit{ghaṭīs}.$$

Rationale. Since

Moon's true daily motion in minutes

$$= \frac{\text{Moon's mean daily motion in } \textit{yojanas} \times \mathrm{R}}{\text{Moon's true distance in } \textit{yojanas}},$$

1. *Cf. SiŚi*, I, vi. 6(a-b).

therefore, $\dfrac{\text{Earth's semi-diameter}}{\text{Moon's true distance in } yojanas}$

$$= \frac{\text{Earth's semi-diameter} \times \text{Moon's true daily motion in minutes}}{\text{Moon's mean daily motion in } yojanas \times \text{R}}.$$

Similarly,

$$\frac{\text{Earth's semi-diameter}}{\text{Sun's true distance in } yojanas}$$

$$= \frac{\text{Earth's semi-diameter} \times \text{Sun's true daily motion in minutes}}{\text{Sun's mean daily motion in } yojanas \times \text{R}}$$

Therefore, from vs. 15,

lambana

$$= \frac{dṛggati \times \text{Earth's semi-diameter} \times \text{motion-difference of Sun and Moon}}{\text{Planets' mean daily motion in } yojanas \times \text{R}} \text{ minutes}$$

$$= \frac{dṛggati \times \text{Earth's semi-diameter} \times 60}{\text{Planets' mean daily motion in } yojanas \times \text{R}} \; ghaṭīs$$

$$= \frac{4 \times dṛggati}{\text{R}} \; ghaṭīs,$$

because

planets' mean daily motion in *yojanas* = 15 × Earth's semi-diameter.[1]

Alternative rationale.

When the *dṛggati* is maximum and equals the radius, the *lambana* is also maximum and is equal to 4 *ghaṭīs*; and when the *dṛggati* equals zero, the *lambana* is also equal to zero. Hence the *lambana* varies directly as the *dṛggati*. Consequently,

$$lambana = \frac{4 \times dṛggati}{\text{R}} \; ghaṭīs.$$

In fact two proportions are used : (See *SiŚi*, I, vi. 3-4, com.)

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1. Planets' mean daily motion = 7905 *yojanas* approx. and 15 × Earth's semi-diameter = 15 × 527 = 7905 *yojanas*. See *supra*, chap. 1, sec. 7, vs. 14 and chap. 1, sec. 8, vs. 3.

Proportion 1. When the Rsine of the Sun's distance from the central ecliptic point equals the radius, the *lambana* is maximum and equal to 4 *ghaṭīs*, what will be the *lambana* corresponding to the Rsine of the Sun's distance from the central ecliptic point at the time of geocentric conjunction ? The result is the so called mean *lambana*.

This *lambana* is the mean *lambana* and not the true *lambana*, because the maximum *lambana* is 4 *ghaṭīs* only when the *vitribhaśaṅku* is equal to the radius. Hence one more proportion is needed, viz.

Proportion 2. When the *vitribhaśaṅku* is equal to the radius, the true *lambana* is equal to the mean *lambana*, what will be the *lambana* corresponding to the mean *lambana* obtained above ? The result is the true *lambana*.

Thus :

$$\text{true } lambana = \frac{\text{mean } lambana \times vitribhaśaṅku}{\text{R}}$$

$$= \frac{\text{Rsin (Sun} - vitribhalagna) \times 4}{\text{R}} \times \frac{vitribhaśaṅku}{\text{R}}\ ghaṭīs$$

$$= \frac{dṛggati \times 4}{\text{R}}\ ghaṭīs.$$

The word "true (*sphuṭa*)" used in the text as an adjective of the term "*lambana*" is meant to distinguish the true *lambana* from the mean *lambana*. The term "true *lambana*" used below is also used in the same sense.

Methcd 3

17. Or, multiply the true *dṛggati* by the difference between the true daily motions of the Sun and the Moon and divide by 15 times the radius : the result, reduced to *ghaṭīs*, as in the case of *tithi*, gives the *lambana*, in terms of *ghaṭīs*.

$$Lambana \text{ in minutes} = \frac{dṛggati \times \text{motion-difference of Sun and Moon}}{15 \times \text{R}}$$

and, multiplying the right hand side by 60 and dividing by the motion-difference of the Sun and Moon, as in the case of *tithi*,

$$lambana \text{ in } ghaṭīs = \frac{dṛggati \times 4}{\text{R}}.$$

Rationale. We have proved above (under vs. 16) that

lambana in minutes

$$= \frac{d\d{r}ggati \times \text{Earth's semi-diameter} \times \text{motion-difference of Sun and Moon}}{\text{Planets' mean daily motion in } yojanas \times \text{R}}$$

But planets' mean daily motion in *yojanas* = 15 × Earth's semi-diameter.

$$\therefore \; lambana \text{ in minutes } = \frac{d\d{r}ggati \times \text{motion-difference of Sun and Moon}}{15 \times \text{R}}.$$

Method 4

18. Or, multiply the *dṛggati* by the Rsine of the (Sun's) greatest declination (i. e., by Rsin 24°) and divide by the radius, and then increase the resulting quotient by one-thirtythird of itself : the result (thus obtained) is loudly stated as the true *lambana* in terms of respirations.

$$Lambana = \left(1 + \frac{1}{33}\right)\frac{d\d{r}ggati \times \text{Rsin } 24^\circ}{\text{R}} \text{ respirations.}$$

Proof. From vs. 16,

$$lambana = \frac{4 \times d\d{r}ggati}{\text{R}} \; gha\d{t}\bar{\imath}s$$

$$= \frac{4 \times d\d{r}ggati}{\text{R}} \times \frac{21600}{60} \text{ respirations}$$

$$= \frac{1440 \times d\d{r}ggati}{\text{R}} \text{ respirations}$$

$$= \frac{(1398 + 42) \times d\d{r}ggati}{\text{R}} \text{ respirations}$$

$$= \frac{1398 \times d\d{r}ggati}{\text{R}} + \frac{42 \times d\d{r}ggati}{\text{R}} \text{ respirations}$$

$$= \frac{1398 \times d\d{r}ggati}{\text{R}} + \frac{1398 \times d\d{r}ggati}{\text{R} \times 33} \text{ respirations}$$

$$= (1 + 1/33)\,\frac{d\d{r}ggati \times \text{Rsin } 24^\circ}{\text{R}} \text{ respirations,}$$

because Rsin 24° = 1398′ 13″ or 1398′ approx. See *supra*, chap. II, sec. 1, vss. 49(c-d)-50.

Method 5

19. (Severally) multiply the *dṛkkṣepa* and the *dṛgjyā*, or the Rsines of the corresponding altitudes, by 4 and divide the (resulting products) by the radius : whatever result in *ghaṭīs* etc. is obtained as the square-root of the difference of their squares is called the true *lambana* by the learned.[1]

$$(1)\quad Lambana = \sqrt{\left[\left(\frac{4 \times \text{Sun's } dṛgjyā}{R}\right)^2 - \left(\frac{4 \times dṛkkṣepa}{R}\right)^2\right]}\ ghaṭīs.$$

$$(2)\quad Lambana = \sqrt{\left[\left(\frac{4 \times vitribhaśaṅku}{R}\right)^2 - \left(\frac{4 \times \text{Sun's } śaṅku}{R}\right)^2\right]}\ ghaṭīs.$$

These results are obviously equivalent to the formula of vs. 16.

Method 6

20. Of the three quantities *dṛkkṣepajyā*, *dṛgjyā* and *madhyajyā* (treated as the first, middle and last respectively), add the difference of the squares of the first and the last to the difference of the squares of the middle and the last, and then take the square-root of the sum. This square-root too gives the so called *dṛggati* (which yields the *lambana* as before).

$$Dṛggati = \sqrt{[(madhyajyā)^2 - (dṛkkṣepajyā)^2 + [(dṛgjyā)^2 - (madhyajyā)^2]}$$

The right hand side is evidently equal to

$$\sqrt{(dṛgjyā)^2 - (dṛkkṣepajyā)^2}$$

which is the value of the *dṛggati*.[2]

LAMBANA FOR MIDDAY OR SMALLER *LAMBANA*

21. The *lambana* should also be calculated from the smaller *dṛggati*, as before. When midday occurs prior to the Sun's position at the central ecliptic point, it should be subtracted from the time of midday; and if midday occurs later, then it should be added to that.

Method 7

22. Multiply the minutes of the Earth by 4 and divide by the radius, and diminish the resulting *nāḍīs* by the smaller *lambana* for midday : this result also is called larger *lambana* or true *lambana*.

---

1. *Cf. SiŚi*, I, vi. 6 (c-d)-7 (a-b).
2. See *SiŚi*, I, vi, 5 (c-d).

$$\text{Larger } lambana = \frac{\text{Earth} \times 4}{\text{R}} - \text{smaller } lambana \text{ for midday.}$$

Since

$$\text{Earth} = \text{smaller } d\d{r}ggati + \text{larger } d\d{r}ggati,$$

it simplifies to the formula

$$\text{larger } lambana = \frac{\text{larger } d\d{r}ggati \times 4}{\text{R}} \; n\bar{a}\d{d}\bar{\imath}s,$$

which is true. See *supra*, vs. 16.

Method 8

23. Or, obtain the product of the difference of the earth-segments (i. e., larger and smaller *dṛggatis*) and 4, and divide that by the radius; then (severally) add the resulting *ghaṭikās* to and subtract them from the *ghaṭīs* of the earth. The greater and smaller results (thus obtained) divided by 2 give the true *lambana* and the midday *lambana* (respectively).

$$\text{True } lambana \text{ or larger } lambana = \tfrac{1}{2}\left[\text{earth-}gha\d{t}\bar{\imath}s + \frac{(\text{difference of earth-segments}) \times 4}{\text{R}}\right]$$

$$Lambana \text{ for midday or smaller } lambana = \tfrac{1}{2}\left[\text{earth-}gha\d{t}\bar{\imath}s - \frac{(\text{ fference of earth-segments}) \times 4}{\text{R}}\right]$$

These formulae, stated in full, are :

$$\text{True } lambana \text{ or larger } lambana = \tfrac{1}{2}\left[\frac{(\text{larger } d\d{r}ggati + \text{smaller } d\d{r}ggati) \times 4}{\text{R}} + \frac{(\text{larger } d\d{r}ggati - \text{smaller } d\d{r}ggati) \times 4}{\text{R}}\right]$$

$$Lambana \text{ for midday or smaller } lambana = \tfrac{1}{2}\left[\frac{(\text{larger } d\d{r}ggati + \text{smaller } d\d{r}ggati) \times 4}{\text{R}} - \frac{(\text{larger } d\d{r}ggati - \text{smaller } d\d{r}ggati) \times 4}{\text{R}}\right]$$

The term "earth" means

larger *dṛggati* + smaller *dṛggati*

(see *supra*, vs. 11), and likewise "earth-*ghaṭīs*" means

$$\frac{(\text{larger } drggati + \text{smaller } drggati) \times 4}{R} \text{ } ghat\bar{\imath}s.$$

Method 9

24. Severally diminish the *dṛkkṣepa-śaṅku* ("Rsine of the altitude of the central ecliptic point") by the (smaller and larger) earth-segments; multiply (each result) by 4 and divide by the radius. By the resulting *ghaṭīs* (severally) diminish the same (*dṛkkṣepa-śaṅku*) as converted into *nāḍīs*. The results obtained are again the (smaller and larger) *lambanas*.

$$\text{Smaller } lambana = drkk\d{s}epa\text{-}\acute{s}a\dot{n}ku \text{ in } n\bar{a}\d{d}\bar{\imath}s - \frac{(drkk\d{s}epa\acute{s}a\dot{n}ku - \text{smaller } drggati) \times 4}{R}$$

$$\text{Larger } lambana = drkk\d{s}epa\acute{s}a\dot{n}ku \text{ in } n\bar{a}\d{d}\bar{\imath}s - \frac{(drkk\d{s}epa\acute{s}a\dot{n}ku - \text{larger } drggati) \times 4}{R},$$

where

$$drkk\d{s}epa\acute{s}a\dot{n}ku \text{ in } n\bar{a}\d{d}\bar{\imath}s = \frac{drkk\d{s}epa\acute{s}a\dot{n}ku \times 4}{R}.$$

Methods 10 and 11

25(a-c). The smaller *lambana* increased by the difference of the (larger and smaller) *lambanas* gives the larger *lambana*, and the larger *lambana* diminished by that difference gives the smaller *lambana*. The two *lambanas* may also similarly be obtained by subtraction from the sum of the two *lambanas*.

Larger *lambana* = smaller *lambana* + difference of larger and smaller *lambanas*

Smaller *lambana* = larger *lambana* − difference of larger and smaller *lambanas*

and

Larger *lambana* = sum of larger and smaller *lambanas* — smaller *lambana*

Smaller *lambana* = sum of larger and smaller *lambanas* — larger *lambana.*

Method 12

25(d)-26. (Severally) multiply and divide the square of the (Sun's) own *nāḍīnara* and the square of the (*madhyanāḍī*) *śaṅku* by the square of the *dṛkkṣepanāḍīnara* and subtract (the quotients) from the square of the *dṛkkṣepanāḍīnara*. The square-roots of the results obtained are known as the true and smaller *lambanas* (respectively).

True *lambana* or larger *lambana*

$$= \sqrt{(dṛkkṣepanāḍīnara)^2 - \frac{(dṛkkṣepanāḍīnara)^2 \times (\text{Sun's } nāḍīnara)^2}{(dṛkkṣepanāḍīnara)^2}}$$

$$= \sqrt{(dṛkkṣepanāḍīnara)^2 - (\text{Sun's } nāḍīnara)^2}$$

Smaller *lambana*

$$= \sqrt{(dṛkkṣepanāḍīnara)^2 - \frac{(dṛkkṣepanāḍīnara)^2 \times (madhyanāḍīnara)^2}{(dṛkkṣepanāḍīnara)^2}}$$

$$= \sqrt{(dṛkkṣepanāḍīnara)^2 - (madhyanāḍīnara)^2}$$

where

$$dṛkkṣepanāḍīnara = \frac{dṛkkṣepaśaṅku \times 4}{R}$$

$$\text{Sun's } nāḍīnara = \frac{\text{Sun's } śaṅku \times 4}{R}$$

$$\text{and } madhyanāḍīnara = \frac{madhyalagnaśaṅku \times 4}{R}.$$

The above methods give the *lambana* for the time of geocentric conjunction of the Sun and Moon. It provides a rough approximation for the *lambana* for the time of apparent conjunction. The next method gives a better approximation for the *lambana* for the time of apparent conjunction, and for practical purposes may be used as *lambana* for the time of apparent conjunction. The error will be negligible.

Method 13

27. (Severally) multiply the *dṛṅnara* ("Rsine of the Sun's altitude") and the *dṛggati* by the Rsine of the Sun's greatest declination and divide by the *dṛkkṣepaśaṅku* ("Rsine of the altitude of the central ecliptic point"): the results are the upright (*agra* or *koṭi*) and the base (respectively). Now if it is day, diminish the radius by the upright; and if it is night, increase the radius by the upright. By the square-root of the sum of the squares of that (difference or sum) and the base, divide the product of the *dṛkkṣepaśaṅku* and the base: the number of *asus* ("respirations"), thus obtained, gives the true *lambana* which should be applied once (i.e., not repeatedly) to the time of geocentric conjunction of the Sun and Moon, as before.[1]

That is : $$\text{upright} = \frac{\textit{dṛṅnara} \times \text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}$$

$$\text{base} = \frac{\textit{dṛggati} \times \text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}$$

$$\text{hypotenuse} = \sqrt{(\text{R} \mp \text{upright})^2 + (\text{base})^2}$$

and $$\textit{lambana} = \frac{\textit{dṛkkṣepaśaṅku} \times \text{base}}{\text{hypotenuse}} \textit{ asus.}$$

What is meant in the rule is not "the Rsine of the Sun's greatest declination" but "the Rsine of the greatest *lambana*." But both being equal, it is immaterial which of the two is mentioned.

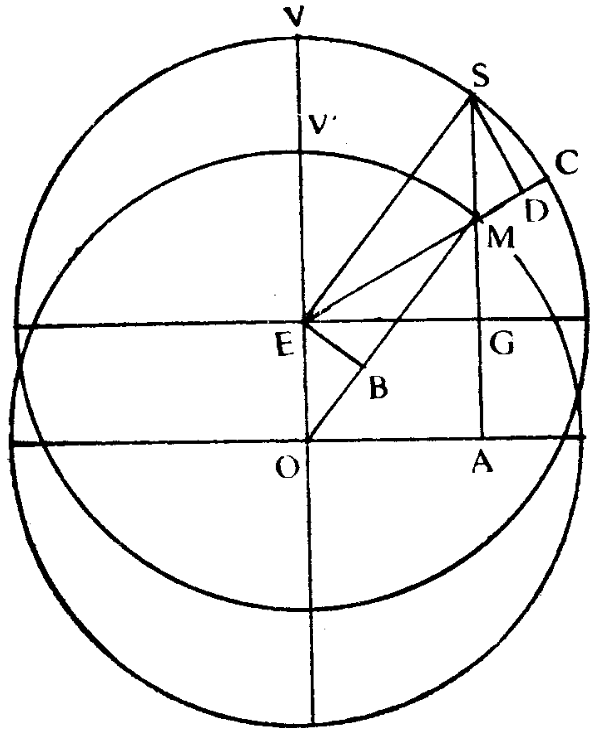
The rationale of this rule is as follows :

We shall suppose that the observer is stationed at the centre of the Earth and that *lambana* is caused due to the deflection of the Moon's orbit.

Consider the figure below. E is the Earth and the circle centred at E is the Sun's orbit; the other circle centred at O is the Moon's deflected orbit. V is the *vitribhalagna* of the Sun's orbit and V′ the *vitribhalagna* of the Moon's orbit. They are in the same direction from E, so that when the Sun is at V and the Moon at V′ the *lambana* is zero. OE is equal to the Rsine of the maximum *lambana*.

---

1. The method given by Bhāskara II in his *SiŚi*, I, vi. 8-9 is equivalent to this method.



In drawing the above figure, the consideration is that the *lambana* is zero when the Sun and the Moon are at the *vitribhalagna* and that the *lambana* is maximum when they are at the distance of 90° from the *vitribha-lagna*. The maximum *lambana* is taken to be equal to 24°, because

$$\text{maximum } lambana = 4 \ ghaṭīs = \frac{4 \times 360}{60} \text{ or 24 degrees.}$$

Let S and M be the positions of the Sun and the Moon at the time of geocentric conjunction when the distance of the Sun and the Moon from the *vitribhalagna* is the same. Then as seen from the observer at E the *lambana* is equal to as many *asus* as there are minutes in the arc SC. This is obtained as follows :

Join OM and drop EB perpendicular to OM. Also let SMA be parallel to VO. Then the triangles EBO and MOA are similar and we have

$$OB = \frac{MA \times EO}{OM}$$

or

$$\text{upright} = \frac{R\cos(\angle VES) \times R\sin 24^\circ}{R}$$

$$= \frac{R\cos(S \sim V) \times R\sin 24^\circ}{R}$$

$$= \frac{\text{Rcos } (S \sim V) \times \textit{dṛkkṣepaśaṅku}}{\text{R}} \times \frac{\text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}$$

$$= \frac{\text{Rsin (Sun's altitude)} \times \text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}$$

$$= \frac{\textit{dṛṅnara} \times \text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}, \quad (1)$$

where S and V are the longitudes of the Sun and the *vitribhalagna* or central ecliptic point;

$$\text{EB} = \frac{\text{OA} \times \text{EO}}{\text{OM}}$$

or

$$\text{base} = \frac{\text{Rsin } (S \sim V) \times \text{Rsin } 24^\circ}{\text{R}} \quad (i)$$

$$= \frac{\text{Rsin } (S \sim V) \times \textit{dṛkkṣepaśaṅku}}{\text{R}} \times \frac{\text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}$$

$$= \frac{\textit{dṛggati} \times \text{Rsin } 24^\circ}{\textit{dṛkkṣepaśaṅku}}; \quad (2)$$

and

$$\text{EM} = \sqrt{(\text{OM} \mp \text{OB})^2 + \text{EB}^2}$$

or

$$\text{hypotenuse} = \sqrt{(\text{R} \mp \text{upright})^2 + (\text{base})^2}. \quad (3)$$

Hence from the similar triangles SDM and EMG, we have

$$\text{SD} = \frac{\text{EG} \times \text{SM}}{\text{EM}}$$

i. e.,

$$\text{Rsin } (\textit{lambana}) = \frac{\text{Rsin } (S \sim V) \times \text{Rsin } 24^\circ}{\text{hypotenuse}}. \quad (4)$$

But this Rsin (*lambana*) has been obtained with the assumption that the maximum *lambana* = 24° which is the case when *dṛkkṣepaśaṅku* = R. In general, however, the Rsine of the maximum *lambana*

$$= \frac{\textit{dṛkkṣepaśaṅku} \times \text{Rsin } 24^\circ}{\text{R}},$$

so that, in fact,

$$\text{OE} = \frac{\textit{dṛkkṣepaśaṅku} \times \text{Rsin } 24^\circ}{\text{R}}.$$

Hence, the true value of

$$\text{Rsin}\,(\textit{lambana}) = \frac{\text{Rsin}\,(S \sim V)}{\text{hypotenuse}} \times \frac{\textit{dṛkkṣepaśaṅku} \times \text{Rsin}\,24°}{\text{R}}$$

$$= \frac{\text{base} \times \textit{dṛkkṣepaśaṅku}}{\text{hypotenuse}}, \text{ using (i)},$$

or, approximately,

$$\textit{lambana} = \frac{\text{base} \times \textit{dṛkkṣepaśaṅku}}{\text{hypotenuse}}. \tag{5}$$

Note. In the above rationale, the observer is supposed to be stationed at the centre of the Earth and *lambana* is supposed to be caused by the deflection of the Moon's orbit.

Bhāskara II's form. The hypotenuse EM can also be expressed in the alternative form as

$$\text{EM} = \sqrt{(\text{SG} - \text{SM})^2 + \text{EG}^2}$$

$$= \sqrt{[\text{Rcos}\,(S \sim V) - \textit{para}]^2 + [\text{Rsin}\,(S \sim V)]^2},$$

where SM, designated as *para* by Bhāskara II, is equal to

$$\frac{\textit{dṛkkṣepaśaṅku} \times \text{Rsin}\,24°}{\text{R}} \text{ or } \frac{\textit{dṛkkṣepaśaṅku} \times 13}{32}.$$

Therefore,

$$\textit{lambana} = \frac{\text{EG} \times \text{SM}}{\text{EM}}$$

$$= \frac{\text{Rsin}\,(S \sim V) \times \textit{para}}{\sqrt{[\text{Rcos}\,(S \sim V) - \textit{para}]^2 + [\text{Rsin}\,(S \sim V)]^2}}$$

$$= \frac{\text{Rcos}\,(L \sim S) \times \textit{para}}{\sqrt{[\text{Rsin}\,(L \sim S) - \textit{para}]^2 + [\text{Rsin}\,(L \sim S)]^2}}, \tag{6}$$

where L denotes the longitude of the rising point of the ecliptic and S that of the Sun.

Formula (6) is Bhāskara II's form for the *lambana* in one step (*sakṛt lambana*). See *SiŚi*, I, vi. 8-9.

The term *para* is evidently the short form of *paramalambana*.

A NOTE ON THE ABOVE METHOD

28. To begin with, obtain the above-mentioned *lambana* (by applying the rule) once. After its subtraction or addition (as the case may be), the process should not be iterated. Thus when this method is used, the *lambana* is fixed with lesser effort; if any other method is used, it is fixed with greater effort.

What is meant is that when the method just stated is used, there is no need for iterating the process. The *lambana* obtained by applying the rule once itself will give a fairly good approximation for the desired *lambana* In the case of the other methods stated before, iteration of the process is necessary until the *lambana* is fixed in value.

Method 14

29-30. Divide the *dṛkkṣepa* and the *dṛgjyā* ("Rsine of the Sun's zenith distance") by their own *śaṅkus* (i. e., the former by the *dṛkkṣepa-śaṅku* and the latter by the Sun's *śaṅku*) and multiply (each quotient) by 12 : the results are the corresponding (gnomonic) shadows, in terms of *aṅgulas*. Then multiply each result by the hypotenuse of shadow for the other. Square the two results and find the square-root of the difference (of those squares). Divide the resulting *aṅgulas* by the Sun's distance as multiplied by the "multiplier" (which is stated below) : then are obtained the *nāḍīs* of the *lambana*. One-fourth of the product of the two hypotenuses of shadow as divided by the Sun's distance gives the "multiplier" to be used here.

Lambana (in terms of *nāḍīs*)

$$= \frac{\sqrt{\left(\frac{\text{Sun's } dṛgjyā \times 12 \times h_2}{\text{Sun's } śaṅku}\right)^2 - \left(\frac{dṛkkṣepa \times 12 \times h_1}{dṛkkṣepaśaṅku}\right)^2}}{\text{Sun's distance} \times \text{multiplier}},$$

where

$$h_1 = \frac{\text{R} \times 12}{\text{Sun's } śaṅku}; \quad h_2 = \frac{\text{R} \times 12}{dṛkkṣepaśaṅku}$$

and $$\text{multiplier} = \frac{h_1\, h_2}{4 \times \text{Sun's distance}}.$$

The above formula is equivalent to the formula of vs. 16, because the right hand side of the above formula is equal to

$$= \frac{h_1 h_2 \sqrt{\left(\dfrac{\text{Sun's } d\underline{r}gjy\bar{a} \times 12}{\text{Sun's } \acute{s}a\dot{n}ku \times h_1}\right)^2 - \left(\dfrac{d\underline{r}kk\d{s}epa \times 12}{d\underline{r}kk\d{s}epa\acute{s}a\dot{n}ku \times h_2}\right)^2}}{\text{Sun's distance} \times \text{multiplier}}$$

$$= \frac{h_1 h_2 \sqrt{\left(\dfrac{\text{Sun's } d\underline{r}gjy\bar{a}}{R}\right)^2 - \left(\dfrac{d\underline{r}kk\d{s}epa}{R}\right)^2}}{\text{Sun's distance} \times \dfrac{h_1 h_2}{4 \times \text{Sun's distance}}}$$

$$= \frac{4\sqrt{(\text{Sun's } d\underline{r}gjy\bar{a})^2 - (d\underline{r}kk\d{s}epa)^2}}{R}$$

$$= \frac{4 \times d\underline{r}ggati}{R}.$$

OTHER FORMS FOR *DṚGGATI*

31. The Rsine of the difference between the longitudes of the Sun and the central ecliptic point (lit. rising point of the ecliptic minus 3 signs) multiplied by the Rsine of the altitude of the central ecliptic point (*dṛkkṣepaśaṅku*) and divided by the radius gives the *dṛggati*. The same *dṛggati* is also accurately obtained by multiplying the same (Rsine of the difference between the longitudes of the Sun and the central ecliptic point) by 12 and dividing by the hypotenuse of the (gnomonic) shadow when the Sun is at the central ecliptic point.[1]

Let the figure below represent the Celestial Sphere, centred at the observer O, in which DEN is the horizon and Z the zenith, VS the ecliptic and K its pole. V is the *vitribhalagna* ("central ecliptic point"), VC the *vitribhaśaṅku*. AB is the gnomon of 12 *aṅgulas* and BO its shadow when the Sun is at V. AO is the hypotenuse of this shadow. S is the Sun, ZS the Sun's zenith distance and ZF the *dṛggaticāpa*. Then,

$$d\underline{r}ggati = \text{Rsin ZF}$$

$$= \frac{\text{Rsin VS} \times \text{Rsin ZK}}{R}$$

$$= \frac{\text{Rsin VS} \times \text{Rsin VD}}{R}. \qquad (1)$$

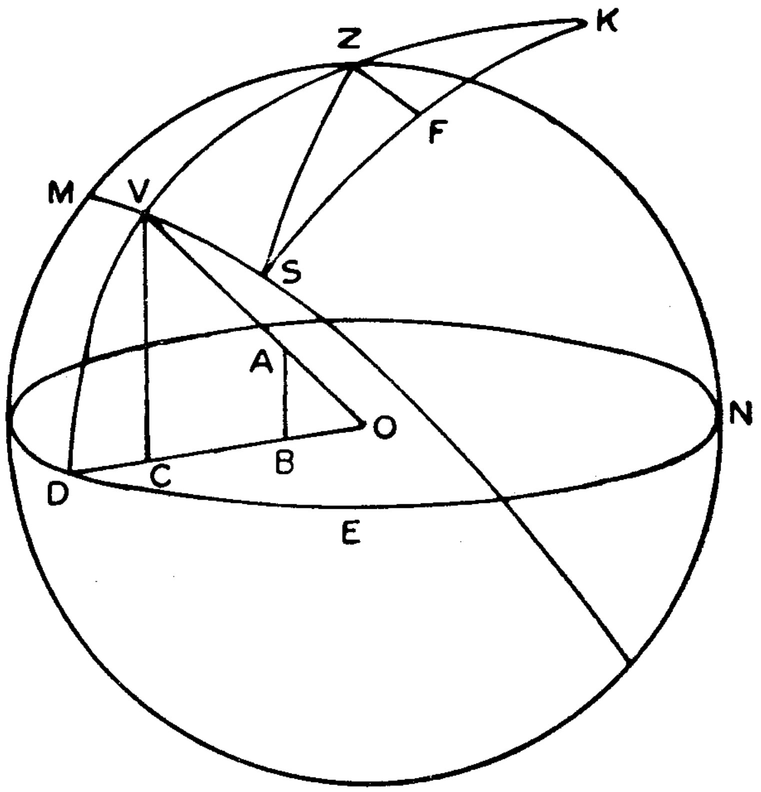
1. This rule is the same as found to occur in *BrSpSi*, v. 4, 6; and *SiŚi*, I, vi. 4, 5 (a-b). Also see *SiŚe*, vi. 4.

Since $\frac{\text{Rsin VD}}{\text{R}} = \frac{\text{VC}}{\text{VO}} = \frac{\text{AB or 12}}{\text{AO}}$,

therefore, we also have

$$d\d{r}ggati = \frac{\text{Rsin VS} \times 12}{\text{AO}}. \tag{2}$$

Hence the above rule.



Method 15 (Alternative to Method 13)

32. Diminish the longitude of the rising point of the ecliptic by three signs : (the result is the longitude of the central ecliptic point). Diminish that by the longitude of the Sun (or *vice versa*, according as the Sun is to the west or east of the central ecliptic point) : (the result is the arc of the ecliptic intervening between the Sun and the central ecliptic point). Find the Rsine and the Rcosine thereof; and divide (each of them) by $\frac{32}{13}$: (the results are the base and the upright). Diminish or increase the radius by the upright according as it is day or night. Find the sum of the squares of that (difference or sum) and the base ; and by the square-root thereof divide the base as multiplied by the Rsine of the altitude of the central ecliptic point (*dṛkkṣepaśaṅku*) : (the result is the *lambana*, in terms of *asus*).

$$Lambana = \frac{dṛkkṣepaśaṅku \times base}{\text{hypotenuse}}\ asus,$$

where
$$\text{base} = \frac{\text{Rsin}\,(S \sim V)}{32/13}$$

$$\text{upright} = \frac{\text{Rcos}\,(S \sim V)}{32/13}$$

and
$$\text{hypotenuse} = \sqrt{(\text{R} \mp \text{upright})^2 + (\text{base})^2},$$

where S and V are the longitudes of the Sun and the *vitribhalagna* or central ecliptic point, as before.

This rule is equivalent to that given in vs. 27, and has been obtained by replacing (Rsin 24°)/R by 13/32.

REMARK

33. The ***lambana*** **obtained here in this way is also** ***sakṛt*** **(i. e., obtained directly by applying the process once). The** ***lambana*** **may also be calculated from the** ***dṛggati*** **as before (see vs. 16). The** ***lambana*** **for midday may (similarly) be obtained from the Rsine of the portion of the ecliptic lying between the meridian ecliptic and central ecliptic points.**

A note on *Dṛggati*

The formula stated for *dṛggati* in vs. 10 above by Vaṭeśvara gives the value of Rsin ZB (see figure under vs. 13). The Hindu astronomers, however, take Rsin VS for the *dṛggati* and likewise Rsin ZB as the approximate value of Rsin VS. Vaṭeśvara too is of this opinion. He calls Rsin VS by the name *bṛhad-dṛggati* (larger *dṛggati*) and Rsin MV by the name *laghu-dṛggati* (smaller *dṛggati*), although the formulae stated by him for these larger and smaller *dṛggatis* give Rsin ZB and Rsin ZA respectively. Vaṭeśvara goes one step further. Treating the spherical triangle ZMS as a plane triangle, he takes[1]

VS = larger *dṛggati*

MV = smaller *dṛggati*

MS = "Earth" or Base

1. See Vaṭeśvara's *Gola*, ch. 3, vs. 24.

ZM = *madhyajyā*

ZS = *dṛgjyā*

and ZV = *dṛkkṣepa*,

and using the right-angled triangles ZVS and ZVM, he takes

$$VS = \sqrt{[ZS^2 - ZV^2]} \text{ and } MV = \sqrt{[MZ^2 - ZV^2]}$$

or larger *dṛggati* $= \sqrt{[(dṛgjyā)^2 - (dṛkkṣepa)^2]}$ (1)

and smaller *dṛggati* $= \sqrt{[(madhyajyā)^2 - (dṛkkṣepajyā)^2]}$. (2)

Other Hindu astronomers too have taken *dṛggati*, *dṛkkṣepa* and *dṛgjyā* as the sides of a right-angled plane triangle.

Since ZVS and ZVM are not plane triangles, formulae (1) and (2) are incorrect. Brahmagupta, therefore, has criticised Āryabhaṭa I for obtaining the *dṛggati* from formula (1) by treating the triangle ZVS as plane. He writes:

"*Dṛkkṣepajyā* is the base and *dṛgjyā* the hypotenuse; the square-root of the difference between their squares is the *dṛṅnatijyā* (*dṛggatijyā* or *dṛggati*). —This configuration is also improper."[1]

We have interpreted the rules stated by Vaṭeśvara correctly.[2]

1. *BrSpSi*, xi. 27.

2. Correction : Read [R cos $(L \sim S)]^2$ in place of [Rsin $(L \sim S)]^2$ in the denominator of formula (6) on p. 476.

Section 2

Nati or Parallax in Latitude

PRELIMINARY CALCULATIONS

1. One should obtain the longitude of the central ecliptic point, the Rsine of the zenith distance of the meridian ecliptic point and, by the subtraction or addition of the celestial latitudes of the Sun and Moon (at the central ecliptic point), the *dṛkkṣepa*, and so on, for the Sun as well as for the Moon. Whatever is obtained for the Sun should also be obtained for the Moon in the same way.

MOON'S *DṚKKṢEPA*

2. The arc corresponding to the *dṛkkṣepa* should be diminished or increased by the Moon's latitude at the central ecliptic point, according as they are of unlike or like directions. The Rsine of that (sum or difference) gives the Moon's *dṛkkṣepa*.[1] The other (elements) for the Sun and Moon should be obtained from their own means.

The direction of the *dṛkkṣepa* is north or south, according as the central ecliptic point is towards the north or south of the zenith.[2]

Āryabhaṭa II [*MSi*, vi. 11 (a-b)] and Bhāskara II (*SiŚi*, I, v. 18-19, com.) have criticised the addition or subtraction of the Moon's latitude at the central ecliptic point to or from the *dṛkkṣepa*. Bhāskara II says :

> "When the local latitude is 24°, the Sun and Moon at rising are at the first point of Libra, and the longitude of the Moon's ascending node is 6 signs, then the ecliptic is vertical and coincides with the prime vertical. So the Moon, deflected from the Sun due to parallax, remains on the ecliptic and does not leave it, and likewise there is no *nati*. But the correction of the Moon's latitude at the central ecliptic point (to the *dṛkkṣepa*) does give *nati*; this is of no use." (See Bhāskara II's commentary on *SiŚi*, I, vi. 18-19).

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1. Same rule occurs in *BrSpSi*, v. 9-10; *SiŚi*, I, vi. 11. Bhāskara II says that the rule stated in *SiŚi*, I, vi. 11 is Brahmagupta's rule, which he has mentioned; he does not agree with it.
2. See *BrSpSi*, v. 8; *SiŚi*, I, vi. 10 (c-d).

NATI OR PARALLAX IN LATITUDE

3. (The *dṛkkṣepa* of the Sun or Moon) should be multiplied by its own *mandakalākarṇa* and divided by the radius : then is obtained the true *dṛkkṣepa*. The true *dṛkkṣepas* (of the Sun and the Moon, obtained in this way) should be multiplied by the semi-diameter of the Earth and divided by their own (true) distances in *yojanas* : the resulting quotients are the *natis* (of the Sun and the Moon).

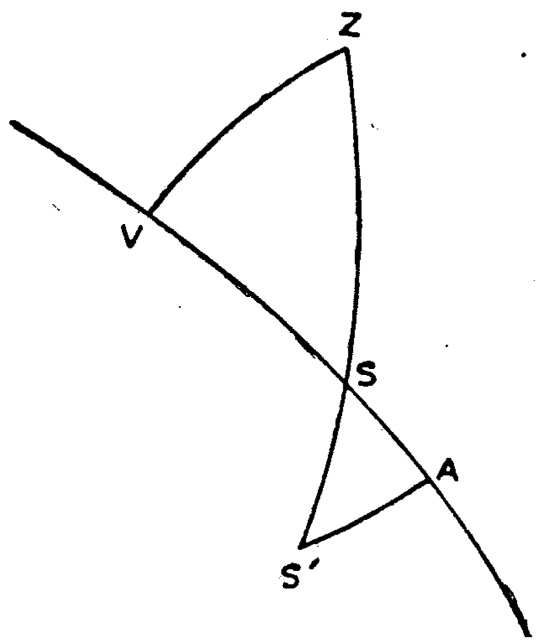
$$\text{Sun's true } dṛkkṣepa = \frac{\text{Sun's } dṛkkṣepa \times \text{Sun's } mandakarṇa \text{ in minutes}}{\text{R}}.$$

$$\text{Moon's true } dṛkkṣepa = \frac{\text{Moon's } dṛkkṣepa \times \text{Moon's } mandakarṇa \text{ in mins.}}{\text{R}}.$$

$$\text{Sun's } nati = \frac{\text{Sun's true } dṛkkṣepa \times \text{Earth's semi-diameter in } yojanas}{\text{Sun's true distance in } yojanas}.$$

$$\text{Moon's } nati = \frac{\text{Moon's true } dṛkkṣepa \times \text{Earth's semi-diameter in } yojanas}{\text{Moon's true distance in } yojanas}.$$

Rationale. Consider the adjoining figure. Let Z be the zenith. VSA the ecliptic, V the central ecliptic point, S the Sun, S′ the apparent Sun due to parallax, and S′A the perpendicular from S′ on the ecliptic. Then comparing the triangles SS′A and SZV,



S′A or Sun's *nati*

$$= \frac{\text{Rsin ZV} \times \text{Rsin SS}'}{\text{Rsin SZ}}$$

$$= \frac{\text{Sun's } dṛkkṣepa \times \text{Rsin SS}'}{\text{Rsin SZ}}.$$

But

$$\text{Rsin SS}' = \frac{\text{Sun's mean horizontal parallax in z. d.} \times \text{Rsin SZ}}{\text{R}}$$

$$= \frac{\text{Earth's semi-diameter in } yojanas \times \text{R}}{\text{Sun's mean distance in } yojanas} \times \frac{\text{Rsin SZ}}{\text{R}}$$

Therefore

$$\text{Sun's } nati = \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Earth's semi-diameter in } yojanas}{\text{Sun's mean distance in } yojanas}$$

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's true distance in } yojanas}{\text{Sun's mean distance in } yojanas} \times \frac{\text{Earth's semi-diameter in } yojanas}{\text{Sun's true distance in } yojanas}$$

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's } mandakar\d{n}a \text{ in minutes}}{R} \times \frac{\text{Earth's semi-diameter in } yojanas}{\text{Sun's true distance in } yojanas}$$

$$= \frac{\text{Sun's true } d\d{r}kk\d{s}epa \times \text{Earth's semi-diameter in } yojanas}{\text{Sun's true distance in } yojanas}.$$

Similarly,

$$\text{Moon's } nati = \frac{\text{Moon's true } d\d{r}kk\d{s}epa \times \text{Earth's semi-diameter in } yojanas}{\text{Moon's true distance in } yojanas}.$$

Method 2.

4. Or, multiply the (true) *dṛkkṣepas* (of the Sun and the Moon) by their true daily motions (in minutes) and divide by the radius and also by 15 : the results are their true *natis* in their own eccentric orbits.[1]

$$\text{Sun's } nati = \frac{\text{Sun's true } d\d{r}kk\d{s}epa \times \text{Sun's true daily motion}}{15 \times R}$$

$$\text{Moon's } nati = \frac{\text{Moon's true } d\d{r}kk\d{s}epa \times \text{Moon's true daily motion}}{15 \times R}.$$

Rationale. We have

$$\text{Sun's } nati = \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's mean horizontal parallax in z. d.}}{R}$$

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's mean motion for 4 } gha\d{t}\bar{\imath}s}{R}$$

1. *Cf. LBh*, v. 11; *ŚiDVr*, vi. 11 (c); *SiŚi*, I, vi. 11 (c-d)-12 (a-b).

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's mean daily motion in minutes}}{15 \times R}$$

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's mean daily motion in minutes}}{\text{Sun's true daily motion in minutes}}$$

$$\times \frac{\text{Sun's true daily motion in minutes}}{15 \times R}$$

$$= \frac{\text{Sun's } d\d{r}kk\d{s}epa \times \text{Sun's } mandakar\d{n}a \text{ in minutes}}{R}$$

$$\times \frac{\text{Sun's true daily motion in minutes}}{15 \times R}$$

$$= \frac{\text{Sun's true } d\d{r}kk\d{s}epa \times \text{Sun's true daily motion in minutes}}{15 \times R}.$$

Similarly for the Moon's *nati.*

Method 3

5. **Or, multiply the mean *dṛkkṣepas* (of the Sun and the Moon) by the Earth's semi-diameter and divide by their own mean distances in *yojanas* : then are obtained the *natis* (of the Sun and the Moon).**

$$\text{Sun's } nati = \frac{\text{Sun's mean } d\d{r}kk\d{s}epa \times \text{Earth's semi-diameter in } yojanas}{\text{Sun's mean distance in } yojanas}$$

$$\text{Moon's } nati = \frac{\text{Moon's mean } d\d{r}kk\d{s}epa \times \text{Earth's semi-diameter in } yojanas}{\text{Moon's mean distance in } yojanas}.$$

Method 4

6. **Or, multiply the (mean) *dṛkkṣepas* (of the Sun and the Moon) by their own mean daily motions (in minutes) and also by $\frac{1}{15}$ and divide by the radius : then are obtained the *natis* (of the Sun and the Moon) in terms of minutes, in their respective order.**[1]

1. *Cf. BrSpSi,* v. 11; *SiŚe,* vi. 8 (a-b); *SiŚi,* I, vi. 11 (c-d).
Brahmagupta (*BrSpSi,* v, 12 (a-b)) gives the following alternative forms :

$$\text{Sun's } nati = \frac{ravid\d{r}kk\d{s}epacch\bar{a}y\bar{a} \times \text{Sun's mean daily motion}}{15 \times ravid\d{r}kk\d{s}epacch\bar{a}y\bar{a}kar\d{n}a}$$

$$\text{Moon's } nati = \frac{candrad\d{r}kk\d{s}epacch\bar{a}y\bar{a} \times \text{Moon's mean daily motion}}{15 \times candrad\d{r}kk\d{s}epacch\bar{a}y\bar{a}kar\d{n}a}.$$

These formulae are analogous to the similar formulae for the *lambana* in the previous section. See sec. 1, vs. 31.

$$\text{Sun's } nati = \frac{\text{Sun's (mean) } d\d{r}kk\d{s}epa \times \text{Sun's mean daily motion in mins.}}{15 \times R}.$$

$$\text{Moon's } nati = \frac{\text{Moon's (mean) } d\d{r}kk\d{s}epa \times \text{Moon's mean daily motion in mins.}}{15 \times R}.$$

All the four methods given above are equivalent.

MOON'S TRUE *NATI* AND MOON'S TRUE LATITUDE

7. The sum or difference of the *natis* (of the Sun and the Moon), according as they are of unlike or like directions, gives the Moon's *nati* relative to the Sun's disc. That (Moon's relative *nati*) added to or subtracted from the Moon's instantaneous latitude according as the two are of like or unlike directions, gives Moon's true latitude.[1]

The direction of the *nati* is taken to be north or south according as the meridian or central ecliptic point is towards the north or south of the zenith. See *ŚiDVṛ*, vi. 11 (b) or *MSi*, vi. 12 (a-b). Thus the direction of the *nati* is the same as that of the *dṛkkṣepa.*

1. *Cf. PSi*, viii. 14; ix. 25; *MBh*, v. 31; *LBh*, v. 12; *BrSpSi*, v. 12 (c-d)-13; *KK*, I, v. 4; *ŚiDVṛ*, vi. 11(d); *ŚūSi*, v. 12; *MSi*, vi. 12; *SiŚe*, vi. 8(c-d); *SiŚi*, I, vi. 12 (a-b), 14(c-d).

Section 3 : *Sthityardha* and *Vimardārdha*

STHITYARDHA AND *VIMARDĀRDHA*

Method 1

1-2. Calculate the *sthityardha* and the *vimardārdha* as in the case of a lunar eclipse.[1] Subtract them from and add them to the time of geocentric conjunction (*karaṇāgata-tithi*). Then using the iteration process, determine the corresponding *lambana-nāḍīs*; and then obtain the true times of contact and separation. From the difference between the times of contact and the middle of the eclipse and from the difference between the times of the middle of the eclipse and separation, obtain the (*spārśika* and *maukṣika*) *sthityardhas*.

Similarly are obtained the (true) times of immersion and emersion; and from the difference between the times of immersion and the middle of the eclipse and the difference between the times of the middle of the eclipse and emersion are obtained the (*spārśika* and *maukṣika*) *vimardārdhas*.[2]

The time of the middle of the eclipse is evidently the time of apparent. conjunction of the Sun and Moon.

The rule stated in the text may be fully described as follows :

Time of apparent or true conjunction.

First of all calculate the time of geocentric conjunction (*gaṇitāgata* or *karaṇāgata darśānta* or *amānta*). Then calculate the *lambana* for that time; and treating it as the *lambana* for the time of apparent conjunction, obtain the time of apparent conjunction by the formula :

time of apparent conjunction = time of geocentric conjunction
$\pm$ *lambana* for the time of apparent conjunction, (1)

+ or − sign being taken according as the conjunction occurs to the west or east of the central ecliptic point.

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1. *Cf. BrSpSi*, v. 13 (c-d); *KK*, I, v. 4 (c-d).
2. *Cf. BrSpSi*, v. 16-17; *KK*, I, v. 5; *ŚiDVr*, vi. 12; *MSi*, vi. 13-15; *ŚiŚi*, I, vi. 15-16.

Next, calculate the *lambana* for the time of apparent conjunction (thus obtained); and then obtain the time of apparent conjunction by formula (1) again.

Then calculate the *lambana* for the time of apparent conjunction (just obtained), and obtain the time of apparent conjunction by formula (1) again.

Repeat this process until the *lambana* for the time of apparent conjunction is fixed. Applying this *lambana* in formula (1), get the correct time of apparent conjunction. This is called the time of *spaṣṭa darśānta* or *spaṣṭa amānta*, and also the time of the middle of the eclipse.

Spārśika and maukṣika sthityardhas (calculated as in the case of a lunar eclipse).

Calculate the semi-diameters of the Sun and Moon and also the Moon's true latitude (i. e., the Moon's latitude corrected for *nati*) for the time of apparent conjunction; and then taking them as the semi-diameters of the Sun and Moon and the Moon's true latitude for the time of the first contact, calculate the *spārśika sthityardha* by the formula :

$$\text{spārśika sthityardha} = \frac{\sqrt{(S+M)^2 - \beta_1^2}}{d} \text{ ghaṭīs}, \qquad (2)$$

where S, M are the semi-diameters of the Sun and Moon, β_1 the Moon's true latitude for the time of the first contact, and d the difference between the true daily motions of the Sun and Moon in terms of degrees. (In practice one uses the semi-diameters of the Sun and Moon for the time of apparent conjunction, because the semi-diameters of the Sun and Moon for the time of the first contact are practically the same as those for the time of geocentric or apparent conjunction).

Thereafter, find the time of the first contact by the formula :

$$\text{time of first contact} = \text{time of geocentric conjunction} - \text{spārśika sthityardha}. \qquad (3)$$

Next, calculate the Moon's true latitude for the time of the first contact (thus obtained); and then find the *spārśika sthityardha* by formula (2), and thereafter the time of the first contact by formula (3).

Then calculate the Moon's true latitude for the time of the first contact

(just obtained); then calculate the *spārśika sthityardha* by formula (2), and thereafter the time of the first contact by formula (3) again.

Repeat this process until the *spārśika sthityardha* and the time of the first contact are fixed.

Similarly, find the *maukṣika sthityardha* and the time of separation; and also the *spārśika* and *maukṣika vimardārdhas* and the times of immersion and emersion.

The *sthityardhas* and *vimardārdhas* which are thus obtained are called *madhyama* (or mean) *sthityardhas* and *vimardārdhas,* because they are still uncorrected for *lambana.*

Lambanas for the times of apparent first contact and separation.

Calculate the *lambana* for the time of the first contact obtained above; and treating it as the *lambana* for the time of apparent first contact, obtain the time of apparent first contact by the formula:

time of apparent first contact = time of first contact $\pm$ *lambana* for the time of apparent first contact, (4)

$+$ or $-$ sign being taken according as the first contact takes place to the west or east of the central ecliptic point.

For the time of apparent first contact, thus obtained, calculate the *lambana* afresh and applying it in formula (4) obtain the time of apparent first contact again.

Repeat this process until the *lambana* for the time of apparent first contact is fixed.

Similarly, find the *lambanas* for the times of apparent separation, immersion and emersion.

Spārśika and maukṣika sthityardhas, corrected for lambana.

The *madhyama spārśika* and *madhyama maukṣika sthityardhas,* corrected for *lambana,* are called *sphuṭa* (or true) *spārśika* and *sphuṭa maukṣika sthityardhas.* They are obtained by the formulae :

true *spārśika sthityardha* = time of apparent conjunction
— time of apparent first contact

true *maukṣika sthityardha* = time of apparent separation
− time of apparent conjunction.

Similarly,

true *spārśika vimardārdha* = time of apparent conjunction
− time of apparent immersion

true *maukṣika vimardārdha* = time of apparent emersion
− time of apparent conjunction.

Method 2

3(a-b). The *spārśika* and *maukṣika sthityardhas* become true when they are increased by (i) the difference between the *lambanas* for the first contact and the middle of the eclipse and (ii) the difference between the *lambanas* for the middle of the eclipse and the last contact, (respectively). The *spārśika* and *maukṣika vimardārdhas* become true when they are increased by (i) the difference between the *lambanas* for immersion and the middle of the eclipse and (ii) the difference between the *lambanas* for the middle of the eclipse and emersion, (respectively).

3(c-d)-4(a-b). (But this is the case) when the *lambana* for the first contact is subtractive and greater than the *lambana* for the middle of the eclipse (which is also subtractive), or additive and less than the *lambana* for the middle of the eclipse (which is also additive); and the *lambana* for the last contact is additive and greater than the *lambana* for the middle of the eclipse (which is also additive), or subtractive and less than the *lambana* for the middle of the eclipse (which is also subtractive).

4(b-d). In the contrary case, the (*spārśika* or *maukṣika*) *sthityardha* (or *vimardārdha*) becomes true when it is diminished by the corresponding *lambana*-difference.

5(a-b). When one *lambana* is additive and the other subtractive, in that case the true *sthityardha* (or *vimardārdha*) is obtained by adding the sum of those *lambanas*.[1]

This method is essentially the same as the previous one. The difference is that the *madhyama spārśika sthityardha* and the *lambana* for the time

1. *Cf. SuSi*, iv 18-20. Also see *BrSpSi*, v. 18; *ŚiDVr*, vi. 16 (a-b); *SiŚe*, vi. 14 (a-b); *SiŚi*, I, vi. 18-19.

of apparent first contact having been obtained, the *spaṣṭa* or true *spārśika sthityardha* is obtained by the addition or subtraction of the difference or sum of the *lambanas* for the times of apparent conjunction and apparent first contact to the *madhyama spārśika sthityardha*, in the manner prescribed in the text.

Let T be the time of geocentric conjunction, T' the time of apparent conjunction, t_1 the time of the first contact, and t'_1 the time of apparent first contact. Let L be the *lambana* for the time T' and l_1 the *lambana* for the time t'_1. Let s be the mean *spārśika sthityardha*. Then, if the solar eclipse occurs to the east of the central ecliptic point,

$$T' = T - L$$

$$t'_1 = T - s - l_1.$$

Therefore,

$$\text{true } spārśika \ sthityardha = T' - t'_1$$

$$= s + (l_1 - L), \quad \text{if } l_1 > L$$

$$= s - (L - l_1), \quad \text{if } l_1 < L.$$

If the solar eclipse occurs to the west of the central ecliptic point, then

$$T' = T + L$$

$$t'_1 = T - s + l_1.$$

Therefore,

$$\text{true } spārśika \ sthityardha = T' - t'_1$$

$$= s + (L - l_1), \quad \text{if } l_1 < L$$

$$= s - (l_1 - L), \quad \text{if } l_1 > L.$$

In case l_1 is subtractive and L additive, then

$$T' = T + L$$

$$t'_1 = T - s - l_1.$$

Therefore,

$$\text{true } spārśika \ sthityardha = s + (L + l_1).$$

Similarly, in the other cases.

IṢṬAGRĀSA FOR *IṢṬAKĀLA*

5(c-d)-7. Multiply the difference between the (true) *spārśika sthityardha* and the *iṣṭakāla*, the so called *vīṣṭa*, by the *madhyama sthityardha* and divide by the *spaṣṭa sthityardha* : the result is the truer than the true *vīṣṭa*.

Multiply that by the difference, in terms of degrees, between the true daily motions of the Sun and the Moon : this is the *bhuja* ("base"). The Moon's (true) latitude at the extremity of that (*bhuja*) is the *koṭi* ("upright"). The square-root of the sum of their squares is known as the hypotenuse for the given time.

Subtract that from the sum of the semi-diameters of the eclipsed and eclipsing bodies : the remainder obtained is the *iṣṭagrāsa*.[1]

The *iṣṭakāla* is the time elapsed since the beginning of the eclipse. We shall denote it by *i*.

Let *t* be the given time (measured since sunrise), and suppose that, at that time, MC (See the figure) is the ecliptic, M_0 the Moon, M_1 its apparent position (due to parallax), M its position on the ecliptic, S the Sun and S_1 its apparent position (due to parallax). M_0M, M_1B and S_1C are perpendicular to the ecliptic and S_1A perpendicular to M_1B. Then, in the right-angled triangle M_1AS_1,

S_1A is the *bhuja* (base)

M_1A is the *koṭi* (upright)

M_1S_1 is the *karṇa* (hypotenuse).

Now *bhuja* S_1A = BC

= MS + SC − MB

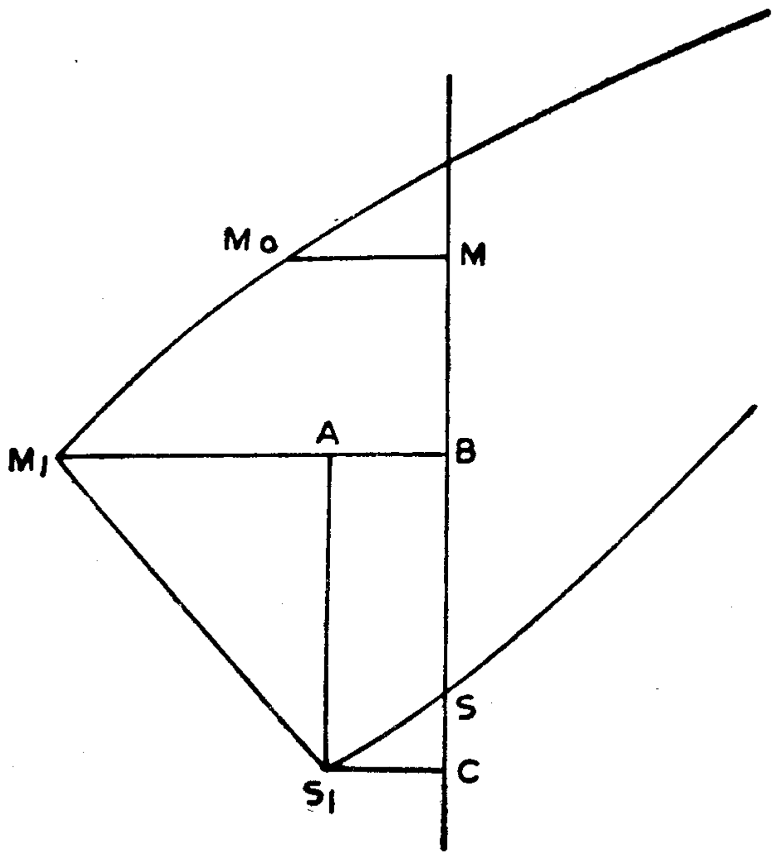
= MS − (MB − SC)

= MB − (Moon's *lambana* − Sun's *lambana*)

= MB − *lambana* at time *t*.

∴ MB = *bhuja* + *lambana* at time *t*.

1. *Cf. BrSpSi*, v. 14-15; *KK*, I, v. 6; *ŚiDVr*, vi. 13; *SiŚe*, vi. 11-12; *SūSi*, v. 15-17.



Let T be the time of geocentric conjunction, T' the time of apparent conjunction, and L *ghaṭīs* the *lambana* at time T'. Let t_1 be the time of the first contact, t'_1 the time of the apparent first contact, and l_1 *ghaṭīs* the *lambana* at time t'_1. Let l *ghaṭīs* be the *lambana* for the given time t. Let s be the mean *spārśika sthityardha* and s' the true *spārśika sthityardha*. Then (assuming that the eclipse occurs towards the east of the central ecliptic point)

$$T' = T - L$$

$$t'_1 = T - s - l_1$$

$$t = T - \textit{ghaṭīs} \text{ corresponding to MS}$$

$$= T - (\textit{bhujaghaṭīs} + l).$$

$$\therefore \textit{iṣṭakāla } i = (T - \textit{bhujaghaṭīs} - l) - (T - s - l_1)$$

$$= s + (l_1 - l) - \textit{bhujaghaṭīs}.$$

$$\therefore \textit{bhujaghaṭīs} = s - i + (l_1 - l).$$

Now we apply the proportion: When $(l_1 - L)$ is the *lambana*-diff-

erence corresponding to s', what will be the *lambana*-difference corresponding to i ? The result is $l_1 - l$. Thus

$$l_1 - l = \frac{i\,(l_1 - L)}{s'}.$$

Therefore

$$bhujaghaṭīs = s - i + \frac{i\,(l_1 - L)}{s'}$$

$$= \frac{ss' - i\,(s' - l_1 + L)}{s'}.$$

But
$$s' = T' - t'_1$$

$$= (T - L) - (T - s - l_1)$$

$$= s + l_1 - L,$$

so that $s = s' - l_1 + L.$

$$\therefore\ bhujaghaṭīs = \frac{ss' - is}{s'}$$

$$= \frac{s\,(s' - i)}{s'}\ ghaṭīs.$$

Hence $\quad$ *bhuja* $S_1A = \dfrac{s\,(s' - i)}{s'} \times d$ minutes,

where d denotes the difference, in terms of degrees, between the true daily motions of the Sun and the Moon.

Also, evidently,

koṭi M_1A = Moon's true latitude

and *karṇa* M_1S_1 = distance between apparent Sun and Moon.

Hence $\quad$ *iṣṭagrāsa* = sum of semi-diameters of Sun and Moon — *karṇa*.

The method for finding the *iṣṭakāla* from the *iṣṭagrāsa* has been omitted here by Vaṭeśvara. The method is just the reverse of what has been stated above. The interested reader is referred to the *Brāhma-sphuṭa-siddhānta* (v. 18-19) of Brahmagupta, the *Śiṣya-dhī-vṛddhida* [vi. 16(c-d)] of Lalla, the

Sūrya-siddhānta (iv. 22-23), the *Mahā-siddhānta* (v. 15) of Āryabhaṭa II, the *Siddhānta-śekhara* (vi. 14) of Śrīpati and the *Siddhānta-śiromaṇi* (vi. 18-19) of Bhāskara II.

CORRECTION TO MOON'S DIAMETER

8-9. Divide the Moon's true daily motion by 300, and subtract the quotient from half the minutes of the Moon's diameter : (the result is the true semi-diameter of the Moon in terms of minutes).

Since, in a solar eclipse, the Sun is bright and the Moon transparent, therefore prior to constructing the diagram of a solar eclipse one should compute the (*spārśika* and *maukṣika*) *vimardārdhas*, and the (*spārśika* and *maukṣika*) *sthityardhas* by making use of this true (semi-diameter of the) Moon's disc. But in the case of a lunar eclipse, this is not to be used.

Subtraction of

$$\frac{\text{Moon's true daily motion}}{300}$$

from the Moon's semi-diameter in terms of minutes is prescribed here in the case of a solar eclipse, because (*vide infra*, sec. 4, vs. 31) in the case of a solar eclipse, eclipse amounting to one-twelfth of the Sun's diameter is not visible to the naked eye. One can easily see that

$$\frac{\text{Moon's true daily motion}}{300} \quad \text{and} \quad \frac{\text{Sun's diameter in minutes}}{12}$$

are both approximately equal to $2\frac{2}{3}$ minutes.

VALANA ETC.

10(a) Calculation of the *valana* etc. is to be done as in the case of a lunar eclipse.

REVERSAL OF DIRECTIONS EXPLAINED

10(b-d). Since in its own eclipse the Moon enters into the Earth's shadow and in the eclipse of the Sun it enters into the Sun, this is why reversal of directions is made in the case of the two eclipses (while making their diagrams).

Section 4 : *Parilekha* or Diagram

INTRODUCTION

1. Since the various phases of an eclipse become clear from its diagram, therefore I shall (now) describe, in clear terms, the method of constructing the diagram of the eclipses of the Sun and the Moon.

CONSTRUCTION OF THREE CIRCLES

2. By means of a compass construct on the ground (a circle denoting) the eclipsed body with radius equal to half its diameter measured in *aṅgulas*, another circle with radius equal to the sum of the semi-diameters of the eclipsed and eclipsing bodies, and still another circle, called the radius-circle, with radius equal to the Rsine of three signs.[1]

These circles are known as *grāhya-vṛtta*, *mānaikyārdha-vṛtta*, and *trijyā-vṛtta*, respectively.

3(a-c). The centres of these circles lie at the same place; the east-west line is also one and the same. So also is the north-south line drawn with the help of a fish-figure of that (east-west line).

LAYING OFF OF *VALANA* IN THE *TRIJYĀ-VṚTTA*

3(d)-5. In the so called *trijyā-vṛtta* ("radius-circle"), the *spārśika* and *maukṣika digvalanas* for the eclipses of the Moon and the Sun, respectively, should be laid off on the eastern side, whereas the *maukṣika* and *spārśika digvalanas* (for the eclipses of the Moon and the Sun, respectively) should be laid off on the western side. They should be laid off like the Rsine towards the south or north (in their own directions on the eastern side) and towards the north or south in the contrary directions (on the western side). Thereafter one should lay off the *digvalana* for the middle of the eclipse from the north or south point (towards the east or west), in the manner prescribed for it. Then one should draw three lines (*sūtras*) having their extremities at those points (i. e., at the ends of the *spārśika*, *madhya* and *maukṣika digvalanas*) and going to the centre. Starting from them, one should lay off in the *sthiti-vṛtta* (= *mānaikyārdha-vṛtta*).

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1. *Cf. KK*, II, iv. 6; *ŚiDVṛ*, v. 30 (a-b); *SiŚe*, v. 25 (a-b).

The *digvalana* for the middle of the eclipse is laid off in accordance with the following rules.

Rule for lunar eclipse :

When the direction of the Moon's latitude for the middle of the eclipse is north, then the *digvalana* for the middle of the eclipse should be laid off from the south point towards the east or west, according as the direction of the *digvalana* for the middle of the eclipse is north or south.

When the direction of the Moon's latitude for the middle of the eclipse is south, then the *digvalana* for the middle of the eclipse should be laid off from the north point towards the west or east, according as the direction of the *digvalana* for the middle of the eclipse is north or south.

Rule for solar eclipse :

When the direction of the Moon's latitude for the middle of the eclipse is north, then the *digvalana* for the middle of the eclipse should be laid off from the north point towards the west or east, according as the direction of the *digvalana* for the middle of the eclipse is north or south.

When the direction of the Moon's latitude for the middle of the eclipse is south, then the *digvalana* for the middle of the eclipse should be laid off from the south point towards the east or west, according as the direction of the *digvalana* for the middle of the eclipse is north or south.

LAYING OFF OF MOON'S LATITUDE IN THE *MĀNAIKYĀRDHA-VṚTTA*

6 The (Moon's) latitudes for the first and last contacts should be laid off from their own *sūtras* (in the *mānaikyārdha-vṛtta*) like the Rsines towards their own directions if the eclipse be solar, or towards the contrary directions if the eclipse be lunar.[1] The (Moon's) latitude for the middle of the eclipse should be laid off from the centre along its own *sūtra* (in its own direction in the case of a solar eclipse and in the contrary direction in the case of a lunar eclipse).

CONTACT AND SEPARATION POINTS AND *MADHYAGRĀSA*

7. Taking the (three) ends of the (Moon's) latitudes as centre and the semi-diameter of the eclipsing body as radius, one should construct by

1. *Cf.* *ŚiDVr*, v. 29 (c-d); *SūSi*, vi. 8; *SiŚe*, v. 25, 27-29.

means of a compass three circles (touching or) cutting the circle of the eclipsed body. Then are clearly seen the points of contact and separation as well as the phase at the middle of the eclipse.[1]

8. Or, (the points) where the circles constructed by taking the ends of the (Moon's) latitudes for the first and last contacts as centre touch the circle of the eclipsed body are said to be the points where contact and separation actually take place.

9. Or, taking that point as centre which lies at a distance equal to the radius (of the *trijyā-vṛtta*) minus the (Moon's) latitude for the middle of the eclipse from the extremity of the *digvalana* (for the middle of the eclipse) on its own direction-line (i. e., on the line joining the extremity of the *digvalana* to the centre), one should cut (the circle of) the eclipsed body by means of a compass with radius equal to the semi-diameter of the eclipsing body : the *madhyagrāsa* (i. e., the portion eclipsed at the middle of the eclipse) will then be clearly exhibited.

PATH OF ECLIPSING BODY AND *IṢṬAGRĀSA*

10. With the three points (lying at the ends) of the latitudes, construct a pair of fishes. Taking the point of intersection of the strings passing through their heads and tails as centre, construct a large circle going through the ends of the latitudes. This is the path of the eclipsing body.[2] Where the extremity of the hypotenuse (stretched from the centre) touches it, with that point as centre draw a circle by means of a compass with radius equal to the semi-diameter of the eclipsing body. Then will be obtained the *iṣṭagrāsa* ("the measure of the eclipse for the desired time").[3]

11. Similarly, (the points of) contact and separation should also be determined from the hypotenuses lying between (the centres of) the discs (of the eclipsed and eclipsing bodies at those times).

DIAGRAM FOR TOTAL ECLIPSE

12 (a-c). When the eclipse is total, one should lay off the *digvalanas* for the times of immersion, middle of the eclipse, and emersion as well

1. Rule given in vss. 3(d)-7 is the same as stated in *ŚiDVṛ*, v. 30-33. Rule given in vss. 2-7 is the same as stated in *KK*, II, iv. 6-10; 11-13. Also *cf. MSi*, viii, 1-7 (a-b).
2. *Cf. BrSpSi*, xvi, 39-40 (a-b); *ŚiDVṛ*, v. 34 (a-b); *SiŚe*, v. 34 (a-b).
3. *Cf. ŚiDVṛ*, v. 34 (c-d); *SiŚe*, v. 34 (c-d).

as the Moon's latitudes for those times. The *iṣṭagrāsa* is also exhibited in the same way.

THE ECLIPSE TRIANGLE

12(d)-13(a-b). The Moon's latitude for that time is the upright. Along the *bāhusūtra* ("ecliptic") lies the base. Touching (one end of) the base and extending from the centre upto the extremity of the upright lies the hypotenuse.

DIFFERENCE BETWEEN LUNAR AND SOLAR ECLIPSES

13(b-d). In the case of the Moon, the eclipse begins towards the eastern side (of the body) and the base for the time of separation lies towards the west. In the case of the Sun, the case is just the reverse; this is why the portion of the Sun intercepted by the circle drawn with radius equal to half the diameter of the eclipsing body lies on the reverse side.

PARILEKHA IN THE *TRIJYĀ-VṚTTA*

14-15. The Moon's latitudes for the times of contact and separation should be multiplied by the radius and divided by half the sum of the diameters of the eclipsed and eclipsing bodies; and then they should be laid off in the *trijyā-vṛtta* ("radius-circle") towards their own directions (as before). The Moon's latitude for the middle of the eclipse (as it is) should be laid off from the centre (as before). The eclipsing body should then be drawn in order to know the amount of eclipse etc. for the desired time, in the eclipsed body. The *Parilekha* for the times of immersion etc. pertaining to the desired eclipse is as before.

The larger *digvalanas* for those times (i. e., the *digvalanas* as obtained by calculation) should be laid off in the circle (i. e., in the *trijyā-vṛtta*), (as before).

This *Parilekha* is essentially the same as before; only the Moon's latitudes for the times of contact and separation are now laid off in the *trijyā-vṛtta* (instead of in the *mānaikyārdha-vṛtta*) after being multiplied by the radius and divided by the radius of the *mānaikyārdha-vṛtta*

PARILEKHA IN THE *MĀNAIKYĀRDHA-VṚTTA*

16-17. The Rsines of the *digvalanas* (for the times of contact, middle of the eclipse, and separation etc,) should be multiplied by the sum of the semi-diameters of the eclipsed and eclipsing bodies and (the products obtai-

ned) should be divided by the radius : (the Rsines of the *digvalanas* are thus reduced to the *mānaikyārdha-vṛtta*). They should then be laid off in the *mānaikyārdha-vṛtta*. The Moon's latitudes for the times of contact and separation should then be laid off from the extremities of the corresponding Rsines of the *digvalanas*. The Moon's latitude for the middle of the eclipse should be laid off from the centre towards its own direction (as before).

The *Parilekha* in the *mānaikyārdha-vṛtta* for the time of immersion (or emersion) or for the desired time is similar.

Every other thing (relating to *Parilekha* in the *mānaikyārdha-vṛtta*) is just the same as in the case of *Parilekha* in the *trijyā-vṛtta*.[1]

PARILEKHA IN THE *GRĀHYA-VṚTTA*

18-20. (The *digvalanas* for the times of contact and separation should be multiplied by the semi-diameter of the eclipsed body and divided by the radius : the resulting reduced *digvalanas* should be laid off in the *grāhya-vṛtta*, as before). The Moon's latitudes for the times of contact and separation should be multiplied by the semi-diameter of the eclipsed body and (the products obtained) should be divided by half the sum of the diameters of the eclipsed and eclipsing bodies. They should then be laid off from the extremities of those *digvalanas*. The Moon's latitude for the middle of the eclipse, as it is, should be laid off from the centre; (but before doing this) the corresponding *digvalana* (as reduced to the *grāhya-vṛtta*) should be laid off (as before). Taking the extremity of the Moon's latitude for the middle of the eclipse as centre one should then cut the eclipsed body by means of a compass with radius equal to the semi-diameter of the eclipsing body : this will give the measure of the eclipse. The other two extremities of the Moon's latitudes for the times of contact and separation are the true positions of contact and separation. One should now lay off from the centre two threads of length equal to half the sum of the diameters of the eclipsed and eclipsing bodies, passing through them (i e., passing through the positions of contact and separation).

21-22. With the help of two fishes constructed by taking the (three points) lying at the extremities of those threads and at the extremity of the Moon's latitude for the middle of the eclipse as centre,

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1. *Cf. SiŚe*, v. 38.

one should draw a circle passing through those (three) points : this is known as the path of the eclipsing body. Then one should stretch from the centre threads equal to the hypotenuses for the times of contact and separation as also those for the times of immersion and emersion towards east and west respectively in the case of a lunar eclipse or towards the contrary directions in the case of a solar eclipse, so as to touch the path of the eclipsing body. Taking the points thus obtained as centre, one should again draw the figure of the eclipsing body and determine the other things (such as the points of the four contacts etc.) in the manner described before.[1]

LAYING OFF OF *KOṬI* IN THE *TRIJYĀ-VṚTTA* OR *MĀNAIKYĀRDHA-VṚTTA*

23. **Similarly, the *koṭi* (i. e., the Moon's latitude for the given time) as multiplied by the radius or by half the sum of the eclipsed and eclipsing bodies and divided by the *karṇa* ("hypotenuse") for that time should be laid off in the prescribed manner in the *Trijyāvṛtta-parilekha* or the *Mānakyārdhavṛtta-parilekha* respectively.**

The *koṭi* having been laid off in the *trijyā-vṛtta* or *mānaikyārdha-vṛtta*, a line should be drawn joining the centre and the extremity of the *koṭi*. The point where this line intersects the path of the eclipsing body should be taken as the position of the centre of the eclipsing body at the given time.

IṢṬAGRĀSA FROM *IṢṬANĀḌĪS*

24. **The distance (in *aṅgulas*) between two celestial latitudes of the eclipsing body (one for the beginning of the eclipse and the other for the middle of the eclipse), (measured along the path of the eclipsing body), multiplied by the *iṣṭanāḍīs* (i. e., *nāḍīs* elapsed since the beginning of the eclipse) and divided by the (*spārśika*) *sthityardha-nāḍīs* gives the *aṅgulas* (corresponding to the *iṣṭanāḍīs*, measured along the path of the eclipsing body). Laying off these *aṅgulas* appropriately on the path of the eclipsing body, one should find the *iṣṭagrāsa* ("measure of eclipse for the given time").**[2]

This is the procedure to be adopted when the *iṣṭanāḍīs* are *spārśika*, i. e., when they denote the *nāḍīs* elapsed since the beginning of the solar eclipse. In case the *iṣṭanāḍīs* are *maukṣika*, i. e., when they denote the *nāḍīs* to

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1. *Cf. BrSpSi*, xvi. 19-22.
2. Same rule occurs in *SiŚe*, v. 36.

elapse before the end of the solar eclipse, one should take the celestial latitudes of the Moon for the middle and end of the eclipse in place of those for the beginning and middle of the eclipse, and the *maukṣika-sthityardha* in place of *spārśika sthityardha.*

Both these cases are implied in the rule stated by the author.

CELESTIAL LATITUDE FROM *IṢṬANĀḌĪS*

25. Multiply the difference between the celestial latitudes of the eclipsing body for the beginning and middle of the eclipse by the *iṣṭaghaṭikās* (i.e., the *ghaṭikās* elapsed since the beginning of the eclipse) and divide (the resulting product) by (the *ghaṭikās* of) the (*spārśika*) *sthityardha*; and add the quotient to the celestial latitude (of the eclipsing body) for the beginning of the eclipse if it is less than that for the middle of the eclipse or subtract that quotient from the celestial latitude (of the eclipsing body) for the beginning of the eclipse if it is greater than that for the middle of the eclipse : what is now obtained is called the *iṣṭaśara*, i. e., the celestial latitude (of the eclipsing body) for the given time.

Let β_1, β be the celestial latitudes of the eclipsing body for the beginning and the middle of the eclipse and G the *ghaṭīs* of the *spārśika sthityardha.* Also let g be the *ghaṭīs* elapsed at the given time since the beginning of the eclipse. Then the celestial latitude β' of the eclipsing body, at the given time, is given by the formula :

$$\beta' = \beta_1 \pm \frac{(\beta \sim \beta_1) \times g}{G},$$

where $+$ or $-$ sign is taken according as β_1 is less than or greater than β.

IṢṬAGHAṬĪKĀS FROM *IṢṬAKARṆA*

26. Multiply the difference between the celestial latitudes (for the middle of the eclipse and the beginning or end of the eclipse, as the case may be) by the length of the path of the eclipsing body up to the point where the *iṣṭakarṇa* (the given hypotenuse) touches it (as measured from the position of the eclipsing body at the beginning or end of the eclipse) and divide by the length of the path of the eclipsing body up to the point where the hypotenuse for the middle of the eclipse touches it; subtract the result from or add that to the celestial latitude (for the beginning or end of the eclipse, according as it is greater or less than that for the middle of the eclipse). Then is obtained the celestial latitude (for the desired time), the so called upright, without the use of the process of iteration. From that find out the *iṣṭaghaṭikās* (i. e., the *ghaṭīs* elapsed

since the beginning of the eclipse or to elapse before the end of the eclipse).[1]

The value of the celestial latitude β' for the desired time being known, the *iṣṭaghaṭīs* g may be obtained by the formula :

$$g = \frac{G\,(\beta' \sim \beta_1)}{\beta \sim \beta_1},$$

where β_1, β are the celestial latitudes for the beginning or end and the middle of the eclipse and G the *ghaṭīs* corresponding to the *spārśika* or *maukṣika sthityardha.*

ECLIPSED AND ECLIPSING BODIES FROM *STHITYARDHA-NĀḌĪS*

27. Add the square of the *nāḍīs* of the *sthityardha* as multiplied by the degrees of the difference between the daily motions (of the Sun and the Moon) to the square of the celestial latitude (for the time of contact); take the square-root thereof ; and multiply that by 2. (Severally) diminish and increase that by the difference between the diameters of the eclipsed and the eclipsing bodies and divide (each result) by 2 : the results are the measures (of the diameters) of the eclipsed and the eclipsing bodies. (Or, the square-root multiplied by 2) diminished by the diameter of the eclipsed body gives the diameter of the eclipsing body (*āvṛti* or *āvaraṇa*) and the same diminished by the diameter of the eclipsing body gives the other (i. e., the diameter of the eclipsed body).[2]

Let E be the centre of the eclipsed body, EA the ecliptic, E′ the centre of the eclipsing body at the time of contact. E′A is perpendicular to EA. Then

EA, i. e., *sthityardha* in minutes = (*sthityardhanāḍīs* × *gatyantarakalā*)/60

= *sthityardhanāḍīs* × *gatyantarāṁśa*

and E′A = latitude for the time of contact.

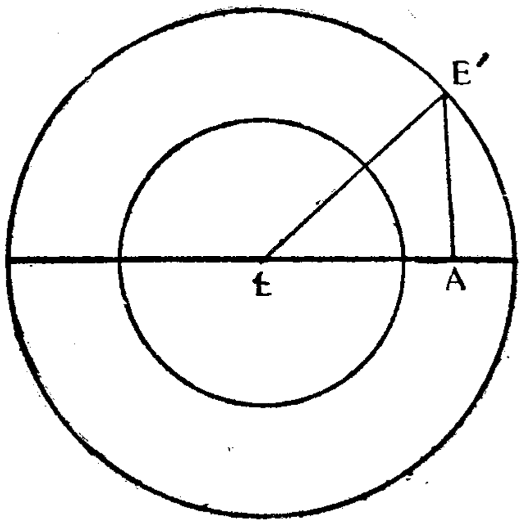
$$\therefore\ EE' = \sqrt{EA^2 + E'A^2}$$

Now, EE′ = semi-diameter of eclipsing body + semi-diameter of eclipsed body.

1. See also *SiŚi*, I, v. 35.
2. A similar rule is given in *SiŚe*, v. 30.

Hence if D denotes the diameter of the eclipsing body and D' the diameter of the eclipsed body, then

$$EE' = \frac{D + D'}{2}.$$



Therefore

$$D' = \frac{2EE' - (D - D')}{2}$$

and $$D = \frac{2EE' + (D - D')}{2}.$$

Also $D = 2EE' - D'$ and $D' = 2EE' - D$

Hence the above rule.

PARILEKHA IN A CIRCLE OF ARBITRARY RADIUS

28-29. The *valanas* corresponding to the desired circle should be obtained by proportion. The *valanas* for the times of contact and separation should be laid off from the centre in their proper directions (and then transferred to their actual positions); the corresponding celestial latitudes should be laid off in their proper directions (as explained before). At the time of middle of the eclipse[1] the two bodies (i. e., the eclipsing and eclipsed bodies) lie at the end of the celestial latitude (*kṣepakānta*) and the centre (*madhya*). At the times of contact and separation they lie at the extremities of the base and the hypotenuse; so also is the case at the desired time. This is how diagrams of eclipses are drawn in one single circle.

COLOUR OF ECLIPSED BODY

30. At the time of contact (with the Shadow) or separation (from the Shadow), the Moon (i. e., the eclipsed part of the Moon) is smoky; when eclipse amounts to half, it is black; when more than half, it is reddish black; and when it is totally eclipsed, it is tawny. The other planet (viz. the Sun) looks blackish red (throughout its eclipse).[2]

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1 Here "the time of middle of the eclipse" means "the time of conjunction in longitude."

2. *Cf. Ā*, iv. 46; *BrSpSi*, iv. 19 (for lunar eclipse), v. 26 (for solar eclipse); *ŚiDVr*, v. 36; *SūSi*, vi. 23; *SiŚe*, v. 40. Also see *PSi*, vi. 9(c-d),10(c-d); *KK*, II, iv. 17; *MSi*, vi. 16(c-d); *KPr*, iv. 22; *SiŚi*, I, v. 36; *KKu*, v. 9(c-d); *GLa*, vi. 6(c-d); *KKau*, v. 27.

INVISIBLE PART OF ECLIPSE

31. Due to the extreme brilliancy of the Sun, one-twelfth of its disc, though eclipsed, is not seen to be so; but due to the transparency of the Moon's disc, even though one-sixteenth of it is eclipsed, it is easily seen to be so in the sky.[1]

Mallikārjuna Sūri (in his com. on *ŚiDVr*, v. 17) adds: "At midday, even though one-eighth of the Sun's disc is eclipsed, it is not seen (by the eye)." In support he quotes the following hemistitch from the *Bhāskarīya-tantra* :

अष्टमांशगृहीतोऽर्कोऽप्यगृहीत इव त्विषा ।

"On account of the (dazzling) brilliance (of the midday Sun), the Sun appears uneclipsed even though one-eighth of its disc is eclipsed."

1. *Cf. Ā*, iv. 47; *BrSpSi*, v. 20; *KK*, II, iv. 18; *ŚiDVr*, vi. 17; *MSi*, vi. 16(a-b); *KPr*, vi. 9; *SiŚe*, v. 41; *SiŚi*, I, v. 37; *KKu*, v. 9(a-b). Āryabhaṭa II says : "One-twelfth of the Sun as well as one-sixteenth of the Moon, though eclipsed, is not seen by the eye."

Section 5

Parvajnāna or Determination of *Parva*

POSSIBILITY OF ECLIPSE

1. When, at the end of a lunar month or lunar fortnight, the sum of the longitudes of the Moon and the Moon's ascending node[1] amounts to 180° or 360° approximately, an eclipse of the Sun or Moon may occur.

The degrees by which the sum of the longitudes of the Moon and the Moon's ascending node falls short of 180° or 360° or by which it is in excess of 180° or 360° constitute the *kṣepaka*.

MOON'S LATITUDE

2. The *kṣepaka* diminished by one-fifteenth of itself and then multiplied by 5 gives the Moon's latitude (at the end of a lunar month or lunar fortnight) in terms of minutes.

The diameters of the Moon, Sun and the Shadow as also the *sthityardha* and *vimardārdha* are obtained as before.

The *kṣepaka* is the distance of the Sun or Moon, at the end of a lunar month or lunar fortnight, from the nearer node of the Moon's orbit.

Let *kṣepaka* = θ degrees = $60\,\theta$ minutes. Then, since $60\,\theta$ is very small, Rsin $(60\,\theta) = 60\,\theta$ mins., approx. Therefore, the Moon's latitude at the end of a lunar month or lunar fortnight

$$= \frac{\text{Rsin}\,\theta^\circ \times 270}{3438} \text{ mins.} = \frac{60\,\theta \times 270}{3438} \text{ mins.}$$

$$= \frac{900\,\theta}{191} \text{ mins.} = \frac{70\,\theta}{15} \text{ mins.}$$

$$= 5\,(\theta - \theta/15) \text{ mins. approx.}$$

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1. It must be remembered that the longitude of the Moon's ascending node is measured westwards by Vaṭeśvara.

Assuming $2\frac{1}{3}$ mins. = 1 *aṅgula*, as done by Lalla, the above formula gives

$$\text{Moon's latitude} = \frac{70\theta}{15} \times \frac{3}{7} \textit{ aṅgulas}$$

$$= 2\theta \textit{ aṅgulas},$$

which agrees with Lalla's value. See *ŚiDVṛ*, vii. 3 (a); also vii. 3 (b-d), 8.

MEASURE OF ECLIPSE

3. Or, (the Moon's mean latitude, obtained above) multiplied by the Moon's mean hypotenuse and divided by the Moon's true hypotenuse, is the Moon's true latitude.

Half the sum of the diameters of the eclipsed and eclipsing bodies diminished by that is stated to be the measure of eclipse at the time of the middle of the eclipse.

(1) Moon's true latitude

$$= \frac{\text{Moon's mean latitude} \times \text{Moon's mean hypotenuse}}{\text{Moon's true hypotenuse}}$$

(2) Measure of eclipse = half the sum of the diameters of the eclipsed and eclipsing bodies — Moon's true latitude.

Rationale of (1).

Moon's true latitude

$$= \frac{\text{Moon's mean latitude} \times R}{\text{Moon's } \textit{mandakarṇa}}$$

$$= \frac{\text{Moon's mean latitude} \times \text{Moon's mean hypotenuse}}{\text{Moon's true hypotenuse}}.$$

STHITYARDHA AND *VIMARDĀRDHA* (FOR LUNAR ECLIPSE)

4. Divide the Moon's mean latitude itself by the degrees of the difference between the true daily motions of the Moon and the Sun. Find the square of that and severally subtract that (square) from 21 and 4, respectively. Whatever are obtained as the square-roots thereof are the values of the *sthityardha* and the *vimardārdha* (in terms of *ghaṭīs*), in the case of a lunar eclipse.

That is: If β be the Moon's mean latitude and d the degrees of the difference between the true daily motions of the Moon and the Sun, then

(1) *Sthityardha* $= \sqrt{21-(\beta/d)^2}$ *ghaṭīs*

(2) *Vimardārdha* $= \sqrt{4-(\beta/d)^2}$ *ghaṭīs*.

Rationale. Let

S = semi-diameter of Shadow, in minutes

M = semi-diameter of Moon, in minutes

d' = Moon's true daily motion, in minutes − Sun's true daily motion, in minutes.

Then

$$Sthityardha = \frac{\sqrt{(S+M)^2-\beta^2}\times 60}{d'}\ ghaṭīs$$

$$= \frac{\sqrt{(S+M)^2-\beta^2}}{d}\ ghaṭīs$$

$$= \sqrt{\left(\frac{S+M}{d}\right)^2-\left(\frac{\beta}{d}\right)^2}\ ghaṭīs. \quad \text{(i)}$$

Similarly,

$$Vimardārdha = \sqrt{\left(\frac{S-M}{d}\right)^2-\left(\frac{\beta}{d}\right)^2}\ ghaṭīs. \quad \text{(ii)}$$

But $S = 41'$ approx., $M = 16'$ approx., and $d = (791 - 59)/60$ degrees approx. Therefore

$$\left(\frac{S+M}{d}\right)^2 = 21 \text{ approx.}$$

and $$\left(\frac{S-M}{d}\right)^2 = 4 \text{ approx.}$$

Hence, from (i) and (ii) we have the desired results.

STHITYARDHAS WITHOUT ITERATION

5-6. In the odd quadrant, respectively add *palas*[1] equal to half (the minutes of) the Moon's latitude (for the time of opposition) to and sub-

1. One *ghaṭī* = 60 *palas* (or *vighaṭikās*).

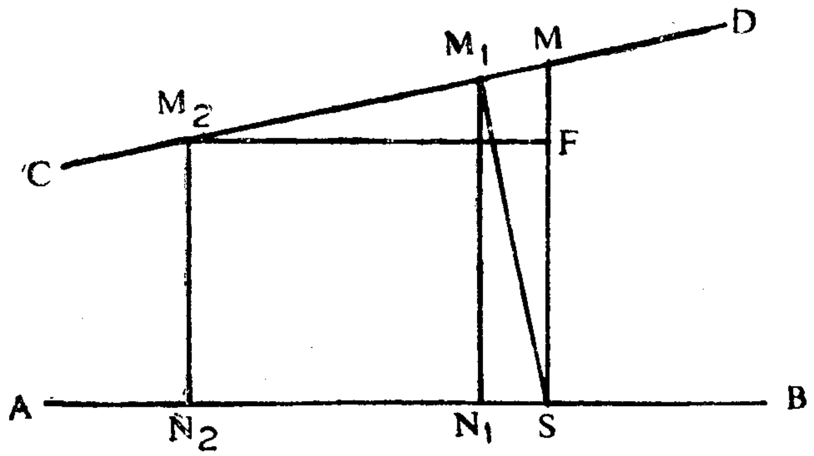
tract the same from the *sthityardha*; and, in the even quadrant, respectively subtract and add the same: then are obtained the *spārśika* and *maukṣika sthityardhas*.

(In the case of the Moon's latitude) : apply the minutes of the celestial latitude as calculated from the *sthityardha* (treated as the Rsine of the *bhuja*) to the Moon's latitude for the time of opposition, reversely (i.e., in the odd quadrant, subtract the minutes of the celestial latitude derived from the *sthityardha* from and add them to the Moon's latitude for the time of opposition; and in the even quadrant, add and subtract the same) : then are obtained the Moon's latitudes for the times of contact and separation, (respectively).

Applying the rule stated above for the *sthityardha*, one should obtain the *vimardārdhas* for immersion and emersion too.

The case of the solar eclipse is similar.

In the figure, let AB be the ecliptic and CD the Moon's orbit relative to the Shadow centred at S on the ecliptic. S and M are the centres of the Shadow and the Moon respectively, at the time of opposition (of the Sun and Moon). SM_1 is the perpendicular dropped from S on the Moon's orbit and M_1N_1 the perpendicular from M_1 on the ecliptic. Then M_1 is the Moon's centre at the middle of the eclipse.



In the triangle MM_1S (regarded as plane),

$$\angle MM_1S = 90^\circ$$

$$MS = \text{Moon's latitude at opposition}$$

and $\angle MSM_1 = i$, inclination of Moon's orbit to the ecliptic.

Therefore, taking Rsin i = 270′,

$$M_1M = \frac{270 \times MS}{3438} \text{ mins.}$$

Since M_1M is almost parallel to N_1S, therefore

$$N_1S = M_1M = \frac{270 \times MS}{3438} \text{ mins.}$$

$$= \frac{270 \times MS}{3438} \times \frac{60 \times 60}{790'\,35'' - 59'\,8''} \; palas$$

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$$= \frac{270 \times 60 \times 60 \times MS}{3438 \times 731} \; palas$$

Siddhanta darpana MS/2.096

$$= \frac{MS}{2\frac{1}{2}} \; palas$$

$$= \frac{MS}{2} \; palas, \text{ according to Vaṭeśvara.}$$

This gives the time-interval between the time of opposition and the middle of the eclipse. Hence the rules for the *spārśika* and *maukṣika sthityardhas* and *vimardārdhas*.

In the same figure, let M_2 be the centre of the Moon at the time of first contact, M_2N_2 the perpendicular from M_2 to the ecliptic and M_2F the perpendicular from M_2 on MS. Evidently therefore

$$M_2N_2 = MS - MF$$

= Moon's latitude at opposition — latitude corresponding to *bhuja* equal to M_2M or N_2S approx.

This gives the Moon's latitude for the time of the first contact in the odd quadrant. Other cases may be explained similarly. Hence the rule for the Moon's latitude.

In place of MS/2 *palas* stated by Vaṭeśvara, Mañjula prescribes

$$\frac{MS}{144} \; ghaṭīs = \frac{MS \times 60}{144} \; palas \text{ or } MS/2\tfrac{2}{5} \; palas.$$

See *LMā* iii. 14. Āryabhaṭa II has followed Mañjula and gives the same correction. See *MSi*, v. 11-12.

A Similar rule has been given by Bhāskara II also. See *KKu*, iv. 11-12 (a-b).

Acyuta (*Karaṇottama*, iv. 10) gives the expression for N_1S in the general form, viz.

$$N_1S = \frac{5MS}{d} \textit{ palas},$$

where d is the difference between the daily motions of the Sun and Moon in terms of degrees.

LAMBANA AND APPARENT CONJUNCTION

7. The product of the *nata-ghaṭikās* of the Sun (for the time of geocentric conjunction of the Sun and Moon) and 6 gives the degrees (between the Sun and the meridian ecliptic point). These added to or subtracted from the Sun's longitude, according as the conjunction occurs in the western or eastern half of the celestial sphere gives the longitude of the meridian ecliptic point.

The difference or sum of the declination of the meridian ecliptic point and the local latitude, according as they are of unlike or like directions, gives the zenith distance of the meridian ecliptic point. 90° minus that is the altitude of the meridian ecliptic point.

8. The Rsine of the hour angle of the Sun for the time of geocentric conjunction of the Sun and the Moon divided by 860, when multiplied by the Rsine of the altitude of the meridian ecliptic point and divided by the radius gives the *lambana* (in terms of *ghaṭīs*). This should be added to or subtracted from the *tithighaṭīs*, according as the *tithi* falls in the western or eastern part of the celestial sphere. This process should be repeated again and again until the *lambana* for the time of apparent conjunction (and likewise the time of apparent conjunction) of the Sun and the Moon is fixed.

This rule is the reproduction of the rule given by Lalla in his *Śiṣya-dhī-vṛddhida*, vii. 5-7.

Rationale.

$$\textit{Lambana} \text{ in } \textit{ghaṭīs} = \frac{\textit{dṛggati} \times 4}{3438}$$

$$= \frac{dṛggati}{860}$$

$$= \frac{\text{Rsin (Sun} \sim \textit{vitribhalagna}) \times \textit{vitribhaśaṅku}}{\text{R} \times 860}$$

$$= \frac{\text{Rsin (Sun} \sim \textit{madhyalagna}) \times \textit{madhyalagnaśaṅku}}{\text{R} \times 860} \text{ approx.}$$

$$= \frac{\text{Rsin (Sun's hour angle)} \times \textit{madhyalagnaśaṅku}}{\text{R} \times 860} \text{ approx.}$$

NATI AND MOON'S TRUE LATITUDE

9. Multiply the Rsine of the zenith distance (of the meridian ecliptic point) by 2 and divide by 141 : then is obtained the *nati* in terms ot minutes.[1] That diminished or increased by the Moon's latitude for that time gives the Moon's true latitude.[2]

Rationale. Using the proportion : When the Rsine of the zenith distance of the meridian ecliptic point is equal to the radius (3438′), the *nati* is equal to 49′, what then is the value of the *nati* corresponding to the given Rsine of the zenith distance of the meridian ecliptic point ? the result is

$$nati = \frac{\text{Rsin } z \times 49'}{3438} \text{ mins.}$$

$$= \frac{\text{Rsin } z \times 2}{141} \text{ mins.,}$$

where z is the zenith distance of the meridian ecliptic point. See Mallikārjuna Sūri's commentary on *ŚiDVṛ*, vii. 8.

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1. *Cf, MSi*, vi. 11(c-d); *SiŚi*, I, vi. 12(c-d). Āryabhaṭa II and Bhāskara II, however, state the formula as :

$$nati = \frac{\text{Rsin (z. d. of central ecliptic point)} \times 2}{141}.$$

The *Sūrya-siddhānta* (v. 11) gives the following two formulae :

$$nati = \frac{\text{Rsin (z. d. of central eeliptic point)}}{70}$$

$$\text{and } nati = \frac{\text{Rsin (z. d. of central ecliptic point)} \times 49}{\text{R}}.$$

For a similar rule, see *SiŚe*, vi. 16(c-d)-17(a-b).

2. *Cf. MSi*, vi. 11(c-d)-12.

General rationale. Using the formula of vs. 6 of ch. V, sec. 2, above, we have

$$nati = \frac{dṛkkṣepa \times (\text{Moon's mean daily motion} - \text{Sun's mean daily motion})}{15 \times R}$$

$$= \frac{\text{Rsin (z. d. of } madhyalagna) \times 731'}{15 \times R} \text{ approx.}$$

$$= \frac{\text{Rsin (z. d. of } madhyalagna) \times 49'}{R} \text{ approx.}$$

$$= \frac{\text{Rsin (z. d. of } madhyalagna) \times 2'}{141} \text{ approx.}$$

The direction of the *nati* is the same as the direction of the zenith distance of the central or meridian ecliptic point. That is to say, the *nati* is north or south, according as the central or meridian ecliptic point is to the north or south of the zenith.

STHITYARDHAS (FOR SOLAR ECLIPSE)

10. Calculate the time of geocentric conjunction (lit. *tithi* to which *lambana* has not been applied). (Severally) diminish and increase it by the *sthityardha* and calculate the corresponding *lambanas*. Apply them to the times of contact and separation, (respectively). Repeat the process again and again until the *lambanas* for the times of contact and separation are fixed. Applying them to the times of contact and separation, as before, one gets the true (or apparent) times of contact and separation.

11. The difference (i) between the (true) times of contact and the middle of the eclipse and (ii) between the (true) times of separation and the middle of the eclipse, are the true (*spārśika* and *maukṣika*) *sthityardhas*. In the same way are obtained the (true *spārśika* and *maukṣika*) *vimardārdhas*. The processes of finding the *valana* etc. are the same as stated before.

SIX-MONTHLY *KṢEPAS* (FOR LONGITUDES)

12-13. One who wants to know the next eclipse (which might occur after six months near the next node) from the current one should add the signs etc. (given below) to the longitudes of the Sun and the Moon for

the middle of the current eclipse, and degrees etc. (given below) to those of the Moon's apogee and ascending node :

Sun	**5 signs 24° 27′ 6″**
Moon	**5 signs 22° 12′ 53″**
Moon's apogee	**19° 42′ 56″**
Moon's ascending node	**9° 22′ 41″.**

These are the motions of the Sun, Moon, Moon's apogee and Moon's ascending node for 177 days (i.e., the whole number of days in 6 lunar months). The same have been given by Brahmagupta, Lalla and Śrīpati. See *BrSpSi,* xvi. 30-32; *KK,* II, iv. 20-22; *ŚiDVṛ,* vii. 9-10; *SiŚe,* vii. 3. Also see *GLā,* vii. 8 (a-b).

SIX-MONTHLY *KṢEPA* (FOR TIME)

14. One who, without going through the above accurate process, adds 2 days and 11 *nāḍīs* (to the time of the current eclipse), knows without any effort the day (and time) at which the eclipse of the Moon or the Sun might occur (after six months).[1]

This rule is based on the fact that there are 177 days and 11 *nāḍīs* in six lunar months. When these are divided by 7 days, the remainder is 2 days and 11 *nāḍīs.*

According to Bhāskara II,[2] there are

29 days 31 *ghaṭīs* and 50 *vighaṭīs*

in 1 lunar month. Therefore in 6 lunar months there are

(29 days 31 *ghaṭīs* 50 *vighaṭīs*) × 6

= 177 days 11 *ghaṭīs.*

1. Also see *GLā,* vii. 8(d).
2. See *SiŚi,* I, i (f). 6, com.

Section 6

Computation with Lesser Tools

This Section teaches how to find the true *tithi*, the Moon's latitude, *lambana* and *nati*, etc., without making direct use of the local latitude, the Sun's declination, and the longitudes of the Sun, Moon and the Moon's ascending node.

COMPUTATION OF TRUE *TITHI*

Method 1. Brahmagupta's method

(Step 1. The so called *kṣaya*)

1. Obtain the product of the *avamaśeṣa* and *yugādhika* (*māsa*) ("the number of intercalary months in a *yuga*"), divide that by the number of civil days in a *yuga* and add the (resulting) quotient to the *adhimāsaśeṣa*; divide that by the number of lunar months in a *yuga*. The (resulting) quotient (which is in terms of days etc. when) regarded as degrees etc. and (severally) added to the longitudes of the apogees of the Sun and the Moon gives the so called *kṣaya* ("subtractive") (for the Sun and the Moon, respectively).

Kṣaya for the Sun = longitude of Sun's apogee + total *adhimāsaśeṣa*

and

kṣaya for the Moon = longitude of Moon's apogee + total *adhimāsaśeṣa*,

where total *adhimāsaśeṣa*

$$= \frac{\dfrac{avamaśeṣa \times yugādhimāsa}{\text{civil days in a } yuga} + adhimāsaśeṣa}{\text{lunar months in a } yuga} \text{ days},$$

days etc. of total *adhimāsaśeṣa* being treated as degrees etc.

(Step 2. Mean anomalies of Sun and Moon)

2. To the *avamaśeṣa* divided by the number of civil days in a *yuga* add the lunar months and lunar days elapsed (since Caitrādi); in another

place multiply that by 13. (Severally) diminish the two results by the *ṛṇa* or *kṣaya* for the Sun and the Moon, respectively. Then are obtained the (mean) anomalies of the Sun and the Moon, respectively.

Sun's mean anomaly = [lunar months and lunar day elapsed since Caitrādi

$$+ \frac{\textit{avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \text{ days}] - \textit{kṣaya} \text{ for the Sun,}$$

Moon's mean anomaly = 13 [lunar months and lunar days elapsed since Caitrādi

$$- \frac{\textit{avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \text{ days}] - \textit{kṣaya} \text{ for the Moon,}$$

months and days etc. being treated as signs and degrees etc.

Rationale. Suppose that m lunar months and d lunar days have elapsed since the beginning of Caitra. Then

$$m \text{ months} + d \text{ days} + \frac{\textit{avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \text{ days} - \text{total } \textit{adhimāsaśeṣa} \text{ days}$$

denotes the time in mean solar months and mean solar days etc. elapsed since the beginning of the current mean solar year up to the mean sunrise on the current lunar day. (For details, see my notes on *Mahā-Bhāskarīya*, i. 13-19)

Let M, D, G, denote, respectively, the mean solar months, the mean solar days, and the mean solar *ghaṭīs* elapsed since the beginning of the current mean solar year up to the mean sunrise on the current lunar day. Then

Mean longitude of the Sun = M signs D degrees G minutes

$$= m \text{ signs } d \text{ degrees} + \text{degrees equal to } \frac{\textit{avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \text{ days}$$

$$- \text{degrees equal to total } \textit{adhimāsaśeṣa} \text{ days}$$

Mean longitude of the Moon = 13 [m signs d degrees + degrees

$$\text{equal to } \frac{\textit{avamaśeṣa}}{\text{civil days in a } \textit{yuga}} \text{ days}] - \text{degrees equal to}$$

total *adhimāsaśeṣa* days.

Therefore,

Sun's mean anomaly = [*m* signs *d* degrees + degrees equal to

$$\frac{avama\acute{s}e\d{s}a}{\text{civil days in a } yuga} \text{ days}]$$

— (degrees equal to total *adhimāsaśeṣa* days + longitude of Sun's apogee)

Moon's mean anomaly = 13 [*m* signs *d* degrees + degrees equal to

$$\frac{avama\acute{s}e\d{s}a}{\text{civil days in a } yuga} \text{ days}]$$

— (degrees equal to total *adhimāsaśeṣa* days + longitude of Moon's apogee).

Hence,

Sun's mean anomaly = [lunar months and days elapsed since

$$\text{Caitrādi} + \frac{avama\acute{s}e\d{s}a}{\text{civil days in a } yuga} \text{ days}]$$

— (total *adhimāsaśeṣa* days + longitude of Sun's apogee)

Moon's mean anomaly = 13 [lunar months and days elapsed since

$$\text{Caitrādi} + \frac{avama\acute{s}e\d{s}a}{\text{civil days in a } yuga} \text{ days}]$$

— (total *adhimāsaśeṣa* days + longitude of Moon's apogee),

months and days etc. being treated as signs and degrees etc.

The rule stated above in vss. 1-2 is exactly the same as given in *BrSpSi*, xiii. 23.

(Step 3. True *tithi* obtained)

3. The corresponding corrections (i. e., the equation of the centre etc. for the Sun and the Moon) computed up to seconds of arc should be applied to the respective (mean) anomalies negatively or positively in the same way as in finding the true positions (of the Sun and the Moon): then are obtained the true anomalies (of the Sun and the Moon).

In the manner stated heretofore, one should compute the correction due to the Sun's ascensional difference (*cara*), equation of the centre (*bhujāphala*), correction due to the Sun's equation of the centre (*bhujāvivara*), and the correction due to the longitude (*deśāntara*) (for the Sun and the Moon).

4. These corrections (reduced to minutes and) divided by 12 are certainly in *ghaṭīs* and should be subtracted from or added to the previously mentioned *avamaghaṭīs*, those for the Moon in the usual manner and those for the Sun reversely. Then is obtained (the part of) the current true *tithi* elapsed at sunrise (in terms of *ghaṭīs*).

5. That subtracted from 60 *ghaṭīs* is the unelapsed part of the current true *tithi*. Those (i.e., the elapsed and unelapsed parts of the current true *tithi*) multiplied by 720 and divided by the difference between the daily motions of the Sun and the Moon give the better values of the same.

The *avamaghaṭīs* are the *ghaṭīs* of the current mean *tithi* lying between the beginning of the current mean *tithi* and sunrise.

$$\text{True } tithi = \frac{\text{Moon's true long. in degrees} - \text{Sun's true long. in degrees}}{12}.$$

Assuming that there are 60 *ghaṭīs* in one *tithi*, we have :

true *tithi* in *ghaṭīs*

$$= \frac{(\text{Moon's true long. in degrees} - \text{Sun's true long. in degrees}) \times 60}{12}$$

$$= \frac{60}{12}[(\text{Moon's mean longitude in degrees} + \text{Moon's corrections in degrees})$$

$$- (\text{Sun's mean longitude in degrees} + \text{Sun's corrections in degrees})]$$

$$= \frac{60}{12}(\text{Moon's mean long. in degrees} - \text{Sun's mean long. in degrees}) + \frac{\text{Moon's corrections in minutes}}{12}$$

$$- \frac{\text{Sun's corrections in minutes}}{12}.$$

Therefore, omitting complete *tithis,*

Ghaṭīs elapsed of the current true *tithi*

$$= \textit{avamaghaṭīs} \text{ (i. e., elapsed } \textit{ghaṭīs} \text{ of the current mean } \textit{tithi})$$

$$+ \frac{\text{Moon's corrections in minutes}}{12}$$

$$- \frac{\text{Sun's corrections in minutes}}{12}.$$

Since one *tithi* has been assumed to be equal to 60 *ghaṭīs,* therefore the unelapsed *ghaṭīs* of the current *tithi* are obtained by subtracting the elapsed *ghaṭīs* from 60 *ghaṭīs.*

The above elapsed and unelapsed *ghaṭīs* have been obtained by assuming one *tithi* as equal to 60 *ghaṭīs.* In fact, one *tithi* is equal to

$$\frac{60 \times 12^\circ}{\text{motion-difference of Sun and Moon in degrees}} \textit{ ghaṭīs}$$

$$\text{or } \frac{60 \times 60 \times 12}{\text{motion-difference of Sun and Moon in mins.}} \textit{ ghaṭīs}.$$

Hence, the accurate value of the true *tithi* in *ghaṭīs*

$$= \frac{(\text{Moon's long. in degrees} - \text{Sun's long. in degrees}) \times 60 \times 60 \times 12}{12 \times (\text{motion-difference of Sun and Moon in mins.})}.$$

$$= \frac{(\text{true } \textit{tithi} \text{ in } \textit{ghaṭīs}) \times 720}{\text{motion-difference of Sun and Moon in mins.}}.$$

Likewise, the accurate value of the *ghaṭīs* elapsed or to elapse of the current true *tithi*

$$= \frac{(\textit{ghaṭīs} \text{ elapsed or to elapse of the current true } \textit{tithi}) \times 720}{\text{motion-difference of Sun and Moon in mins.}}.$$

Hence the rule stated in the text.

The rule stated in vss. 1-5 above is essentially the same as given in *BrSpSi,* xiii. 23-25, and *SiŚe,* iii. 72-74.

Method 2

5(d)-6. Or, dividing the (lunar and solar) corrections (in minutes) by the degrees of difference between the daily motions of the Sun and the Moon obtain the *nāḍīs*, and apply them, as before, to the (*avama*) *nāḍīs* obtained by dividing the *avamaśeṣa* by the number of lunar days (in a *yuga*) and multiplying (the quotient) by 60 : the result is (the accurate value of) the *avamaśeṣa* (in terms of *nāḍīs*) or *nāḍīs* elapsed (at sunrise) (of the current *tithi*).

True *avamaśeṣa* in *nāḍīs*

$$= \frac{avamaśeṣa \times 60}{\text{lunar days in a } yuga} + \frac{\text{Moon's corrections in mins.}}{d'}$$

$$- \frac{\text{Sun's corrections in mins.}}{d'},$$

where d' = difference between the daily motions of the Sun and the Moon in terms of degrees.

Rationale. As shown above (under vss. 3-5), the accurate value of *nāḍīs* elapsed (at sunrise) of the current *tithi* or true *avamanāḍīs*

$$= \left[avamanāḍīs \text{ (lunar)} \right.$$

$$\left. + \frac{\text{Moon's corrections in mins.}}{12} - \frac{\text{Sun's corrections in mins.}}{12} \right] \frac{720}{d},$$

(where d = difference between the daily motions of Sun and Moon in terms of minutes)

$$= avamanāḍīs \text{ (civil)}$$

$$+ \frac{\text{Moon's corrections in mins.}}{d'} - \frac{\text{Sun's corrections in mins.}}{d'}$$

(approx.)

$$= \frac{avamaśeṣa \times 60}{\text{lunar days in a } yuga}$$

$$+ \frac{\text{Moon's corrections in mins.}}{d'} - \frac{\text{Sun's corrections in mins.}}{d'},$$

Method 3.

7. The *ahargaṇa* being multiplied (severally) by the number of lunar years (in a *yuga*), the number of lunar months (in a *yuga*), and the number of lunar days (in a *yuga*) and divided (in each case) by the number of civil days (in a *yuga*), the result is the (mean) *tithi* in terms of lunar years, lunar months and lunar days (respectively). This is rectified (or corrected) by the corrections stated above in the manner stated heretofore.

For details of this method, the reader is referred to the *Mahā-Bhāskarīya* (viii. 1-4) of Bhāskara I, and to my notes thereon.

MEAN ANOMALIES OF SUN AND MOON

(Alternative methods)

8. Multiply the *ahargaṇa* by 24^2 (i. e., 576), then subtract 46088, and then divide by 210389 : then is obtained the Sun's (mean) anomaly in terms of revolutions etc.

$$\text{Sun's mean anomaly} = \frac{576\,A - 46088}{210389}\ \text{revs.},$$

where A is the *ahargaṇa* reckoned from the birth of Brahmā (or from the beginning of Kaliyuga).

Rationale. Sun's mean anomaly = Sun's mean longitude − longitude of Sun's apogee,

where

$$\text{Sun's mean longitude} = \frac{4320000 \times A}{1577917560} = \frac{576\,A}{210389 + 1/125}$$

$$= \frac{576\,A}{210389}\ \text{revs., approx.,}$$

and Longitude of Sun's apogee in the beginning of Śaka 826 (Kali 4005)

$$= \frac{165801 \times \text{years elapsed since Brahmā's birth}}{\text{years of Brahmā's life}}$$

$$= \frac{165801 \times 26782530124005}{72000 \times 1008 \times 4320000}$$

$$= \frac{4440570277090153005}{313528320000000}\ \text{revs.}$$

$$= 14163 + \frac{68680930153005}{313528320000000} \text{ revs.},$$

$$= \frac{68680930153005}{313528320000000} \text{ revs.},$$

neglecting complete revolutions which are not needed,

$$= \frac{46087{\cdot}4}{210389} \text{ revs , approx.}$$

Vaṭeśvara takes 46088 in place of 46087·4.

9. Multiply the *ahargaṇa* by 110 and divide by 3031 : the result is the Moon's (mean) anomaly in terms of revolutions etc.

10. The Sun's and Moon's (mean) anomalies, in revolutions etc., (thus obtained), are reckoned from the birth of Brahmā.

$$\text{Moon's mean anomaly} = \frac{110 \times A}{3031} \text{ revs},$$

where A denotes the *ahargaṇa* reckoned from the birth of Brahmā.

Rationale. According to Vaṭeśvara :

$$yuga = 1577917560 \text{ civil days}$$

$$\text{Moon's revs.} = 57753336$$

$$\text{Revs. of Moon's apogee} = 488211$$

$$\therefore \text{Revs. of Moon's anomaly} = 57753336 - 488211$$

$$= 57265125$$

$$\therefore \text{Moon's mean anomaly} = \frac{57265125\, A}{1577917560} \text{ revs.}$$

$$= \frac{110\, A}{3031} \text{ revs. approx.}$$

COMPUTATION OF *YUTI* OR MOON PLUS MOON'S ASCENDING NODE (FOR USE IN FINDING MOON'S LATITUDE)

General method

11. The *ahargaṇa* multiplied by the sum of the revolutions of the Moon and the Moon's ascending node and divided by the civil days (in a *yuga*) gives the so called *Yuti* in terms of revolutions etc. This is made true by the application of the Moon's equation of the centre (lit. the correction arising from the Moon's anomaly), in the manner stated before.

$$Yuti = \frac{S \times A}{C} \text{ revs.,}$$

where A denotes the *ahargaṇa* reckoned from the birth of Brahmā, C the number of civil days in a *yuga*, and S the sum of the revolutions of the Moon and Moon's ascending node in a *yuga*.

Simplified method

12. Or, the *ahargaṇa* multiplied by 43200 and divided by 1175569 gives the *Yuti*, when 1 minute of arc is diminished (therefrom) every 4463 years. It is made true as before.

$$Yuti = \frac{43200\,A}{1175569} \text{ revs.} - \frac{Y}{4463} \text{ mins.,}$$

where A is the *ahargaṇa* reckoned from the birth of Brahmā and Y the years elapsed.

Rationale. According to Vaṭeśvara,

$$\text{Moon's revs.} = 57753336$$

$$\text{Revs. of Moon's asc. node} = 232234$$

$$\text{Their sum} = 57753336 + 232234 = 57985570$$

$$\therefore \text{ Daily motion of } Yuti = \frac{57985570}{1577917560} \text{ revs.}$$

$$= \frac{43200}{1175569} - \frac{5267}{1175569 \times 157791756} \text{ revs.}$$

$$= \frac{43200}{1175569} \text{ revs.} - \frac{5267 \times 21600}{1175569 \times 432000} \text{mins. per year}$$

$$= \frac{43200}{1175569} \text{ revs.} - \frac{1}{4463 + 4759/5267} \text{mins. per year}$$

$$= \frac{43200}{1175569} \text{ revs.} - \frac{1}{4463} \text{ mins. per year, approx.}$$

$$= \frac{43200}{1175569} \text{ revs.} - 1 \text{ min. every 4463 years.}$$

Kaliyugādi Kṣepas for *Yuti* and Moon's anomaly.

13. To the *Yuti* calculated from the *ahargaṇa* reckoned from the beginning of Kaliyuga, add 6 signs; and from the Moon's anomaly, subtract 3 signs.

This rule is based on the fact that in the beginning of Kaliyuga, the longitudes of the Moon's apogee and the Moon's ascending node were 3 signs and 6 signs, respectively. See *supra*, ch. I, sec. 4, vs. 56 (c-d).

COMPUTATION OF *LAMBANA*

Declinations in *yojanas*

14. 11×5 (=55), 108, 154, 190, 213 and 221 are the declinations in terms of *yojanas* at the end of every half-sign of the *bhuja* (of the Sun's longitude).

The following table gives the declinations for Sun's longitude equal to 15°, 30°, 45°, 60°, 75° and 90° in terms of minutes according to Brahmagupta (*KK*, I, iii. 7) and the same in terms of *yojanas* of the Earth, according to Vaṭeśvara :

Sun's longitude	declination in mins.	declination in *yojanas*
15°	362′	55 *yojanas*
30°	703′	108 ,,
45°	1002′	154 ,,
60°	1238′	190 ,,
75°	1388′	213 ,,
90°	1440′	221 ,,

According to Vaṭeśvara (*supra*, ch. I, sec. 8, vs. 3), Earth's diameter = 1054 *yojanas*, and likewise Earth's circumference = 3312 *yojanas*. So 3312 *yojanas* correspond to 21600 minutes. This is the relation used in converting the minutes of the declinations into the corresponding *yojanas* of the Earth.

DṚKKṢEPA AND *DṚKKṢEPA-ŚAṄKU*

15-16. The elapsed *nāḍīs* of the *tithi* (i. e., the *nāḍīs* elapsed since sunrise at the time of conjunction of the Sun and the Moon) multiplied by six should be added to the degrees of the Sun's anomaly and (the resulting sum should be) diminished[1] by 11. (Then is obtained the longitude of the central ecliptic point in terms of degrees). The *yojanas* corresponding to the declination of that (central ecliptic point) should be added to or subtracted from the *yojanas* lying between the local place and the local equatorial place, according as the Sun is in the six signs beginning with the sign Libra or in the six signs beginning with the sign Aries : the result is the *Doḥ* or *Bhuja* (= *dṛkkṣepa*). One fourth of the Earth's circumference diminished by that is the *Koṭi* (=*dṛkkṣepa-śaṅku*). Twenty three *yojanas* make a *dhanu* (i. e., an arc of 150 mins.). Using this relation one should obtain the Rsines of the *Agra* (= *Koṭi*) and *Doḥ* (= *Bhuja*), as stated before.

This rule is gross and is meant for rough calculation. The explanation is as follows : Since

$6 \times$ *nāḍīs* of the *tithi* = degrees on the ecliptic between the Sun and the eastern horizon, approx.,

$\therefore$ $5 \times$ *nāḍīs* of the *tithi* + degrees of Sun's anomaly $- 11°$

= (longitude of rising point of the ecliptic − longitude of Sun) + (longitude of Sun − 79°) − 11°

= longitude of rising point of the ecliptic − 90°

= longitude of central ecliptic point (in terms of degrees).

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1. Vaṭeśvara adds 11.

Let δ be the *yojanas* of the declination of the central ecliptic point, and ϕ the *yojanas* of the local latitude. Then

$$Dṛkkṣepa \text{ or } Bhuja = \phi + \text{ or } \sim \delta,$$

and $Dṛkkṣepa\text{-}śaṅku$ or $Koṭi = \dfrac{yojanas \text{ of Earth's circumference}}{4} - Bhuja,$

$+$ or $\sim$ sign being taken according as the Sun is in the six signs beginning with Libra or in the six signs beginning with Aries.

As shown above 3312 *yojanas* correspond to 21600 minutes, according to Vaṭeśvara. Hence, one can easily see that 23 *yojanas* correspond to 150′.

It is interesting to note that Vaṭeśvara designates an arc of 150′ as *dhanu*, i. e., elemental arc. It means that he has in his mind a table of 36 Rsines in a quadrant. We know of only one such table, viz. that given in Nīlakaṇṭha's *Jyotirmīmāṁsā* (pp. 48-49). Nīlakaṇṭha has quoted it from some earlier work.

Derivation of *lambana*

17. Multiply the Rsine due to the *nāḍīs* of the (Sun's) hour angle (for the time of geocentric conjunction) by the Rcosine of the *dṛkkṣepa* (*Agrajyā* or *Koṭijyā*) and divide (the resulting product) by the square of half the radius : then are obtained the *nāḍīs* of the *lambana*. Subtract them from or add them to the time of geocentric conjunction according to the rule (prescribed for it), and repeat the process (until the *lambana* is fixed).[1]

That is : If V, z be the longitude and zenith distance of the central ecliptic point, and H the Sun's hour angle at the time of conjunction of the Sun and Moon, then

$$lambana = \frac{\text{R}\sin H \times \text{R}\cos z}{(\text{R}/2)^2} \; ghaṭīs.$$

Rationale. Since (vide *supra*, sec. 1, vs. 16)

$$lambana = \frac{4 \times dṛggati}{\text{R}} \; ghaṭīs$$

1. The same rule occurs in *ŚiDVṛ*, vi. 8.

and (vide *supra*, sec. 1, vs. 31)

$$drggati = \frac{\text{Rsin (Sun} - V) \times \text{Rcos } z}{\text{R}},$$

therefore,

$$lambana = \frac{\text{Rsin (Sun} - V) \times \text{Rcos } z}{(\text{R}/2)^2} \; ghaṭīs$$

$$= \frac{\text{Rsin } H \times \text{Rcos } z}{(\text{R}/2)^2} \; ghaṭīs, \text{ approx.}$$

COMPUTATION OF *NATI* AND MOON'S TRUE LATITUDE

18. Multiply the Rsine of the *dṛkkṣepa* (*Dorguṇa* or *Bhujajyā*) by the difference between the mean daily motions of the Sun and the Moon and divide (the resulting product) by 15 times the radius : the result is the *avanati* (or *nati*) whose direction is the same as that of the *dṛkkṣepa* (i e., the direction of the zenith distance of the central ecliptic point as reckoned from the zenith).[1] This (*avanati* or *nati*) added to or subtracted from the Moon's latitude (according as the two are of like or unlike directions) gives the Moon's true latitude.[2]

That is : *Nati*

$$= \frac{dṛkkṣepajyā \times (\text{Moon's mean daily motion} - \text{Sun's mean daily motion})}{15 \times \text{R}}$$

and

Moon's true latitude = Moon's latitude + or ~ *nati*,

+ or ~ sign being taken according as the Moon's latitude and *nati* are of like or unlike directions.

AKṢAVALANA AND *AYANAVALANA*

19. One should compute the *akṣavalana* in the manner stated before. And, from the Sun's anomaly diminished[3] by 11 degrees one should calculate the Sun's *ayanavalana* which is equal to the declination derived from the Rversed-sine (of the *bhuja*) thereof (by treating it as the Rsine of the Sun's longitude). Its direction is contrary to that of the hemisphere of the Sun's anomaly[4] (minus 11 degrees).

1. See *BrSpSi*, v. 11; v. 23(c-d)-24; *SūSi*, v. 10.
2. *Cf. BrSpSi*, v. 13; *SūSi*, v. 12.
3. Here also Vaṭeśvara adds 11 degrees. 4. Vaṭeśvara takes Moon's anomaly.

$$\text{Rsin (Sun's } \textit{ayanavalana}) = \frac{\text{Rvers (Sun's anomaly} - 11°) \times \text{Rsin } 24°}{\text{R}}$$

$$= \frac{\text{Rvers (Sun's long.} - 90°) \times \text{Rsin } 24°}{\text{R}},$$

because Sun's anomaly = Sun's longitude − 79°, Sun's longitude being tropical.

20. Other things should be computed in the manner (already) stated. The Moon's latitude, etc., should be made use of in the case of a solar eclipse (while constructing its diagram), as per instructions. (See *supra*, sec. 4)

General instruction has been given here by me; the details one should himself think out judiciously.

OTHER APPROXIMATE METHODS

(1) Meridian ecliptic point

21 (a-b). The longitude of the Sun diminished or increased by the signs obtained by dividing (the *ghaṭīs* of) the Sun's hour angle by 5 gives the longitude of the meridian ecliptic point.

That is : Longitude of meridian ecliptic point

$$= \text{Sun's longitude} \pm \left(\frac{\text{Sun's hour angle in terms of } \textit{ghaṭīs}}{5}\right) \text{ signs,}$$

where + or − sign is to be taken according as the Sun is to the west or to the east of the meridian ecliptic point.

This rule is approximate and follows by neglecting the obliquity of the ecliptic.

(2) *Lambana* for Ānandapura (latitude 24°)

21(c-d)-22(a). Multiply that (Sun's hour angle), (in terms of degrees), by 3 and divide by 74 : the result is the *lambana* in terms of *ghaṭīs* which should be subtracted from or added to the *tithi* (i. e., the time of geocentric conjunction of the Sun and Moon), the addition or subtraction of the *lambana* being made in the manner stated before.

22(b-d). (The *lambana* in terms of *ghaṭīs* may be obtained also) by multiplying that (Sun's hour angle, in terms of *ghaṭīs*,) for the time of conjunction by 10 and dividing (the resulting product) by 247—1/4.

The *natikā* (or *nati*) is obtained by using the distances (of the Sun and the Moon), as before.

That is : If H denotes the Sun's hour angle for the time of conjunction, in terms of degrees, then at Ānandapura (lat. 24°),

(1) *lambana* $= \dfrac{H \times 3}{74}$ *ghaṭīs*

(2) *lambana* $= \dfrac{H \times 10}{247-1/4}$ *ghaṭīs*.

Rationale. As in the previous rule, here too, neglecting the obliquity of the ecliptic and taking the latitude of the place to be 24°, we have

$$\textit{dṛggati} = \frac{\text{Rsin } H \times \text{Rsin } 66^\circ}{\text{R}}$$

$$= H \times {\cdot}91 \text{ degrees, approx.,}$$

so that

$$\textit{lambana} = \frac{H \times {\cdot}91 \times 4}{90} \textit{ ghaṭīs}, \text{ approx.}$$

$$= \frac{H \times 3}{74} \textit{ ghaṭīs}, \text{ approx.} \qquad (1)$$

The expression on the right may also be written as :

$$\frac{H \times 10}{247 - 1/4} \textit{ ghaṭīs}, \text{ approx.} \qquad (2)$$

CONCLUSION

23. The approximate method of computation of an eclipse that has been taught (above) without the use of the longitudes of the Moon, the Moon's ascending node and the Sun, the declination, and the Rsines of colatitude and latitude, is very difficult to be excelled by the other astronomers.

Section 7

Examples on Chapters IV and V

1. Those who, by (laying off) the Rsines of the *akṣa* and *ayana valanas* in the radius-circle, know the configuration of the eclipse at its beginning, middle and end are proficient in the construction of the diagram of an eclipse.

2. Or, one who, by reducing (the *valana* etc.) to the circle of radius equal to the sum of the semi-diameters of the eclipsed and eclipsing bodies, or to the circle of radius equal to the semi-diameter of the eclipsed body, knows how to draw the diagram of an eclipse in both of these two circles, is proficient in the graphical representation of an eclipse.

3. Or, one who (diagrammatically) exhibits (the phenomena of) immersion and emersion and the *iṣṭagrāsa* ("eclipse for the given time") with the help of the corresponding hypotenuse, upright and base, or with the help of the path of the eclipsing body, or with the help of the Rsines of the *valanas*, is a proficient astronomer.

4. Or, one who determines the (local) longitude in time from the lunar eclipse, therefrom the (local) longitude in terms of *yojanas*, and the diameters of the eclipsed and the eclipsing bodies from the *grāsa* ("measure of eclipse") at the middle of the eclipse, is a highly proficient astronomer on the earth.

5, Or, one who finds out the *grāsa* from the given time, the *ghaṭīs* of time from the given *grāsa*, and the *ghaṭīs* of the *parvatithi*, respectively, is regarded as the foremost amongst the astronomers.

6 One who knows (how to compute) the eclipses of the Moon and the Sun without the help of the longitudes of the Moon's ascending node, the Moon and the Sun, the local latitude and the declination, his lotus-like feet are always adored by those who are free from envy.

7. One who, by observing the Sun or Moon rising on the horizon, finds out its diameter, and determines the diameter of the Earth from *lambana* or *nati* is (indeed) an astronomer on the earth girdled by the oceans.

Chapter VI

HELIACAL RISING AND SETTING

RISING OR SETTING IN THE EAST OR WEST

1. A planet with lesser longitude (than the Sun) rises in the east if it is slower than the Sun, and sets in the east if it is faster than the Sun; whereas a planet with greater longitude (than the Sun) rises in the west if it is faster than the Sun, and sets in the west if it is slower than the Sun.[1]

2. The Moon, Venus and Mercury rise in the west, whereas Saturn, Mars and Jupiter and also retrograding Mercury and Venus rise in the east. These planets set in the opposite direction[2] **This rising and setting depends on the time-degrees of visibility and the visibility corrections.**

Āryabhaṭa II says :

"Mars, Jupiter, Saturn and Canopus, as well as retrograding Mercury and Venus, rise in the east when their longitudes are less than that of the Sun; when their longitudes are greater than the Sun's longitude, they set in the west.

Mercury and Venus, when in direct motion, as well as the Moon, when less than the Sun, set in the east; when greater than the Sun, rise in the west"[3]

TIME-DEGREES FOR HELIACAL VISIBILITY

3. Venus, with its luminosity lost in the Sun, becomes visible when it is at a distance of 9 time-degrees (from the Sun); Jupiter, Mercury, Saturn and Mars, when they are farther and farther away by 2 time-degrees in succession; the Moon as well as retrograding Mercury, when at a distance of 12 time-degrees;[4] **and retrograding Venus, when it is at a distance of 8 time-degrees.**[5]

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1. *Cf. BrSpSi*, vi. 2; also x. 30, 31; *ŚiDVr*, viii. 1; *SiŚe*, ix. 2; *SiŚi*, I, viii. 4(c-d).
2. *Cf. ŚiDVr*, viii. 1(c-d); *MSi*, ix. 1-2; *SiŚe*, ix. 3; *SiŚi*, I, viii. 5; *TS*, vii. 13. Also see *SūSi*, ix. 2-3.
3. *MSi*, ix. 1-2.
4. Same time-degrees are given in *BrSpSi*, vi. 6; *SiŚe*, ix. 8(a-b).
5. Same time-degrees are given in *ŚiDVr*, viii. 5.

Regarding Venus and Mercury, Brahmagupta further says :

"Owing to its small disc, Venus (in direct motion) rises in the west and sets in the east at a distance of 10 time-degrees (from the Sun); and owing to its large disc, the same planet (in retrograde motion) sets in the west and rises in the east at a distance of (only) 8 time-degrees (from the Sun). Mercury rises and sets in a similar manner when its distance (from the Sun) is 14 time-degrees (in the case of direct motion) or 12 time-degrees (in the case of retrograde motion)."[1]

Śrīpati[2] as well as the author of the *Sūryasiddhānta*[3] has said the same.

The following table gives the time-degrees for heliacal visibility (or heliacal rising and setting) of the planets according to the various Hindu astronomers.

Table 26. Time-degrees for heliacal visibility of the planets

Planet	Time-degrees according to				
	Ā[4], *KK*[5]	*ĀSi*	*BrSpSi*[6], *SūSi*[7] *SiŚe*[8], *SiŚi*[9]	*ŚiDVr*,[10] *VSi*	*MSi*[11]
Moon	12°	12°	12°	12°	12°
Mars	17°	17°	17°	17°	17°
Mercury	13°	13°	14°	13°	13°
Mercury (retro.)		12°	12°	12°	12° 30′
Jupiter	11°	11°	11°	11°	12°
Venus	9°	9°	10°	9°	8°
Venus (retro.)		7°	8°	8°	7° 30′
Saturn	15°	15°	15°	15°	15°

1. *BrSpSi*, vi. 11, 12; *KK*, II, v. 3-4. 2. See *SiŚe*, ix. 9.
3. See *SūSi*, ix. 7. 4. iv. 4. 5. I, vi. 1. 6. vi. 6, 11-12. 7. x. 1; ix. 6-8.
8. ix. 8-9. Śrīpati says that Venus, in retrograde motion, rises when it is separated from the Sun, according to some astronomers, by 4 time-degrees, and according to others by 3 time-degrees. See *SiŚe*, ix. 11(c-d). The former is indeed the opinion of Bhāskara I. See *MBh*, vi. 45.
9. I, viii. 6. 10. viii. 5.
11. ix. 3. Read *Kothā* for *Koḍhā* in S. Dvivedi's edition.

According to the Greek astronomer Ptolemy (c. A. D. 100-178) the distances of the planets, when in the beginning of the sign Cancer (i. e., when the equator and ecliptic are nearly parallel), from the true Sun, at which they become heliacally visible, are : for Mars, 14°30′; for Jupiter, 12°45′; for Saturn, 14°; and for Mercury and Venus, in the west, 11°30′ and 5°40′, respectively. See The *Almagest*, xiii. 7.

INCLINATIONS OF THE PLANETS' ORBITS

4. 7, 11, 5, 9 and 9, each multiplied by 15, are, in minutes, the greatest celestial latitudes of the planets beginning with Mars. Like the declination, the celestial latitude, too, is north or south, north when the sum of the longitudes of the planet and its ascending node (the latter measured westwards) is in the six signs beginning with Aries and south when that sum is in the six signs beginning with Libra.

That is, the orbital inclinations of the planets, according to Vaṭeś-vara, are :

Mars, 105′; Mercury, 165′; Jupiter, 75′; Venus, 135′; and Saturn 135′.

The same according to the other Hindu astronomers are as shown in the following table :

Table 27. Inclinations of the planet's orbits

Planet	Old *SūSi* (SMT)	*Ā*[1], *KK*[2] *ŚiDVṛ*[3], *SūSi*[4]	*BrSpSi*[5], *SiŚe*[6] *SiŚi*[7]	*MSi*[8]
Mars	90′	90′	110′	106′
Mercury	133′	120′	152′	138′
Jupiter	60′	60′	76′	74′
Venus	123′	120′	136′	130′
Saturn	126′	120′	130′	130′

1. i. 8. 2. I, viii. 1(c-d). 3. x. 5(a-b). 4. i. 68-70. 5. ix. 1.
6. xi. 8. 7. I, vii. 1. 8. iii. 39.

The orbital inclinations given by the Greek astronomer Ptolemy and by the modern astronomers are:

Planet	Ptolemy	Modern astronomers (for 1950·00 A. D.)
Mars	60′	111′ 00″
Mercury	420′	421′ 14″
Jupiter	90′	78′ 21″
Venus	210′	203′ 39″
Saturn	150′	149′ 25″

In the case of Mercury and Venus, the Hindu values differ significantly from those of Ptolemy and modern astronomers. It is because the values given by the Hindu astronomers are geocentric whereas those given by Ptolemy and modern astronomers are heliocentric.

COMPUTATION OF CELESTIAL LATITUDE

Method 1

5. To the longitude of the planet's ascending node apply the *śīghra-phala* contrarily to its application to the longitude of that planet: then is obtained the true longitude of the planet's ascending node. Add the true longitude of the planet's ascending node to the longitude of the planet and find the Rsine of the sum; this Rsine multiplied by the planet's greatest celestial latitude and divided by the planet's *śīghrakarṇa* gives the planet's celestial latitude.

$$\text{Planet's latitude} = \frac{\text{Rsin}\,(P + N) \times i}{H},$$

where P is the planet's longitude, N the true longitude of the planet's ascending node (measured westwards), i the planet's greatest celestial latitude and H the planet's *śīghrakarṇa*.

Rationale. In the case of the superior planets (Mars, Jupiter and Saturn) as well as the inferior planets (Mercury and Venus):

Distance of the planet from its ascending node (as measured in its actual heliocentric sphere)

= longitude of true-mean planet + longitude of planet's ascending node[1]

= (longitude of true planet − *śīghraphala*) + longitude of planet's ascending node[2]

= longitude of true planet + (longitude of planet's ascending node − *śīghraphala*)

= longitude of true planet + true longitude of planet's ascending node

$= P + N.$

Hence, in its actual heliocentric sphere,

$$\text{Rsin (planet's latitude)} = \frac{\text{Rsin}\,(P+N) \times \text{Rsin}\,i}{\text{R}}.$$

Since planet's latitude and i both are small, therefore

$$\text{Planet's latitude} = \frac{\text{Rsin}\,(P+N) \times i}{\text{R}} \text{ approx.}$$

Hence, in the geocentric sphere,

$$\text{Planet's latitude} = \frac{\text{Rsin}\,(P+N) \times i}{\text{R}} \times \frac{R}{H} = \frac{\text{Rsin}\,(P+N) \times i}{H},$$

where H is the planet's *śīghrakarṇa*.

The rules given by Brahmagupta[3], Lalla[4], Āryabhaṭa II[5], Śrīpati[6] and Bhāskara II[7] are different in form but essentially the same.

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1. *Cf. SiŚi*, I, vii. 2. Also see Method 2, below.
2. The minus sign before the *śīghraphala* denotes that it is to be applied contrarily to its application to the planet.
3. See *BrSpSi*, ix. 1; *KK*, II, v. 2.
4. See *ŚiDVṛ*, x. 6, 9, 10.
5. See *MSi*, iii. 35, 36.
6. See *SiŚe*, xi. 16, 17.
7. See *SiŚi*, I, vii. 2.

Method 2

6. Or, add the longitude of the planet's ascending node as obtained by the usual method to the true-mean longitude of the planet. Multiply the Rsine of that (sum) by the planet's greatest celestial latitude and divide by the planet's *śīghrakarṇa* : then is obtained the planet's desired celestial latitude.[1]

Method 3

7. Or, subtract the longitude of the planet's ascending node from a circle (i. e., 360°) and then subtract it from the true-mean longitude of the planet. The Rsine of that multiplied by the planet's greatest celestial latitude and divided by the planet's *śīghrakarṇa* gives the planet's desired celestial latitude.

Method 4

8. Or, to the longitude of the planet's ascending node as subtracted from a circle (i. e., 360°) apply the minutes of the *śīghraphala* in the usual way, stated before : then is obtained the true longitude of the ascending node (measured in the positive anticlockwise direction). Subtract that from the true longitude of the planet and find the Rsine thereof. Divide that by the planet's *śīghrakarṇa* and multiply by the planet's greatest celestial latitude : the result is the planet's desired celestial latitude.

One can easily see that all the four methods given above are equivalent.

VISIBILITY CORRECTION *AYANADṚKKARMA*

Āryabhaṭa I's Method

9. Multiply the Rversed-sine of (the *bhuja* of) three signs plus the planet's longitude by the Rsine of the (Sun's) greatest declination and also by the planet's celestial latitude and divide by the square of the radius : the result obtained gives the minutes of the *dṛk* (i. e., *ayanadṛkkarma*).[2]

That is, if λ be the planet's (tropical) longitude and B the *bhuja* of $(90° + \lambda)$, then

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1. *Cf. BrSpSi*, ix. 9, 10; *SiŚe*, xi. 15; *SiŚi*, I, vii. 2.
2. *Cf. Ā*, iv. 36; *SiŚe*, ix. 4.

$$ayanadṛkkarma = \frac{\text{Rvers } B \times \text{Rsin } 24^\circ \times \beta}{R^2}, \qquad (1)$$

β being the planet's celestial latitude.

For the rationale of this formula, the reader is referred to my notes on *Ā*, iv. 36.

Brahmagupta[1] modified this formula by replacing Rvers *B* by Rsin ($90^\circ + \lambda$) but his commentators interpreted Rsin ($90^\circ + \lambda$) of his formula as meaning Rvers *B*. It seems that Vaṭeśvara adopted Āryabhaṭa I's formula under the pressure of the general trend.

Āryabhaṭa II[2] followed Brahmagupta and gave the formula :

$$ayanadṛkkarma = \frac{\text{Rcos } \lambda \times \text{Rsin } 24^\circ \times \beta}{R^2},$$

where λ is the planet's tropical longitude and β the planet's latitude.

Brahmagupta, in one place[3], besides replacing Rvers *B* by Rsin ($90^\circ + \lambda$), seems to have multiplied formula (1) by 1800 and divided it by the *asus* of rising at Laṅkā (i. e., by the *asus* of right ascension) of the sign occupied by the planet.

Śrīpati[4], while retaining the use of Rversed-sine, has also multiplied formula (1) by 1800 and divided it by the *asus* of rising at Laṅkā of the sign occupied by the planet.

Bhāskara II[5] has criticised the use of Rversed-sine and has applauded Brahmagupta for having replaced Rversed-sine by Rsine. He has also demonstrated by means of an example the absurdity of the use of the Rversed sine. He rejects all the earlier formulae for the *ayanadṛkkarma* and in place of them prescribes the following two[6] :

1. See *BrSpSi*, vi. 3; xi. 66.
2. See *MSi*, vii. 2, 3.
3. See *BrSpSi*, x. 17.
4. See *SiŚe*, ix. 6.
5. See *ṢiŚi*, II, ix. 16-17 ff.
6. See *SiŚi*, I, vii. 4, 5(a-b).

(1) $ayanadṛkkarma = \frac{ayanavalana \times \beta}{\text{Rcos } \delta} \times \frac{1800}{T}$ mins.,

where δ is the planet's declination and T the time (in terms of *asus*) of rising at Laṅkā of the sign occupied by the planet ;

(2) $ayanadṛkkarma = \frac{ayanavalana \times \beta}{\text{Rcos } (ayanavalana)}$ mins., approx.

ALTERNATIVE FORMS

10. Or, multiply the Rversed-sine of (the *bhuja* of) three signs plus the planet's longitude by the planet's latitude and divide by 8454 : the result is the so called *dṛk* (i.e., *ayanadṛkkarma*). Subtract it from or add it to the planet's longitude according as the *ayana* and celestial latitude (of the planet) are of like or unlike directions.[1]

$$Ayanadṛkkarma = \frac{\text{Rvers } B \times \beta}{8454}, \tag{2}$$

where B is the *bhuja* of three signs plus the planet's (tropical) longitude, and β the planet's celestial latitude.

This formula is equivalent to formula (1) above. For, according to Vaṭeśvara (ch. II. sec. 1, vs. 50), Rsin 24° = 1398′ 13″ and R^2 = 11818047′ 35″, so that R^2/Rsin 24° = 8454 approx.

11. Subtract the Rsine of the planet's longitude from the radius and multiply the difference by the Rsine of 24° and also by the planet's latitude and divide by the square of the radius : (the result is the *ayanadṛkkarma*). Subtract it from or add it to the planet's longitude according as the planet's *ayana* and the planet's latitude are of like or unlike directions.[2]

$$Ayanadṛkkarma = \frac{(\text{R} - \text{Rsin } \lambda) \times \text{Rsin } 24^\circ \times \beta}{\text{R}^2}, \tag{3}$$

where λ, β are the planet's (tropical) longitude and latitude, respectively.

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1. *Cf. SiŚe*, ix. 5. For similar rules see *KK*, I, vi. 2; *KR*, v. 3; *ŚiDVṛ*, viii. 3(a-b); *MSi*, vii. 3. It is noteworthy that the rule for the subtraction or addition of the *ayanadṛkkarma* stated above by Vaṭeśvara is the same as prescribed by Bhāskara II. See *SiŚi*, I, viii. 2.
2. A similar formula occurs in *ŚiDVṛ*, viii. 2.

This formula also is equivalent to formula (1) above. For, the *bhuja* of $90° + \lambda$ is $90° - \lambda$, so that Rvers B = R − Rcos $(90° - \lambda)$ = R − Rsin λ.

12. Or, the radius diminished by the Rsine of the planet's longitude should be multiplied by the planet's latitude and divided by 8454 : the result (called *ayanadṛkkarma*) should be applied to the planet's longitude, as before. Then is obtained the so called *dṛglagna*.

$$Ayanadṛkkarma = \frac{(\text{R} - \text{Rsin}\ \lambda) \times \beta}{8454}, \tag{4}$$

where λ and β are the planet's tropical longitude and latitude, respectively.

Formula (4) is evidently equivalent to formula (2), because Rvers B = R − Rsin λ.

13. Or, find the Rsine of declination from the Rversed-sine of (the *bhuja* of) three signs plus the planet's longitude (treating it as the Rsine of the planet's longitude). Multiply it by the planet's latitude and divide by the radius. Apply the result (known as *ayanadṛkkarma*) in the manner stated above.

Or, multiply the difference between the Rsine of the Sun's greatest declination and the Rsine of declination of the planet, by the planet's latitude and divide by the radius. Apply the result (known as *ayanadṛkkarma*) to the planet's longitude in the manner stated above.

$$Ayanadṛkkarma = \left(\frac{\text{Rvers}\ B \times \text{Rsin}\ 24°}{\text{R}}\right) \times \frac{\beta}{\text{R}} \tag{5}$$

$$Ayanadṛkkarma = \left(\text{Rsin}\ 24° - \frac{\text{Rsin}\ \lambda \times \text{Rsin}\ 24°}{\text{R}}\right) \times \frac{\beta}{\text{R}}, \tag{6}$$

where B is the *bhuja* of three signs plus the planet's (tropical) longitude and β the planet's latitude.

Formula (5) is the same as formula (1) and formula (6) is equivalent to it, because Rvers B = R − Rsin λ.

14. The product of the Rsine of the (Sun's) greatest declination and the planet's latitude being divided by the radius gives the "multiplier" in terms of minutes of arc. The product of the radius and the planet's latitude divided by 8454 is also the "multiplier".

15. Severally multiply the product of the Rversed-sine of (the *bhuja* of) three signs plus the planet's longitude and the Rsine of the (Sun's) greatest declination by the two multipliers (stated in vs. 10) and divide (each product) by the Rsine of 24° and the radius : the quotient (in each case gives the *ayanadṛkkarma* which) should be applied to the planet's longitude in the mannar stated above.

$$Ayanadṛkkarma = \frac{\text{Rvers } B \times \text{Rsin } 24° \times \text{multiplier}}{\text{R} \times \text{Rsin } 24°},$$

where $$\text{multiplier} = \frac{\text{Rsin } 24° \times \beta}{\text{R}} \tag{7}$$

$$\text{or } \frac{\text{R} \times \beta}{8454}, \tag{8}$$

B and β having the same meanings as stated above.

16. Severally multiply (the product of) the Rsine of the (Sun's) greatest declination and the Rsine of the planet's longitude by the multipliers (stated in vs. 10) and divide (each product) by the radius and the Rsine of 24°. The quotient (in each case) should be subtracted from the (corresponding) multiplier and the result (called *ayanadṛkkarma*) should be applied to the longitude of the planet in the manner stated above. Then is obtained the *dṛgvilagna*.

$$Ayanadṛkkarma = \text{multiplier} - \frac{(\text{Rsin } \lambda \times \text{Rsin } 24°) \times \text{multiplier}}{\text{R} \times \text{Rsin } 24°},$$

where $$\text{multiplier} = \frac{\text{Rsin } 24° \times \beta}{\text{R}} \tag{9}$$

$$\text{or } \frac{\text{R} \times \beta}{8454}. \tag{10}$$

17. Multiply the Rsine of the planet's latitude by the product of the Rsine of the (Sun's) greatest declination and the Rsine of the (Sun's) greatest declination minus the Rsine of the planet's own declination and divide (the resulting product) by the radius and the Rsine of 24° : the quotient (called *ayanadṛkkarma*) should be applied to the planet's longitude in the manner stated above.

$$Ayanadṛkkarma = \frac{\text{Rsin}\ \beta \times \text{Rsin}\ 24^\circ \times (\text{Rsin}\ 24^\circ - \text{Rsin}\ \lambda \times \text{Rsin}\ 24^\circ/\text{R})}{\text{R} \times \text{Rsin}\ 24^\circ} \quad (11)$$

This simplifies to formula (3).

VISIBILITY CORRECTION *AKṢADṚKKARMA*

Method 1. Āryabhaṭa I's Method.

18. Multiply the planet's latitude by the equinoctial midday shadow and divide by 12. (Then is obtained the so called *akṣadṛkkarma*.) Apply it to the planet's longitude corrected for the *ayanadṛkkarma*, in the case of its rising (on the eastern horizon) or setting (on the western horizon) as a negative or positive correction, respectively, provided the planet's latitude is north; or, as a positive or negative correction, respectively, provided the planet's latitude is south.[1] (Then is obtained the longitude of that point of the ecliptic which rises or sets with the planet.)

$$Akṣadṛkkarma = \frac{\text{planet's latitude} \times palabhā}{12}.$$

Method 2. Āryabhaṭa I's Alternative Method

19. Or, obtain the minutes (of the *akṣadṛkkarma*) by dividing the product of the planet's latitude and the Rsine of the latitude (of the place) by the Rsine of the colatitude; and apply them as stated above.[2]

$$Akṣadṛkkarma = \frac{\text{planet's latitude} \times \text{Rsin}\ \phi}{\text{Rcos}\ \phi},$$

where ϕ is the latitude of the place.

Both the formulae, stated above, are equivalent. For their rationale the reader is referred to my notes on *Ā*, iv. 35. As pointed out there, these formulae are approximate.

Method 3. Brahmagupta's Method

20. The latter visibility correction (viz. the *akṣadṛkkarma*) for a planet which has been stated with the help of the planet's latitude is not very accurate. So another visibility correction (to replace it) is being stated now which will make computation conform with observation.

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1. *Cf. KK*, I, vi. 3; *BrSpSi*, vi 4; *KR*, v. 2; *ŚiDVṛ*, viii. 3(d)-4; *MSi*, vii. 4; *SiŚe*, ix. 7(b-d); *TS*, vii. 1-2(a-b). This rule occurs in *SMT* also.
2. *Cf. Ā*, iv. 35; *MBh*, vi. 1-2(a-b); *LBh*, vi. 1-2; *ŚiDVṛ*, viii. 3(c); *SiŚe*, ix. 7(a-b).

21. Add the declination and latitude of the planet if they be of like directions; otherwise, take their difference. Then is obtained the true declination of the planet.[1] From that (true declination) and also from the mean declination (of the planet) obtain the *asus* of the ascensional difference in the manner stated.

22-23(a). Take their sum or difference according as they are of unlike or like directions;[2] and apply the result to the planet's longitude, corrected for *ayanadṛkkarma*, in the case of rising of the planet (on the eastern horizon) as a negative correction if the planet's latitude be north, or as a positive correction if the planet's latitude be south; and reversely, in the case of setting of the planet (on the western horizon). Then is obtained the true longitude of that point of the ecliptic which rises when the planet rises (called planet's *udayavilagna*) or the true longitude of that point of the ecliptic which sets when the planet sets (called the planet's *astavilagna*).[3]

The following is the rationale of the above rule :

In Fig 1. TBB′ is the equator and P its north pole; TG′ is the ecliptic and K its north pole. G is the actual position of a planet and G′ its projection on the ecliptic, called the local position of the planet. G′A is the perpendicular dropped from G′ on the great circle PGB. GG′ is the planet's

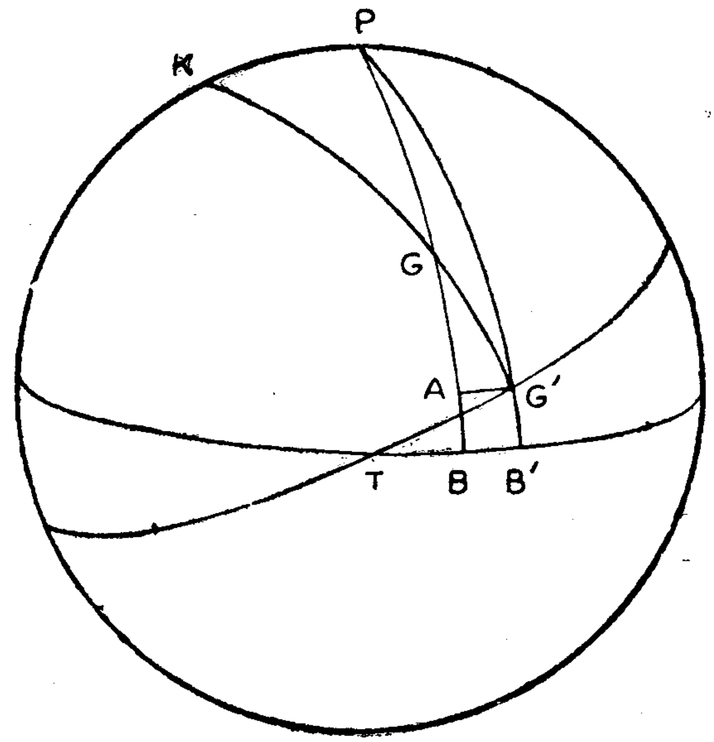


Fig. 1

1. *Cf. KK*, I, iii. 7(c-d); *MSi*, iii. 31(a-b).
2. The ascensional differences corresponding to the mean and true declinations are of unlike or like directions according as the mean and true declinations are of unlike or like directions. See *infra*, chap. VIII, sec. 2, vs. 22(a-b).
3. *Cf. BrSpSi*, x. 13-15, 18-20.

latitude and G′B′ the planet's mean (or local) declination. Then, since the planet's latitude GG′ is small, GB = G′B′ + GG′, approx.

i.e., planet's true declination = planet's mean declination + planet's latitude. (1)

Now, in Fig. 2 below, SEN is the horizon and Z the zenith. X is the actual position of a planet at its rising, and Y its projection on the ecliptic. TE is the equator and P its north pole; TY is the ecliptic and K its north pole. U is the point where the diurnal circle through Y meets the horizon.

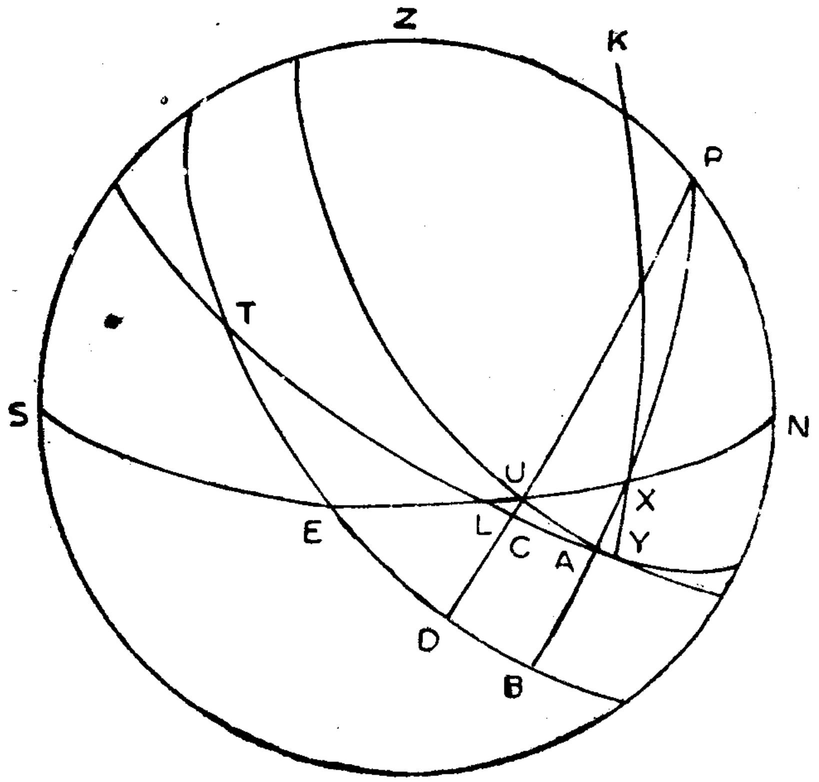


Fig. 2

PUD is the hour circle of U and PXB the hour circle of X. L is the point where the ecliptic intersects the horizon, i. e., the rising point of the ecliptic. A is the point where the hour circle of X intersects the ecliptic. Then AY is the *ayanadṛkkarma* and LA is the *akṣadṛkkarma*. According to Method 3 of Vaṭeśvara,

LA = DB approx. = EB − ED

= asc. diff. due to planet's true declination XB
− asc. diff. due to planet's mean declination UD. (2)

Formulae (1) and (2) are both approximate. They were modified by Bhāskara II[1].

1. See *SiŚi*, I, vii. 3; also II, ix. 10; and I, vii. 6, 7.

The longitude of a planet when corrected for the visibility corrections for rising gives the longitude of that point of the ecliptic which rises when the planet rises, and the longitude of a planet when corrected for the visibility corrections for setting gives the longitude of that point of the ecliptic which sets when the planet sets. These two points have been called above by the names *udayavilagna* and *astavilagna.*

The term *astavilagna* or *astalagna* (for a planet), however, is more generally used in the sense of the longitude of that point of the ecliptic which rises when the planet sets. In the next chapter Vaṭeśvara has also used this term in this sense. See *infra,* chap. VII, sec. 1, vs. 11.

TIME OF HELIACAL RISING OR SETTING

Method 1

23(b-d). In the case of rising (or setting) in the east, find the *asus* of rising of the untraversed part of the sign occupied by the planet (computed for sunrise and corrected for the visibility corrections for rising), and those of the traversed part of the sign occupied by the Sun (at sunrise); and in the case of rising (or setting) in the west, in the reverse order.[1] Add them to the *asus* of rising of the intervening signs. (Then are obtained the *asus* of rising of the part of the ecliptic lying between the planet corrected for the visibility corrections and the Sun, at sunrise. These divided by 60 give the time-degrees between the planet corrected for the visibility corrections and the Sun).

24. (To obtain the time-degrees corresponding to the untraversed and traversed parts), one should multiply the untraversed and traversed degrees by the *asus* of rising of the corresponding signs and divide (the products) by 30 and 60 (i. e., by 1800). The time degrees divided by 6 give the corresponding *ghaṭīs.*[2]

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1. What is meant is that, in the case of rising or setting in the west, one should compute (for sunset) the longitude of the planet corrected for the visibility corrections for setting and also the longitude of the Sun. Both of them should be increased by six signs. One should then find the *asus* of rising of the traversed part of the sign occupied by the planet (increased by six signs) as also the *asus* of rising of the untraversed part of the sign occupied by the Sun (increased by six signs).

2. See *supra,* ch. III, sec. 8, vs. 26.

25. When the time-degrees between the planet corrected for the visibility corrections and the Sun are greater than the time-degrees for the planet's visibility, it should be understood that the planet is in (heliacal) rising; if less, it is not so.[1]

Mallikārjuna Sūri explains the procedure as follows :

"In the case of rising in the east, one should find the *gamya asus* of the planet (for sunrise) corrected for the visibility corrections for rising, and the *gata asus* of the Sun (for sunrise); these should be added to the *asus* of rising of the (complete) signs lying between them. The resulting *asus* are the *asus* lying between the (visible) planet and the Sun. In the case of rising in the west, find the *gata asus* of the planet (for sunset) corrected for the visibility corrections for setting and increased by six signs, and the *gamya asus* of the Sun (for sunset) increased by six signs; and these should be added to the *asus* of rising of the (complete) signs lying between them. The resulting *asus* are the *asus* lying between the (visible) planet and the Sun. When divided by 60, they become time-degrees. When these time-degrees are greater than the above mentioned time-degrees for the planet's visibility, it should be understood that the planet is visible; when less, the planet is invisible."[2]

So also writes Āryabhaṭa II:

"(In the case of rising or setting in the east, calculate for sunrise, the longitude of the Sun and the longitude of the planet corrected for the visibility corrections;) in the case of rising or setting in the west, calculate for sunset, the longitude of the Sun and the longitude of the planet corrected for the visibility corrections, each increased by six signs. (In each case) multiply the intervening degrees[3] by the *asus* of rising of the *drekkāṇa* in which they are situated, and divide by 600: the quotient gives the desired time-degrees. When these time-degrees are greater than the time-degrees for the planet's visibility, the setting of the planet is to occur; when less, the planet has already set. In the case of rising, the rule is just the contrary (i. e., when the time-degrees obtained above are greater than the time-degrees for the planet's visibility, the planet has already risen; when less, the rising of the planet is to occur)."[4]

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1. *Cf. ŚiDVṛ*, viii. 6; also viii. 7; *MSi*, ix. 4-5.
2. See Mallikārjuna Sūri's com. on *ŚiDVṛ*, viii. 6.
3. It is supposed that these degrees are less than 10.
4. See *MSi*, ix. 4-5, Āryabhaṭa II, in this rule, has assumed the Sun and the planet to be in the same *drekkāṇa*.

Method 2

26. Multiply the time-degrees (for the planet's visibility) by the number of minutes in a sign and divide by the *asus* of rising of the sign (occupied by the Sun and the planet): the quotient gives the degrees of the ecliptic (corresponding to those time-degrees). When these degrees are greater than the degrees lying between the Sun and the (visible) planet (i. e., between the Sun and the planet corrected for the visibility corrections, calculated for sunrise in the case of rising or setting in the east; or between the Sun and the planet corrected for the visibility corrections, each increased by 6 signs, calculated for sunset in the case of rising or setting in the west), the planet is in heliacal setting; when less, it is visible.[1]

27. Divide that difference by the difference of the daily motions of the Sun and the planet when the planet is in direct motion, and by the sum of the daily motions of the Sun and the planet when the planet is in retrograde motion : the result is the time (in days) which has to elapse before the planet will rise or set or elapsed since the rising or setting of the planet.[2]

Method 3

28. The Moon is visible when it is separated from the Sun, in the manner stated above, by 2 *ghaṭīs*; Venus, Jupiter, Mercury, Saturn, and Mars rise heliacally, when separated from the Sun by 1½ *ghaṭīs* increasing successively by 1/3 of a *ghaṭī*.[3]

29(a-c). Venus, in retrograde motion, rises when it is separated from the Sun by 1⅓ *ghaṭīs*; and Mercury, in retrograde motion, when separated from the Sun by 2 *ghaṭīs*.[4]

29(d)-31. When the *nāḍīs* between the (visible) planet and the Sun exceed (them), the planet is in heliacal rising; when fall short, it is in heliacal setting.

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1. See *SūSi*, ix.16.
2. Similar rules are found in *ŚiDVr*, viii. 7-8; *KP*, vi. 6-7.
3. *Cf. BrSpSi*, x. 32; *ŚiDVr*, viii. 5 (a-b); *SiŚe*, ix. 12-13.
4. *Cf. ŚiDVr*. viii. 5 (c).

The *asus* of the excess or defect divided by the difference between the daily motions of the Sun and the planet (when the planet is in direct motion) or by the sum of their daily motions when the planet is in retrograde motion, give the days elapsed since or to elapse before the heliacal rising or setting of the planet.[1] One should calculate the longitude of the Sun and the visible planet for the approximate time of rising or setting (thus obtained), and then iterate the process until those days are fixed.[2]

The *ghaṭīs* of visibility, stated in vss. 28-29, have been derived from the time-degrees for visibility, stated in vs. 3 above, by dividing them by 6. For, as stated in vs. 24(b) above, time-degrees divided by 6 give the corresponding *ghaṭīs*.

DIRECTION FOR OBSERVATION

32. Holding the instrument with one of its extremities at the top of the gnomon which is set to move with the planet and the other as many *aṅgulas* away from the foot of the gnomon as the tip of the planet's shadow is, the king should be shown through it the heliacal setting, planetary conjunction and its duration, the heliacal rising, and other things which are difficult to perform (by other astronomers).[3]

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1. *Cf. BrSpSi*, vi. 7; also x. 33.
2. *Cf. BrSpSi*, vi. 7; *SiŚe*, ix. 10; *SiŚi*, I, vii. 8(c-d)-10.
3. Similar statements are found to occur in *BrSpSi*, vii. 17; *SiŚe*, iv. 86; *SiŚi*, I, iii. 109.

Chapter VII

ELEVATION OF LUNAR HORNS

Section 1

(1) Diurnal Rising and Setting of the Moon

INTRODUCTION

1. From the longitudes of the Sun and the Moon (the latter corrected for the visibility corrections) calculate the time of rising of the Moon in the night, in the dark half of the month, in the manner stated before,[1] and the time of setting of the Moon in the night, in the light half of the month, contrarily. This, however, is not (always) done from the degrees of the difference between the longitudes of the Moon and the Sun (but from those stated below).

In what follows, the Moon corrected for the visibility corrections for rising (in the case of its rising) or for setting (in the case of its setting) will be called "the visible Moon."

TIME OF MOONRISE OR MOONSET (FIRST QUARTER)

2. In the light half (I Quarter) of the month, the calculation of the time of rising of the Moon in the day is prescribed to be made from the positions of the Sun and the (visible) Moon at sunrise, in the manner stated before;[2] and that of the time of setting of the Moon (in the night) from the positions of the Sun and the (visible) Moon at the end of the day (i. e., sunset), both increased by six signs.

3. In the case of setting of the Moon at night (in the I Quarter of the month), one should find the *asus* (of oblique ascension) intervening between the Sun and the (visible) Moon (for sunset), both increased by six signs, making use of the visibility corrections (for setting, in the case of the Moon), and apply the process of iteration on those *asus* of the difference between the Sun and the (visible) Moon, both increased by six signs.

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1. *Vide supra*, ch. VI. vss. 23-24.
2. Also see *MSi*, x. 2-3.

That is, in order to find the time of moonrise in the I Quarter of the month, proceed as follows: Compute the (tropical) longitudes of the visible Moon and the Sun for sunrise. Then find out the *asus* (A_1) due to oblique ascension of that part of the ecliptic which lies between the visible Moon and the Sun. Then A_1 *asus* denote the first approximation to the time of moonrise (reckoned since sunrise). Then calculate the displacements of the Sun and the Moon for A_1 *asus* and add them respectively to the longitudes of the visible Moon and the Sun (for sunrise); and then find out the *asus* (A_2) due to the oblique ascension of that part of the ecliptic which lies between the positions of the Sun and the (visible) Moon, thus obtained. Then A_2 *asus* give the second approximation to the time of moonrise. Iterate this process until the time of moonrise is fixed. *Asus* may be converted into *ghaṭīs* by dividing them by 360.

The time thus obtained is in civil reckoning. If, however, use of the Moon's displacement alone be made at every stage, the time obtained would be in sidereal reckoning.

Next, in order to find the time of moonset in the I Quarter of the month, proceed as follows: Calculate the (tropical) longitudes of the visible Moon and the Sun for sunset and increase both of them by six signs. Then find out the *asus* due to oblique ascension of that part of the ecliptic which lies between the resulting positions of the Sun and the Moon. These *asus* would give the first approximation to the time of moonset (reckoned since sunset). To get the nearest approximation to the time of moonset, iterate the process, in the manner stated above, until the time of moonset is fixed.

It should be noted that while finding the time of moonset in the first quarter of the month, the longitudes of the Sun and the visible Moon, for sunset, are increased by six signs and then the *asus* of oblique ascension of that part of the ecliptic which lies between them are obtained. This is done because the time of setting of the part of the ecliptic lying between the Sun and the visible Moon, at sunset, is equal to the time of rising of the diametrically opposite part of the ecliptic. What is actually needed is the time of setting of that part of the ecliptic which lies between the Sun and the visible Moon, at sunset. Since this is equal to the time of rising of the diametrically opposite part of the ecliptic, one has to find the diametrically opposite part of the ecliptic by adding six signs to the longitudes of the Sun and the visible Moon, for sunset. But if one uses the table giving the times of setting of the signs (instead of the table giving the times of rising of the

signs), the addition of six signs to the longitudes of the Sun and the visible Moon, for sunset, is not needed. This is true in the other cases also.

Table 28. Times of setting of the signs

Sign	Time of setting in *asus* at the equator	Time of setting in *asus* at the local place	Sign
1. Aries	1669	1669 + a	12. Pisces
2. Taurus	1796	1796 + b	11. Aquarius
3. Gemini	1935	1935 + c	10. Capricorn
4. Cancer	1935	1935 − c	9. Sagittarius
5. Leo	1796	1796 − b	8. Scorpio
6. Virgo	1669	1669 − a	7. Libra

(a, b, c are the ascensional differences of Aries, Taurus and Gemini, respectively).

FOURTH AND SECOND QUARTERS

4. In the dark half (IV Quarter) of the month, the (time of) rising of the Moon, when the night is yet to end, should be calculated by the process of iteration from the positions of the Sun and the (visible) Moon (for sunrise); and in the light half (II Quarter) of the month, the (time of) rising of the Moon, when the day is yet to end, should be calculated by the process of iteration from the position of the Sun (for sunset) increased by six signs and the position of the (visible) Moon (for sunset).

5. In the dark half (IV Quarter) of the month, the time of setting of the Moon, when the day is yet to elapse, should be obtained from the positions of the Sun and the (visible) Moon (for sunset), each increased by six signs.

In the light half (II Quarter) of the month, the same time (of setting of the Moon), when the night is yet to elapse, should be obtained from the position of the (visible) Moon (for sunrise) increased by six signs and the position of the Sun (for sunrise).

That is, to find the time of moonrise in the dark half (IV Quarter) of the month proceed as follows: Calculate the longitudes of the Sun and the

visible Moon for sunrise. Then calculate the *asus* due to the oblique ascension of that part of the ecliptic which lies between the calculated Sun and the visible Moon. This would give the first approximation to the time of moonrise (to elapse before sunrise). To obtain the nearest approximation to the time of moonrise, iterate the process until the time of moonrise is fixed.

To find the time of moonrise in the light half (II Quarter) of the month, calculate the longitude of the visible Moon for sunset and also calculate the longitude of the Sun for sunset, and increase the latter longitude by six signs. Then find the *asus* of the oblique ascension of that part of the ecliptic that lies between the calculated positions of the Sun (as increased by six signs) and the visible Moon. This would give the first approximation to the time of moonrise (to elapse before sunset). To obtain the nearest approximation to the time of moonrise, iterate the above process until the time of moonrise is fixed.

To find the time of moonset in the dark half (IV Quarter) of the month proceed as follows: Calculate the positions of the Sun and the visible Moon for sunset and increase them by six signs. Then find the *asus* due to the oblique ascension of that part of the ecliptic that lies between those positions. This would give the first approximation to the time of moonset (to elpase before sunset). To obtain the nearest approximation to the time of moonset, iterate the above process until the time of moonset is fixed.

To obtain the time of moonset in the light half (II Quarter) of the month, calculate the position of the visible Moon for sunrise and increase it by six signs and also (calculate) the position of the Sun for sunrise. Then find out the *asus* due to the oblique ascension of that part of the ecliptic that lies between those positions. This would give the first approximation to the time of moonset (to elapse before sunrise). To obtain the nearest approximation to that time, iterate the process until the time of moonset is fixed.

To obtain the time of rising of the Moon in the III Quarter of the month, one should proceed in the manner stated in the *Mahā-Bhāskarīya* (vi. 28-31, 32-34).

MOON'S DISPLACEMENT AND PROCESS OF ITERATION CONTEMPLATED ABOVE

6. (In order to obtain the Moon's displacement) the *nāḍīs* of the period elapsed or to elapse, (during day or night), should be multiplied

by the Moon's daily motion and divided by 60. The resultiug displacement (of the Moon) should be subtracted from or added to the longitude of the Moon (as the case may be). Applying the visibility corrections, the time elapsed or to elapse should be obtained afresh. The desired time (of moonrise or moonset) should then be determined by the process of iteration.[1]

The process of iteration has been already explained above If use of the Moon's displacement alone is made in the above process at every stage, the time obtained would be in sidereal reckoning. If the time of moonrise or moonset is required in civil reckoning, one should make use of the displacements of the Sun and the Moon both.

Regarding the subtraction and addition of the Moon's displacement, Bhāskara II, (in his com. on *ŚiDVr*, viii. 11), says :

"If moonrise occurs before sunset (when some part of the day is yet to elapse), then the Moon's displacement should be subtracted from the Moon's longitude for sunset; and if moonrise occurs after sunset (when some part of the night has already elapsed), it should be added to the Moon's longitude (for sunset). Similarly, if moonset occurs after sunrise (when some part of the day has already elapsed), the Moon's displacement should be added to the Moon's longitude (for sunrise); and if moonset occurs before sunrise (when some part of the night is yet to elapse), it should be subtracted from the Moon's longitude (for sunrise)."

MOONRISE ON FULL MOON DAY

7. When the true longitude of the Moon (for sunset), (corrected for the visibility corrections for rising), becomes equal to the longitude of the Sun (for sunset) increased by six signs, then the Moon, in its full phase, resembling the face of a beautiful lady, rises (simultaneously with the setting Sun), and goes high up in the sky, rendering by its light the circular face of the earth freed from darkness and making the lotuses on the earth close themselves on account of hatred for the (mutual) love of the Cakravāka birds.[2]

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1. *Cf. ŚiDVr*, viii. 11.
2. *Cf. SiŚe*, ix. 14.

Lalla says :

"When (at sunset) the longitude of the Moon (corrected for the visibility corections for rising) is equal to the longitude of the Sun increased by six signs, then the Moon rises simultaneously with the setting Sun; when greater, it rises in the night (after sunset); and if less, it rises in the day (before sunset)."[1]

"When (at sunrise) the longitude of the Moon (corrected for the visibility corrections for setting), increased by six signs, is equal to the longitude of the rising Sun, then it sets simultaneously with the rising Sun; when greater, it sets in the day (after sunrise); and when less, it sets in the night (before sunrise)."[2]

RISING MOON AND SETTING SUN ON FULL MOON DAY

8. On the full moon day, the Sun and the Moon, stationed in the zodiac at a distance of six signs, appear on the evening horizon like the two huge gold bells (hanging from the two sides) of Indra's elephant.

The practice of hanging huge bells from the two sides of an elephant is still prevalent. As the elephant moves, the bells ring and herald his arrival.

(2) Moon's Shadow

SUITABLE TIME FOR CALCULATION

9. When the Moon's longitude (corrected for the visibility corrections) is less than the longitude of the rising point of the ecliptic, or greater than the longitude of the setting point of the ecliptic, it is visible in the clear sky.[3] One should then calculate (the length of) the Mooh's shadow (i. e., the shadow cast by the gnomon due to moonlight) and the central width of the lunar horn.

So also says Lalla :

"When the longitude of the Moon, corrected for the visibility corrections, is less than the longitude of the rising point of the ecliptic or greater

1. *ŚiDVr*. viii. 9. Also *cf. LBh*, vi. 20; *SiŚe*, ix. 14.
2. *ŚiDVr*. viii. 10. Also *cf. SiŚe*, ix. 15.
3. *Cf. BrSpSi*, viii. 3; x. 21; *ŚiDVr*, viii. 12 (a-b).

than the longitude of the setting point of the ecliptic, the Moon is visible in the sky."[1]

Brahmagupta says :

"If the instantaneous rising point of the ecliptic is greater than the rising point of the ecliptic at the time of moonrise, or less than the setting point of the ecliptic at the time of mooset, increased by six signs, the Moon is visible. The Moon being visible, one should calculate its shadow."[2]

The process for finding the Moon's shadow is described below in vss. 10-17, and the method for finding the central width of the lunar horn (i. e., the measure of the illuminated part of the Moon) is stated below in vss. 23-24.

MOON'S TRUE DECLINATION AND DAY-RADIUS

10. Find out the declination of the Moon from the longitude of the Moon for that time and decrease or increase it by the Moon's own celestial latitude according as the two are of unlike or like directions : the result is the Moon's true declination. From this find the (Moon's) day-radius (i. e., Rsine of the Moon's true codeclination), etc , as in the case of the Sun.[3]

(1) Moon's true declination = Moon's declination + Moon's latitude, or Moon's declination $\sim$ Moon's latitude,

according as the Moon's declination and the Moon's latitude are of like or unlike directions.

(2) Moon's day-radius = $\sqrt{R^2 - [\text{Rsin (Moon's true declination)}]^2}$.

MOON'S DAY AND MOON'S ASCENSIONAL DIFFERENCE

11. The longitude of that point of the ecliptic which rises with the Moon (the so called *dṛgudayavilagna*), when increased by six signs,[4] gives the longitude of that point of the ecliptic which rises when the Moon sets (the so called *dṛgastalagna*).[5] Find the oblique ascension of

1. *ŚiDVṛ*, viii. 12 (a-b).
2. *BrSpSi*, viii. 3.
3. *Cf. BrSpSi*, vii. 5; *ŚiDVṛ*, ix. 2; *SiŚe*, x. 7; *SiŚi*, I, vii. 13 (a-b).
4. After the increase of 6 signs Mallikārjuna Sūri prescribes the application of the visibility corrections for setting also. See Mallikārjuna Sūri's com. on *ŚiDVṛ*, x-15-16.
5. This definition of the Moon's *dṛgastalagna* is approximate.

that part of the ecliptic that lies between the two (viz. *dṛgudayavilagna* and *dṛgastalagna*) with the help of the oblique ascensions of the signs : (this is the length of the Moon's day).[1] The difference between half of it in terms of *ghaṭīs* and 15 *ghaṭīs* is the Moon's ascensional difference.

Stated more explicitly :

(1) Moon's day = time of rising at the local place of the portion of the ecliptic that lies between the Moon's *dṛgudayavilagna* and the Moon's *dṛgastalagna*

= time of rising of the untraversed portion of the sign occupied by the Moon's *dṛgudayavilagna* + time of rising of the traversed portion of the sign occupied by the Moon's *dṛgastalagna* + time of rising of the intermediate signs.

(2) Moon's ascensional difference = $\frac{1}{2}$ (Moon's day) $\sim$ 15 *ghaṭīs*.

The above formula for the Moon's day is only approximate, as the Moon's motion from its rising to its setting has been neglected. The correct formula was stated by Āryabhaṭa II. According to Āryabhaṭa II[2]:

Planet's day = time of rising of the untraversed portion of the sign occupied by the planet's *udayalagna* + time of rising of the traver-portion of the sign occupied by the planet's *astalagna* + time of rising of the intermediate signs,

where

planet's *udayalagna* = rising point of the ecliptic at the time of planet's rising

= planet's true longitude at the time of its rising + planet's visibility corrections for rising,

planet's *astalagna* = rising point of the ecliptic at the time of planet's setting

---

1. Same has been stated by Lalla. See *ŚiDVṛ*, x, 15-16.
2. See *MSi*, x. 4-5. Also see S. Dvivedi's commentary on it.

= planet's true longitude at the time of rising of the planet + planet's motion for half its day + planet's visibility corrections for setting + 6 signs,

the process of iteration being applied to get the nearest approximation for the planet's day.

It is noteworthy that according to Āryabhaṭa II, and Bhāskara II as well, a planet's *astalagna* means "the rising point of the ecliptic at the time of planet's setting" and a planet's *udayalagna* means "the rising point of the ecliptic at the time of planet's rising."

Āryabhaṭa II says: "On account of the provector wind, a planet always rises when the rising point of the ecliptic is equal to the planet's *udayalagna*, and sets when the rising point of the ecliptic is equal to the planet's *astalagna*."[1]

Āryabhaṭa II further remarks: "The *udayalagna* in the case of the Seven Sages (*Saptarṣis*) and the other stars remains fixed for quite a few years; but that is not so in the case of the Moon, etc., on account of their motion (along the ecliptic)."[2]

As regards the Moon's ascensional difference, Lalla gives the following formula:[3]

Moon's true ascensional difference

= Moon's mean ascensional difference ± *akṣadṛkkarma* for the Moon,

+ or − sign being taken according as the Moon's mean declination and latitude are of like or unlike directions.

This formula is the same as formula (2) given on p. 543 above.

PLANET'S NYCHTHEMERON

12. Sixty (*ghaṭīs*) increased by the *asus* (of oblique ascension) of the planet's true daily motion is stated by the learned scholars with

1. *MSi*, x. 6.
2. *MSi*, x. 8.
3. See *ŚiDVr*, ix. 3.

specialized knowledge of astronomy as the more accurate measure of the planet's day-and-night.

MOON'S SHADOW

Method 1

13. Find the *asus* of day to elapse at the time of moonrise and add them to the *asus* of night elapsed (at the time of computation) : (this gives the *asus* elapsed since moonrise). (From them obtain, as in the case of the Sun,[1] the shadow cast by the gnomon (*nara*) and by the Rsine of the Moon's altitude (*candradīpa*) due to the Moon's motion, as also the *bhuja* etc.

The *bhuja* is the distance of the foot of the perpendicular dropped from the Moon on the plane of the horizon from the east-west line.

Method 2

14. Or, from the portion of the ecliptic lying between the rising point of the ecliptic and the visible Moon (i. e., the Moon corrected for the visibility corrections for rising), (when the Moon is in the eastern half of the celestial sphere), or from the portion of the ecliptic lying between the rising point of the ecliptic and the visible Moon (i. e., the Moon corrected for the visibility corrections for setting), increased by six signs, (when the Moon is in the western half of the celestial sphere), find the *asus* of the *unnatakāla* (i. e., *asus* elapsed since moonrise in the eastern half of the celestial sphere or to elapse before moonset in the western half of the celestial sphere) with the help of the times of rising of the signs for the local place. From these *asus* one should obtain[1] the Moon's shadow etc. as in the case of the Sun.[2]

In his commentary on *SiŚi*, I, vii. 13, Bhāskara II says that although the shadow due to a planet or a star is not perceptible still it should be calculated because it is useful in observing the planet or star through the cavity of the *Nalaka* (the observer's Tube).

MOON'S HOUR ANGLE

15. When the Moon's longitude for the given time is greater than the longitude of the meridian ecliptic point, computed for that time, then

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1. *Vide supra* chap. III, sec. 10.
2. *Cf. BrSpSi*, viii. 4; also viii. 1; *ŚiDVr*, viii. 12 (c-d); *SiŚi*, I, vii. 11-12.

the Moon is in the eastern half of the celestial sphere; when the Moon's longitude is less than the longitude of the meridian ecliptic point for that time, then the Moon is in the western half of the celestial sphere. The *nāḍīs* (of right ascension) that lie between the two (i. e., the Moon and the meridian ecliptic point) are the *nāḍīs* to elapse before or elapsed since the Moon's meridian transit. These are known as the *nāḍīs* of the Moon's hour angle, as before.

ALTITUDE OF MOON'S UPPER OR LOWER LIMB

16. In the light half of the month, the altitude of the Moon should be increased or diminished by the minutes of the Moon's true semi-diameter according as the Moon is in the eastern or western half of the celestial sphere ; in the dark half of the month, the altitude of the Moon should be diminished or increased by the minutes of the Moon's true semi-diameter, according as the Moon is in the eastern or western half of the celestial sphere.

In the light half of the month, the western part of the Moon receives light from the Sun and the eastern part is dark. So when the Moon is in the eastern half of the celestial sphere, its upper limb is visible, and when the Moon is in the western half of the celestial sphere its lower limb is visible. In the dark half of the month, reverse is the case. Hence the above rule.

Brahmagupta and Śrīpati have obtained the altitude of the Moon's upper limb above the visible horizon, whereas Bhāskara II the altitude of the Moon's centre above the visible horizon.[1]

Vaṭeśvara, too, in the case of heavenly bodies in general, obtained the altitude above the visible horizon,[2] but here he does not say so explicitly. Perhaps it is presumed. Lalla, on the other hand, obtains the altitude of the Moon corrected for parallax in longitude.[3]

TIME OF MOON'S MERIDIAN PASSAGE

17. Thus when the *asus* of the *unnatakāla* for the given time (i. e., the *asus* elapsed since moonrise in the eastern half of the celestial sphere

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1. See *BrSpSi*, viii. 6; *SiŚe*, x. 32; *SiŚi*, 1, vii. 14-15 com.
2. See *supra*, ch. III, sec. 11, vs. 7.
3. See *ŚiDVr*, ix. 4.

or to elapse before moonset in the western half of the celestial sphere), calculated methodically, are equal to those of half the Moon's day, the Moon is then on the meridian.[1]

(3) Elevation of Lunar Horns (*Śṛṅgonnati*)

THE *ŚṚṄGONNATI* TRIANGLE

Method 1

18-19(a-b). Find the Moon's *bāhu* for the desired time in the manner stated before; find also the Sun's *bāhu* (for the same time). Their sum or difference, according as they are of unlike or like directions, is the true *bāhu* (or true *bhuja*). Its direction is the same as that of the Moon's *bāhu*, except when the difference is obtained reversely by subtracting the Moon's *bāhu* from the Sun's *bāhu*; in that case its direction is contrary to that of the Moon's *bāhu*.[2] **(The true *bhuja* is the base of the *Śṛṅgonnati* triangle.)**

19(c-d)-20. Now, in the case of the Moon as well as in the case of the Sun, find the square-root of the difference of the square of the Rsine of the (own) zenith distance and the square of the own *bāhu*. (Then are obtained the Moon's *koṭi* and the Sun's *koṭi*.) Their sum or difference, according as the Moon and the Sun are in the different or same halves of the celestial sphere (eastern or western), is the "first" result. (This denotes the east-west distance between the Sun and the Moon.)

The difference or sum of the Rsines of the altitudes of the Moon and the Sun, according as the Moon and the Sun are both above the horizon or one above and the other below (lit. according as it is day or night) is the "second" result. (This denotes the vertical distance between the Sun and the Moon.)

21. The square-root of the sum of the squares of the "second" and the "first" results is the *agrā* (or the upright of the *Śṛṅgonnati* triangle), which lies between the Moon and the end of the true *bhuja*. The square-root of the sum of the squares of the *agrā* and the true *bhuja* is the hypotenuse (of the *Śṛṅgonnati* triangle) which lies between the Sun and the Moon.[3]

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1. *Cf. ŚiDVṛ*, ix. 5 (a-b); *SiŚe*, x. 33 (a-b).
2. *Cf. ŚiDVṛ*, ix. 10-11.
3. The rule given in vss. 18-21 above is the same as given in *BrSpSi*, vii. 6-9; and *SiŚe*, x. 10-13.

That is, the base, the upright and the hypotenuse of the *Śṛṅgonnati* triangle are :

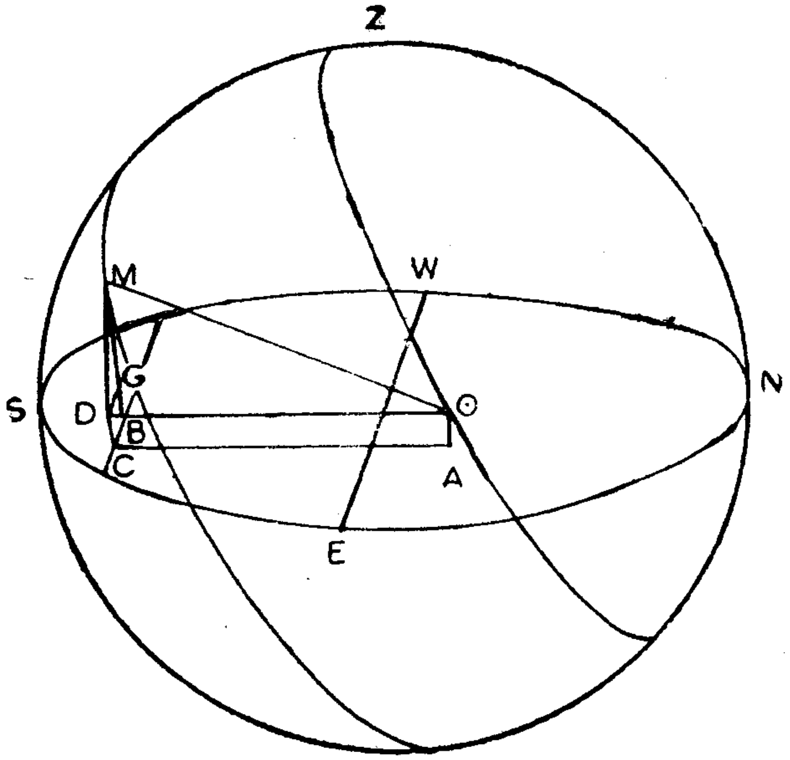
$$\text{base} = b + \text{or} \sim b'$$

$$\text{upright} = \sqrt{(k + \text{or} \sim k')^2 + (\text{Rsin } a + \text{or} \sim \text{Rsin } a')^2}$$

$$\text{hypotenuse} = \sqrt{(\text{base})^2 + (\text{upright})^2},$$

where b, b' ; k, k' ; a, a' are the *bhujas, koṭis* and altitudes of the Sun and Moon respectively, + or $\sim$ sign being taken according as b and b', k and k', a and a' are of unlike or like directions.

The figure below represents the celestial sphere for an observer in latitude ϕ. SENW is the horizon, EW the east-west and NS the north-south lines. ⊙ is the Sun and M the Moon, ⊙A and MB are perpendiculars dropped from ⊙ and M on the plane of the horizon. ⊙D, which is equal to AC, the north-south distance between the Sun and the Moon, is the base of the *Śṛṅgonnati* triangle. DG, which is equal to CB, the east-west distance



between the Sun and the Moon, is the "first" result; and MG, which is the vertical distance between the Sun and the Moon, is the "second" result. MD, which is equal to the square-root of the sum of the squares of the "first" and "second" results, is the upright of the *Śṛṅgonnati* triangle. M⊙, which is the distance between the Sun and the Moon, is the hypotenuse of the *Śṛṅgonnati* triangle. MD⊙ is the *Śṛṅgonnati* triangle.

After stating the above rule (vss 18-21 above) in his *Brāhma-sphuṭa-siddhānta* (vii. 6-9), Brahmagupta adds :

"The base, the upright and the hypotenuse, obtained in this way, are valid when the Moon measured from the Sun is in the first or last quadrant; when the Moon measured from the Sun is in the second or third quadrant, the longitude of the Sun should be increased by half a circle (i.e., 6 signs)."[1]

Śrīpati too has made a similar remark.[2] Bhāskara II, however, does not agree with this. He writes :

"Brahmagupta and others have obtained the elevation of the (Moon's) dark horn, but I do not agree with this. (For) people do not see the elevation of the dark horn clearly."[3]

Bhāskara II[4] has criticised the upright and hypotenuse as conceived by Brahmagupta, Vaṭeśvara and Śrīpati. Those prescribed by Bhāskara II are:[5]

$$\text{upright} = \text{Rsin } a' + \text{ or } \sim \text{Rsin } a$$

$$\text{hypotenuse} = \sqrt{(\text{base})^2 + (\text{upright})^2},$$

$+$ or $\sim$ sign being taken according as a' and a are of unlike or like directions.

Method 2

22. Or, the Rsine (*dorguṇa* or *bhujajyā*) of one-half of Moon's longitude minus Sun's longitude, multiplied by 2, is the hypotenuse, denoting the distance between them (i. e., between the Sun and the Moon). The square-root of the difference between the squares of that (hypotenuse) and the base is the upright, denoting the distance between the Moon and the extremity of the base.[6]

That is, if M denotes the Moon's longitude and $\odot$ the Sun's longitude, then

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1. *BrSpSi*, vii. 10.
2. See *SiŚe*, x. 13(c-d)-14.
3. *SiŚi*, I, ix. 1 (com.).
4. *SiŚi*, I, ix. 10-12.
5. See *SiŚi*, I, ix. 3-5.
6. *Cf. SiŚe*, x. 15.

$$\text{hypotenuse} = 2\ \text{Rsin}\ \frac{M - \odot}{2}.$$

$$\text{upright} = \sqrt{(\text{hypotenuse})^2 - (\text{base})^2}.$$

This rule has been criticised by Bhāskara II, who says :

"The *Śṛṅgonnati* triangle contemplated by Brahmagupta (Vaṭeśvara and Śrīpati) by taking the hypotenuse as equal to twice the Rsine of half the difference between the longitudes of the Sun and the Moon, and the upright as equal to the square-root of the difference between the squares of that (hypotenuse) and the base, is slant. It is not (vertical) like the image in a mirror. So this *Śṛṅgonnati* is not proper : this is my view."[1]

SITA OR MEASURE OF MOON'S ILLUMINATED PART

23. The Rversed-sine of Moon's longitude minus Sun's longitude (when $M - \odot \leqslant 90°$), or the Rsine of the excess of Moon's longitude minus Sun's longitude over 90°, when $M - \odot > 90°$, as increased by the radius, multiplied by the Moon's semi-diameter and divided by the radius is stated to be the measure of the illuminated part of the Moon during the day.

24. The Moon's semi-diameter, multiplied by the degrees of one-half of Moon's longitude minus Sun's longitude (when $\frac{M - \odot}{2} \leqslant 90°$) or by 180° minus those degrees, when $\frac{M - \odot}{2} > 90°$, and divided by 45, gives the measure of the illuminated part of the Moon during the night.

During twilight (*sandhyā*) the measure of the illuminated part of the Moon is equal to half the sum of the measures of the illuminated parts for the day and night.[2]

That is :

$$\textit{Sita}\ \text{during the day} = \frac{\text{Rvers}\ (M - \odot) \times \text{Moon's semi-diameter}}{\text{R}},$$
when $M - \odot \leqslant 90°$;

1. *SiŚi*, I, ix. 4(c-d), com.
2. The same rule occurs also in *BrSpSi*, vii. 11-13; *ŚiDVr*, ix. 13-14; *SiŚe*, x. 15-19 (a-b).

$$= \frac{[R + R\sin\{(M - \odot) - 90^\circ\}] \times \text{Moon's semi-diameter}}{R},$$

when $M - \odot > 90^\circ$;

$$\textit{Sita} \text{ during the night} = \frac{\left(\frac{M - \odot}{2}\right)^\circ \times \text{Moon's semi-diameter}}{45},$$

when $\frac{M - \odot}{2} < 90^\circ$;

$$= \frac{\left[180^\circ - \left(\frac{M - \odot}{2}\right)^\circ\right] \times \text{Moon's semi-diameter}}{45},$$

when $\frac{M - \odot}{2} > 90^\circ$;

$$\textit{Sita} \text{ during twilight} = \frac{\textit{sita} \text{ for day} + \textit{sita} \text{ for night}}{2}.$$

The *Khaṇḍakhādyaka* gives the formula :[1]

$$\textit{śukla} \text{ (or } \textit{sita}) = \frac{(M \sim \odot)^\circ}{15} \textit{ aṅgulas}$$

which follows from the formula :

$$\textit{śukla} = \frac{(M \sim \odot)^\circ \times \text{Moon's semi-diameter}}{45 \times 2}$$

by taking Moon's semi-diameter = 6 *aṅgulas*.[2]

Āryabhaṭa II gives the formula :[3]

$$\textit{śukla} = \frac{(M - \odot)^\circ \times \text{Moon's semi-diameter}}{90}.$$

PARILEKHASŪTRA OR RADIUS OF INNER BOUNDARY OF ILLUMINATION

Method 1

25. The difference between the measure of the Moon's illuminated part (*sita*) and the Moon's semi-diameter is the "divisor". By that (divi-

1. See *KK*, I, vii. 4(a-b). This formula occurs in *SūSi*, x. 9(a-b) also.
2. In *KK*[I, vii. 4 (c-d)], the Moon's diameter has been taken equal to 12 *aṅgulas*.
3. See *MSi*, vii. 7. The author of the *SūSi* gives this formula also. See *SūSi*, x. 9(c-d).

sor) divide the square of the Moon's semi-diameter; to the resulting quotient add the "divisor"; and reduce that (sum) to half. The result thus obtained is the *Parilekhasūtra*.[1]

26. Find the square of the difference between the measure of the Moon's illuminated part (*sita*) and half the measure (diameter) of the Moon, as also the square of half the measure of the Moon. Half the sum of these two, when divided by the difference between the measure of the Moon's illuminated part (*sita*) and half the measure of the Moon, gives the *Parilekhasūtra* relating to the Moon's horns.[2]

The *Parilekhasūtra* is the radius of the circle forming the inner boundary of the Moon's illuminated part.

In the figure below, MD⊙ is the *Śṛṅgonnati* triangle, D⊙ being the base, MD the upright and M⊙ the hypotenuse. ⊙ is the Sun and M the centre of the Moon. The circle centred at M is the Moon's disc, MS the semi-diameter of the Moon and KW the measure of the Moon's illuminated part (shaded in black). The circle SKN is the inner boundary of the illuminated part, its centre is O, and radius called *Parilekha-sūtra*

$$= OS = OK = ON.$$

Let OS $= x$ and MK = Moon's semi-diameter − measure of illuminated part $= y$, say.

Also let d denote the Moon's diameter. Then

$$LM \times MK = SM \times MN$$

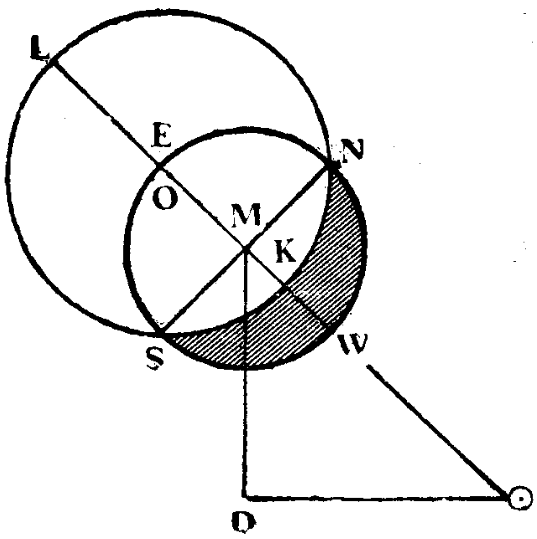
or $(2x - y).y = (d/2)^2$.

$\therefore\ 2xy = y^2 + (d/2)^2$, giving

$$x = (y/2) + (d/2)^2/(2y)$$

$$= \tfrac{1}{2}\left[y + \frac{(d/2)^2}{y} \right] \quad (1)$$

$$= \frac{\tfrac{1}{2}\,[y^2 + (d/2)^2]}{y}, \quad (2)$$



where $y = d/2 - sita$.

1. Similar rules are found to occur in *BrSpSi*, vii. 14; *SiŚe*, x. 20, 21; *SiŚi*, I, ix. 7.
2. *Cf. SiŚe*, x. 22.

Āryabhaṭa II[1] calls OM and ON by the names *koṭi* and *karṇa* respectively and gives the following formulae:

$$koṭi = \tfrac{1}{2}\left[\frac{(d/2)^2}{d/2 - s} - (d/2 - s)\right]$$

$$\text{and } karṇa = \tfrac{1}{2}\left[\frac{(d/2)^2}{d/2 - s} + (d/2 - s)\right],$$

where s stands for *sita.*

Method 2

27-28(a-b). Alternatively, multiply the Moon's semi-diameter by the difference between the measure of the Moon's illuminated portion and half the measure (diameter) of the Moon. Add two times that to the square of the difference between the "divisor" and the Moon's semi-diameter and divide by two times the difference (between the measure of the Moon's illuminated part and half the measure of the Moon). Whatever is thus obtained here is again the *Parilekhasūtra*.

$$Parilekhasūtra = \frac{2 \times \frac{d}{2}\left(\frac{d}{2} - s\right) + \left[\left(\frac{d}{2} - s\right) - \frac{d}{2}\right]^2}{2\left(\frac{d}{2} - s\right)}, \qquad (3)$$

where d is the Moon's diameter and s the measure of the Moon's illuminated part.

Method 3

28(c-d). Or, divide and increase the square of one-fourth of the Moon's diameter by (one-half of) the difference between the Moon's illuminated part and the Moon's semi-diameter. (This again gives the *Parilekha-sūtra*).

In other words:

$$Parilekhasūtra = \tfrac{1}{2}\left(\frac{d}{2} - s\right) + \frac{(d/4)^2}{\frac{1}{2}\left(\frac{d}{2} - s\right)}, \qquad (4)$$

which is exactly the form in which it has been stated by Brahmagupta.[2]

1. *MSi*, vii. 8.
2. See *BrSpSi*, vii. 14; read शशिमानपादवर्गो in place of शशिमानवर्गपादो. The same form occurs in *SiŚe*, x. 20. For other forms, see *SiŚe*, x. 21; *SiŚi*, I, ix. 7.

One can easily see that formulae (1), (2), (3) and (4) are equivalent.

REDUCTION TO *AṄGULAS*

29. The measures of the base, upright and hypotenuse (of the *Śṛṅgonnati* triangle), (in terms of minutes), divided by an optional number (assumed as the number of minutes in an *aṅgula*), give the corresponding measures in true *aṅgulas*. By the (same) number of minutes in an *aṅgula* should also be divided the minutes of the Moon's diameter, the illuminated part of the Moon and the *Parilekhasūtra*.[1]

Lalla assumes 200 for the optional number on one occasion and 9 on the other.[2]

(4) Diagram of Lunar Horns

Method 1

WHEN THE SUN IN NOT ON THE HORIZON

30. Set down a point and assuming it as the Sun, lay off from it the base in its own direction (north or south) and from the (other) extremity thereof lay off the upright towards the east if it is the light half of the month or towards the west if it is the dark half of the month.

31. The line which joins the extremity thereof with the point (assumed as the Sun) is the hypotenuse. Taking the junction of the hypotenuse and the upright as the centre, draw the Moon. Then lay off, along the hypotenuse, the measure of the illuminated part of the Moon, from the west point of the Moon's disc (towards the centre of the Moon) if it is the light half of the month or from the east point of the Moon's disc (towards the centre of the Moon) if it is the dark half of the month.

32. To exhibit the elevation of the bright lunar horns draw (the inner circular boundary of illumination) with radius equal to the *Parilekhasūtra*. In the light half of the month (the maximum limit of) the illuminated part (which yields bright horns) is equal to half the measure of the Moon's disc or to the Rsine (of half of the Moon's diameter).

1. *Cf. SiŚe*, x. 23.
2. See *ŚiDVṛ*, ix. 12 (c-d); xiii. 16. Also see *SiŚe*, x. 23.
3. *Cf. BrSpSi*, xvii. 2-6; *ŚiDVṛ*, ix. 15-18; *SiŚe*, x. 24-25.

What is meant by the statement in the latter half of vs. 32 is that when, in the light half of the month, the illuminated part of the Moon exceeds the semi-diameter of the Moon's disc, one should lay off the unilluminated part of the Moon (reversely) and exhibit the dark lunar horns. Similarly, in the dark half of the month, when the unilluminated part exceeds the semi-diameter of the Moon's disc, one should lay off the illuminated part of the Moon and exhibit the bright lunar horns.

So also writes Mallikārjuna Sūri :[1]

"By the arc of the *Śṛṅgonnativṛtta* (which is drawn with radius equal to the *Parilekhasūtra*), the Moon's disc is divided into two parts. Thus, at the time of *Śṛṅgonnati*, the dark and bright parts are distinctly seen. In the light half of the month, the bright part lies towards the west and the dark part towards the east. Of the two, the elevation of the bright horns is to be known before the eighth lunar date (*aṣṭamī*) only. After the eighth lunar date, one should find the elevation of the dark horns. Similarly, in the dark half of the month, the dark part lies towards the west and the bright part towards the east. Of the two, the elevation of the dark horns should be seen before the eighth lunar date only. After the eighth lunar date, one should see the elevation of the bright horns."[2]

Method 2

WHEN THE SUN IS ON THE HORIZON

33-35(a-b). Or, the sum or difference of the Rsine of the own amplitude of the rising or setting point of the ecliptic and the Moon's *bhuja*, as in the case of conjunction of two planets (in finding the true declination) (i. e., according as the two are of unlike or like directions), is the base with its proper direction. The Rsine of the Moon's altitude for that time is the upright. The square-root of the sum of the squares of that (upright) and the base is the hypotenuse. These (base, upright and hypotenuse) should be abraded by an optional number.

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1. *ŚiDVṛ*, ix. 17-18 com.
2. तद्वृत्तरेखया चन्द्रमण्डलं खण्डितं स्यात् । तत्र शृङ्गोन्नतिकाले कृष्णखण्डं च शुक्लखण्डं च सुव्यक्तं स्यात् । शुक्लपक्षे प्रतीचीनं शुक्लखण्डं प्राचीनं कृष्णखण्डं स्यात् । तयोरष्टम्याः प्रागेव शुक्लशृङ्गोन्नतिः, अष्टम्याः परतः कृष्णशृङ्गोन्नतिरेव विज्ञेया । कृष्णपक्षेऽपीत्थमेव परिलिख्य प्रतीचीनं कृष्णखण्डं प्राचीनं शुक्लखण्डं स्यात् । तयोरष्टम्याः प्रागेव कृष्णशृङ्गोन्नतिः, अष्टम्याः परतः शुक्लशृङ्गोन्नतिः द्रष्टव्या । (Com. on *ŚiDVṛ* ix. 17-18). Also see Mallikārjuna Suri's com. on *ŚiDVṛ*, xiii. 16.

The degrees obtained by subtracting the Sun's longitude from the Moon's longitude give the measure of the Moon's illuminated part in the light half of the month, and those degrees diminished by 180 degrees give the measure of the Moon's unilluminated part in the dark half of the month. These (measures of the Moon's illuminated and unilluminated parts) multiplied by the *aṅgulas* of the Moon's diameter and divided by 180 give the corresponding *aṅgulas*.[1]

35(c-d)-37. Take a point (for the Sun) and from it lay off the base in its own direction. From the extremity of that (base) lay off the upright towards the east if it is the light half of the month or towards the west if it is the dark half of the month. Then draw the hypotenuse as before. At the junction of the two (i. e., the hypotenuse and the upright), draw the disc of the Moon. The hypotenuse is the east-west for the Moon's disc. With the help of a fish-figure drawn on the east-west line, determine the south and north points of the Moon's disc. On the hypotenuse-line, lay off from the west point towards the east, the measure of the Moon's illuminated part in the light half of the month or the measure of the Moon's dark part in the dark half of the month. The construction of the inner circle of illumination, which passes through three points, viz. the point just obtained and the south and north points of the Moon's disc, should then be made with the help of two fish-figures.

Method 3

WHEN THE SUN IS ON THE HORIZON

[Assuming Rsin (Moon's altitude) = 12 *aṅgulas*]

38. Or, in the eastern half of the celestial sphere, take the difference or sum of the declinations of the rising point of the ecliptic and the Moon, and in the western half of the celestial sphere, take the difference or sum of the declinations of the setting point of the ecliptic and the Moon, according as the two declinations are of like or unlike directions. The direction of the resulting quantity should be determined as before.

39. Multiply that by the hypotenuse of the Moon's shadow and also by the hypotenuse of the equinoctial midday shadow, and divide by

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1. *Cf. ŚiDVr*, ix. 13.

the radius multiplied by 12; or, alternatively, multiply that by the hypotenuse of the Moon's shadow and divide by the Rsine of the local colatitude.

40. Find the sum or difference of that result and the equinoctial midday shadow according as the Moon is towards the south or north of the horizon ecliptic point : then is obtained the base (of the *Śṛṅgonnati* triangle). The upright is equal to 12 (*aṅgulas*). The hypotennse is equal to the square-root of the sum of the squares of the base and the upright.[1]

41. The Moon's longitude minus the Sun's longitude (in terms of signs) multiplied by two gives the measure of the Moon's illuminated part (in *aṅgulas*) when the Moon (as measured from the Sun) is in the six signs commencing with Aries. When the Moon (as measured from the Sun) is in the six signs beginning with Libra, that subtracted from 24 gives the measure of the illuminated part (in *aṅgulas*) of the Moon which is assumed to be of 12 *aṅgulas* in diameter.[2]

42. From this (data) the diagram (of the lunar horns) should be made on the ground, cloth, or wooden board, in the manner stated.

The Moon's diameter minus the unilluminated part is the illuminated part, and the same (Moon's diameter) minus the illuminated part is the unilluminated part.

Let δ and δ' be the declinations of the Sun and the Moon respectively, a the altitude of the Moon, and ϕ the latitude of the place. Then, assuming the Sun to be on the horizon and using the symbol $\pm$ in the sense of plus or difference (as the case may be),

$$\text{Moon's } \textit{bhuja} = \text{Moon's } \textit{śaṅkutala} \pm \text{Moon's } \textit{agrā}$$

$$= \frac{\text{Rsin } \phi \times \text{Rsin } a}{\text{Rcos } \phi} \pm \frac{\text{R} \times \text{Rsin } \delta'}{\text{Rcos } \phi}$$

$$\text{Sun's } \textit{bhuja} = \text{Sun's } \textit{agrā} = \frac{\text{R} \times \text{Rsin } \delta}{\text{Rcos } \phi}.$$

$$\therefore \textit{spaṣṭa bhuja} = \text{Moon's } \textit{bhuja} \pm \text{Sun's } \textit{bhuja}$$

$$= \left[\frac{\text{Rsin } \phi \times \text{Rsin } a}{\text{Rcos } \phi} \pm \frac{\text{R} \times \text{Rsin } \delta'}{\text{Rcos } \phi}\right] \pm \frac{\text{R} \times \text{Rsin } \delta}{\text{Rcos } \phi}$$

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1. A similar rule is found to occur in *SūSi*, x. 6-8.
2. In *KK*, I, vii. 4(c-d). Brahmagupta has also taken the Moon's disc to be of 12 *aṅgulas* in diameter.

$$= \frac{\text{Rsin } \phi \times \text{Rsin } a}{\text{Rcos } \phi} \pm \frac{\text{Rsin } \delta' \pm \text{Rsin } \delta}{\text{Rcos } \phi} \times \text{R}.$$

Let Rsin a = 12 *aṅgulas*. Then reducing the *spaṣṭa bhuja* to this unit, we have

$$\textit{spaṣṭa bhuja} = \frac{\text{Rsin } \phi \times 12}{\text{Rcos } \phi} \pm \frac{(\text{Rsin } \delta' \pm \text{Rsin } \delta) \times \text{R} \times 12}{\text{Rcos } \phi \times \text{Rsin } a} \textit{ aṅgulas}.$$

But

(1) $\frac{\text{Rsin } \phi \times 12}{\text{Rcos } \phi}$ = equinoctial midday shadow or *palabhā*

(2) $\frac{\text{R}}{\text{Rsin } a} = \frac{\text{hypotenuse of Moon's shadow}}{12}$

(3) $\frac{12}{\text{Rcos } \phi} = \frac{\text{hyp. of equi. midday shadow or } \textit{palakarṇa}}{\text{R}}$.

Therefore,

spaṣṭa bhuja or base

$$= \textit{palabhā} \pm \frac{(\text{Rsin } \delta' \pm \text{Rsin } \delta) \times (\text{hyp. of Moon's shadow}) \times \textit{palakarṇa}}{\text{R} \times 12} \quad (1)$$

$$= \textit{palabhā} \pm \frac{(\text{Rsin } \delta' \pm \text{Rsin } \delta) \times (\text{hyp. of Moon's shadow})}{\text{Rcos } \phi}. \quad (2)$$

Writing Rsin $\delta' = \delta'$ and Rsin $\delta = \delta$, formulae (1) and (2) may be grossly stated as :

spaṣṭa bhuja or base

$$= \textit{palabhā} \pm \frac{(\delta' \pm \delta) \times (\text{hyp. of Moon's shadow}) \times (\textit{palakarṇa})}{\text{R} \times 12}$$

$$= \textit{palabhā} \pm \frac{(\delta' \pm \delta) \times (\text{hyp. of Moon's shadow})}{\text{Rcos } \phi}.$$

The form stated in the *Sūrya-siddhānta* is :[1]

$$\textit{spaṣṭa bhuja} = \frac{12 \times \text{Rsin } \phi \pm \text{Rsin } (\delta' \pm \delta) \times (\text{hyp. of Moon's shadow})}{\text{Rcos } \phi}.$$

1. See *SūSi*, x. 6-8(a-b).

The formula for the Moon's illuminated part, stated in vs. 41(a-b), is based on the proportion : When (Moon's longitude—Sun's longitude) equals 6 signs, the illuminated part of the Moon (which is assumed to be 12 *aṅgulas* in diameter) is equal to 12 *aṅgulas*, what then would be the measure of the Moon's illuminated part when (Moon's longitude—Sun's longitude) has the given value, say x signs (x being less than 6 in the light half of the month) ? The result is

$$\frac{12 \times x}{6} \text{ or } 2x \textit{ aṅgulas.}$$

In the case of the dark half of the month, let (Moon's longitude — Sun's longitude) be equal to $6 + y$ signs. The above rule gives

$$2\,(6 + y) \textit{ aṅgulas.}$$

But, in this case

$$\text{Sun's longitude} - \text{Moon's longitude} = 6 - y \text{ signs,}$$

so that

$$\text{Moon's illuminated part} = 2\,(6 - y) \textit{ aṅgulas.}$$

This can be written as

$$\text{Moon's illuminated part} = 24 - 2\,(6 + y) \textit{ aṅgulas.}$$

Hence the rule for the dark half of the month.

Method 4

43. Or, a circle having been drawn equal to the Moon's size and the cardinal points having been determined, lay off from the centre the upright towards the east when the Moon is in the eastern half of the celestial sphere (i. e., if it is the dark half of the month) and towards the west when the Moon is in the western half of the celestial sphere (i. e., if it is the light half of the month). From the extremity of that (upright) lay off the base in the direction contrary to its own.

44. Then joining the end of that (base) with the centre of the Moon, draw the hypotenuse : this gives the east and west directions for the Moon. From the fish-figure thereof determine the remaining directions (north and south). The other constructions pertaining to the illuminated or unilluminated part should be made in the manner stated. [1]

1. *Cf. SiŚi*, I, ix. 8-9.

45. Or, (in the Moon supposed to be drawn) at the centre of the hypotenuse-circle (i. e., the circle drawn by taking the hypotenuse for the radius), one might lay off (from the centre) the upright, in the manner stated above, from the extremity thereof the base, and then the hypotenuse, as before, and then other things in the prescribed way.

Method 5

46-48. Or, a circle having been drawn equal to the Moon's size and the cardinal points having been determined, lay off the base, obtained after multiplying it by the Moon's semi-diameter and dividing by its own hypotenuse, towards the west, in the direction contrary to its own (north or south), if the Moon is in the western half of the celestial sphere (i. e., if it is the light half of the month), or in its own direction, if the Moon is in the eastern half of the celestial sphere. It should be laid off from the west point like the *valana*. On the thread forming the hypotenuse-line,[1] one should lay off the measure of the Moon's illuminated part from the western end (towards the centre of the Moon) if it is the light half of the month, or the measure of the Moon's unilluminated part if it is the dark half of the month. Then taking the point where the threads passing through the fish-figures drawn with that point and the north and south points (on the Moon) meet as centre (and the *Parilekhasūtra* as radius) one should draw the inner circle of illumination to determine the elevation of the bright horns of the Moon.[2]

THE ELEVATED HORN

49(a-b). The (Moon's) horn which is in the direction of the base is low (or depressed), whereas that which is in the direction of the Sun is high (or elevated).[3]

THE HALF-ILLUMINATED MOON

49(c-d). When the measure of the Moon's illuminated part happens to be equal to the Moon's semi-diameter, the Moon looks like the forehead of a lady belonging to the Lāṭa-deśa (Southern Gujarat).[4]

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1. The hypotenuse-line is the line joining the end of the base to the Moon's centre, the upright-line (*koṭisūtra*) being east to west.

2. *Cf. BrSpSi*, xvii. 7.

3. *Cf. ŚiDVr*, ix. 19(a-b).

4. *Cf. ŚiDVr*, ix. 19(c-d); *SiŚe*, x. 26.

RISING AND SETTING OF THE ELEVATED HORN

50. The higher horn (of the Moon) rises earlier and sets later, bearing the beauty (seen) at the tip of the Ketaka flower on account of its association with the black bees.[1]

THE CRESCENT MOON

51. The first digit of the Moon appears to the eye like the creeper of Cupid's bow, and gives the false impression of the beauty of the eyebrows of a fair-coloured lady with excellent eyebrows.

Section 2

Examples on Chapter VII

1-2(a-b). One who finds the measure of the illuminated part of the Moon, the time of Moon's rising, and the time of Moon's setting, for every day (of the month); who knows the many ways of exhibiting the Moon by means of a diagram, and depicts the position of the Moon's horns for any time on cloth, wooden board, or wall etc., is versed in (the theory of) moorise.

2(c-d)-3. One who makes others see the Moon or a planet at its first visibility, or the conjunction of two planets, or the eclipsed Sun or Moon, from the upper end of a bamboo, in mirror, oil, or water (below) is indeed (as great as) Brahmā. I bow down to him.

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1. *Cf. ŚiDVṛ*, ix. 20; *SiŚe*, x. 27.

Chapter VIII

CONJUNCTION OF HEAVENLY BODIES

Section 1 : Conjunction of Two Planets

1. *KADAMBAPROTĪYA-YUTI*

When two planets lie on the same secondary to the ecliptic, they have the same longitude. They are then said to be in conjunction in longitude or in conjunction along the same secondary to the ecliptic. This type of conjunction is known as *Kadambaprotīya-yuti*. Āryabhaṭa I and other astronomers who flourished before Brahmagupta studied, as mentioned by Brahmagupta and Śrīpati,[1] this kind of conjunction. In what follows Vaṭeśvara explains how to know the time when two planets are in conjunction in longitude and how to obtain the distance between them at that time.

When at the time of conjunction in longitude the lower planet partly or wholly covers the upper one, the conjunction is called *Bheda*. Vaṭeśvara deals with this type of conjunction in longitude also, though briefly.

EQUALISATION OF CELESTIAL LONGITUDES

Method 1

1. Divide the difference between the longitudes of the two given planets (both moving directly) by the difference between their daily motions : (then are obtained the days elapsed since or to elapse before their conjunction, according as the slower or faster planet is behind).

When the two planets are both retrograding, the result is *vice versa*. (That is, if the difference between the longitudes of two retrograding planets is divided by the difference of their daily motions, the result is the days elapsed since or to elapse before their conjunction according as the faster or slower planet is behind.)

When of the two planets, one with greater longitude is in retrograde motion, the conjunction is to occur; when the one with lesser longitude is in retrograde motion, the conjunction has already taken place.[2]

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1. See *BrSpSi*, ix. 11(c-d); *SiŚe*, xi. 18 (c-d).
2. *Cf. BrSpSi*, ix. 56(a-b); *KK*, I, viii. 3; *ŚiDVṛ*, x. 7-8; *MSi*, xi. 3 (c-d)-4; *SiŚe*, xi. 11-12; *SiŚi*, I, x. 3-4(a-b); *SuSi*, I, x. 1-2.

2. Multiply the minutes of the planets' own daily motions by the number of days (thus obtained) and add the resulting minutes to or subtract them from the planets' own longitudes according as the conjunction is to occur or has already occurred. In the case of planets with retrograde motion, addition and subtraction should be made contrarily. This being done, the longitudes of the two planets become the same from signs to seconds.[1]

Method 2

3. Or, severally multiply the difference between the longitudes of the two planets by the planets' own daily motions and divide (each product) by the difference between the daily motions (of the two planets, if they are both in direct motion or both in retrograde motion) or by the sum of the daily motions (of the two planets) if one is direct and the other retrograde. The resulting minutes being applied to the longitudes of the two planets as before, the two planets become equal.[2]

In case the longitudes of the two planets do not become equal, one should apply the process of iteration.

DIAMETERS OF PLANETS

4. Divide the radius multiplied by 33 (severally) by the Moon's true distance (in minutes) multiplied by 5, 10, 15, 20 and 25 respectively. Then are obtained the measures (of the diameters) of Venus, Jupiter, Mercury, Saturn and Mars respectively, in terms of minutes.[3]

$$\text{Diameter of Venus} = \frac{33\ R}{5 \times \text{Moon's distance in minutes}}\ \text{mins.}$$

$$\text{Diameter of Jupiter} = \frac{33\ R}{10 \times \text{Moon's distance in minutes}}\ \text{mins.}$$

$$\text{Diameter of Mercury} = \frac{33\ R}{15 \times \text{Moon's distance in minutes}}\ \text{mins.}$$

$$\text{Diameter of Saturn} = \frac{33\ R}{20 \times \text{Moon's distance in minutes}}\ \text{mins,}$$

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1. *Cf. SiŚe*, xi. 12(c-d); *SuSi*, I, x. 3 (a-b). Also see *MSi*, xi. 5-6 (a-b).
2. *Cf. BrSpSi*, ix. 6(c-d)-7; *KK*, I, viii. 4; *ŚiDVr*, x, 8(b-d)-9(a-b); *SiŚe*, xi. 13-14.
3. *Cf. ŚiDVr*, x. 2, 4.

$$\text{Diameter of Mars} = \frac{33\text{ R}}{25 \times \text{Moon's distance in minutes}} \text{ mins.}$$

Vaṭeśvara has evidently assumed the linear diameters of the planets at the Moon's distance as follows :[1]

Venus, 330/5 or 66 *yojanas*; Jupiter, 330/10 or 33 *yojanas*; Mercury, 330/15 or 22 *yojanas*; Saturn, 330/20 or 16·5 *yojanas*; and Mars, 330/25 or 13·2 *yojanas*.

The following table gives the mean angular diameters of the planets as given by the various Hindu astronomers and by Tycho Brahe (1546—1631) along with their modern values :

Table 29. Mean angular diameters of the planets

(1)

Planet	Āryabhaṭa I[2] and Lalla[3]	Vaṭeśvara	*SūSi*[4] and Bhaṭṭopala[5]	Tycho Brahe	Modern (mean)
Mars	1′15″·6	1′19″·2	2′	1′40″	14″·3
Mercury	2′6″	2′12″	3′	2′.0″	9″
Jupiter	3′9″	3′18″	3′30″	2′45″	41″
Venus	6′18″	6′36″	4′	3′15″	39″
Saturn	1′34″·5	1′39″	2′30″	1′50″	17″

(2)

Planet	Old *SūSi* (SMT)	Brahmagupta[6] and Śripati[7]	Āryabhaṭa II[8]	Bhāskara II[9]
Mars	4′	4′46″	4′45″	4′45″
Mercury	7′	6′14″	6′15″	6′15″
Jupiter	8′	7′22″	7′15″	7′20″
Venus	9′	9′	9′	9′
Saturn	5′	5′24″	5′15″	5′20″

The values given in (1) are better than those given in (2).

1. *Cf. Ā*, i. 7 (c-d); *MBh*, vi. 56; *ŚiDVr*, x. 2.
2. *Ā*, i. 7.
3. *ŚiDVr*, x. 2-4.
4. vii. 13.
5. *KK*, I, viii. 6, com.
6. *BrSpSi*, ix. 2.
7. *SiŚe*, xi. 9-10.
8. *MSi*, xi. 1.
9. *SiŚi*, I, x. 1.

It is interesting to note that Brahmagupta has used the following empirical formula to calculate the mean angular diameters of the planets :[1]

Planet's mean angular diameter in minutes

$$= \frac{81}{\text{time-degrees of planet's heliacal visibility}}.$$

Āryabhaṭa II, Śrīpati and Bhāskara II seem to have followed Brahmagupta.

DISTANCE BETWEEN TWO PLANETS IN CONJUNCTION

5. Increase the true-mean longitude of the planet by that of its own ascending node for that time. Multiply the Rsine of that by the planet's own greatest celestial latitude and divide by the planet's own *śīghrakarṇa*. Then is obtained the celestial latitude (of the planet for that time).[2]

6. Take the difference or sum of the celestial latitudes of the two planets (which are in conjunction) according as they are of like or unlike directions : the result obtained should be taken as the distance between the two planets.[3]

When the sum of the celestial latitudes is taken, the direction of the sum is the same as that of the celestial latitudes; in the contrary case, the direction is that of the greater celestial latitude.[4]

Āryabhaṭa II[5] correctly says that in the case of the Moon, the celestial latitude should be corrected for parallax in latitude. In the case of the planets, parallax in latitude is small and negligible.

The distance between two planets in conjunction is generally announced in terms of *aṅgulas*.[6] To convert minutes into *aṅgulas*, see Vaṭeśvara's rule stated in vs. 10 below.

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1. See *BrSpSi*, ix. 2.
2. This rule is the same as given above in ch. VI, vs. 6.
3. *Cf. BrSpSi*, ix. 11(a-b); *KK*, I, viii. 6; *ŚiDVr*, x. 11; *MSi*, xi. 7; *SiŚe*, xi. 18 (a-b); *SiŚi*, I, x. 6; *SuSi*, I, x. 4(a-b).
4. Similar statement is made in *ŚiDVr*, x. 12.
5. See *MSi*, xi. 6 (c-d).
6. See *MBh*, vi. 55.

BHEDA OR ECLIPSE OF A PLANET

7. When the distance between the two planets (which are in conjunction) is less than half the sum of the diameters of the two planets, there is eclipse (*bheda*) of one planet by the other.[1] The eclipser is the lower planet. All calculations (pertaining to this eclipse), such as the semi-duration etc., are to be made as in the case of a lunar eclipse.[2]

8. When the Moon eclipses a planet, the time of conjunction should be reckoned from moonrise and for that time one should calculate the *lambana* and the *avanati*.

In case one planet eclipses another planet, the time of conjunction should be reckoned from the (eclipsed) planet's own rising and for that time one should calculate the *lambana* and the *avanati*.

The whole procedure has been explained by Bhaṭṭotpala as follows:[3]

"The planet which lies in the lower orbit is the eclipsing planet (or the eclipser); it is to be assumed as the Moon. The planet which lies in the higher orbit is the eclipsed planet; it is to be assumed as the Sun. Then, assuming the time of conjunction (of the two planets) as reckoned from the rising of the eclipsed planet as the *tithyanta*, calculate the *lagna* for that *tithyanta* with the help of (the longitude of) the eclipsed body, which has been assumed as the Sun, and the oblique ascensions of the signs. Subtracting 3 signs from that, (find the *vitribha-lagna* and then) calculate the corresponding declination (i. e., the declination of the *vitribha-lagna*). Taking the sum of that (declination) and the local latitude when they are of like direction, or their difference when they are of unlike directions, calculate the *lambana* (for the time of conjunction) as in the case of a solar eclipse. When the longitude of the planets in conjunction is greater than (the longitude of) the *vitribha-lagna*, subtract this *lambana* from the time of conjunction; and when the longitude of the planets in conjunction is less than (the longitude of) the *vitribha-lagna*, add this *lambana* to the time of conjunction; and iterate this process: this is how the *lambana* is to be calculated. Then from the longitude of the *vitribha-lagna* which has got iterated in the process of iteration of the *lambana*, severally subtract the ascending nodes of the two planets, and therefrom calculate two celestial latitudes (of the *vitribha-lagna*), as has been done in the case

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1. *Cf. ŚiDVr*, x. 11(d); *SiŚe*, xi. 33(a-b).
2. *Cf. ŚiDVr*, x. 13; *MSi*, xi. 8; *SiŚe*, xi. 33 (c-d); *SiŚi*, I, x. 7-9.
3. See Bhaṭṭotpala's commentary on *KK*, I, viii. 5-6.

of a solar eclipse. Then taking the sum or difference of the declination of the *vitribha-lagna*, the latitude of the *vitribha-lagna*, and the local latitude, each in terms of degrees, (according as they are of like or unlike directions), in the case of both the planets. Then applying the rule: "Multiply the Rsine of those degrees of the sum and difference by 13 and divide by 40 : the result is the *avanati*," calculate the *avanatis* for the two planets. Then calculate the latitude of the eclipsed and the eclipsing planets in the manner stated in the chapter on the rising and setting of the heavenly bodies, and increase or decrease them by the corresponding *avanatis* according as the two are of like or unlike directions : the results are the true latitudes (of the eclipsed and eclipsing planets). Take the sum or difference of those true latitudes according as they are of unlike or like directions : the result of this is the *sphuṭa-vikṣepa*.

Having thus obtained the *sphuṭa-vikṣepa*, one should see whether there exists eclipse-relation between this *sphuṭa-vikṣepa* and the diameters of the discs of the two planets. If the *sphuṭa-vikṣepa* is less than half the sum of the diameters of the two planets, this relation does exist; if greater, it does not. The totality of the eclipse should also be investigated as before. Then (severally) subtract the square of the *sphuṭa-vikṣepa* from the squares of the sum and the difference of the semi-diameters of the eclipsed and eclipsing planets, and take the square-roots (of the results). Multiply them by 60 and divide by the difference or sum of the daily motions of the two planets as before : then are obtained the *sthityardha* and the *vimardārdha*, (respectively). They are fixed (by the process of iteration) as in the case of a solar eclipse. The *sthityardha* and *vimardārdha* having been obtained in this way, they should be corrected for *lambana* (and the true values of *spārśika* and *mauṣika sthityardhas* and *spārśika* and *maukṣika vimardārdhas* should be obtained). Then the time of apparent conjunction should be declared as the time of the middle of the planetary eclipse; this diminished and increased by the (*spārśika and maukṣika*) *sthityardhas* (respectively), as the times of contact and separation (of the two planets); and the same diminished and increased by the (*spārśika* and *maukṣika*) *vimardārdhas* (respectively), as the times of immersion and emersion."

AKṢADṚKKARMA FOR THE TIME OF CONJUNCTION

9. Multiply the *dṛṣṭiphala* (i. e., *akṣadṛkkarma* for rising in the forenoon of the planet or for setting in the afternoon of the planet) by the planet's own hour angle and divide by the semi-duration of the planet's own day : the result should be applied to the longitude of the planets for the

time of their conjunction, as before.[1] In case there is distance (*vivara*) between the planets, one should apply the rule for the northern latitude to the planet lying to the north (of the ecliptic) and the rule for the southern latitude to the planet lying to the south (of the ecliptic).

The term *dṛṣṭiphala* has been used here in the sense of *akṣadṛkkarma.*

The *dṛṣṭiphala* for the time of conjunction of the two planets has been derived from the *dṛṣṭiphala* for the time of planet's rising or setting by the rule of three : When the hour angle is equal to its value at planet's rising or setting, the *dṛṣṭiphala* is equal to its value at planet's rising or setting, what will then be the value of the *dṛṣṭiphala* corresponding to the planet's hour angle at the time of conjunction ? The result is

Planet's *dṛṣṭiphala* for the time of conjunction (when the hour angle is H)

$$= \frac{H \times \textit{dṛṣṭiphala} \text{ for planet's rising or setting}}{\text{hour angle at planet's rising or setting}}$$

$$= \frac{H \times \textit{dṛṣṭiphala} \text{ for planet's rising or setting}}{\text{semi-duration of planet's day}}.$$

The above proportion is motivated by the following facts :

(1) When the hour angle is zero, the *dṛṣṭiphala* is zero.

(2) When the hour angle is equal to the semi-duration of the planet's day, the *dṛṣṭiphala* is equal to its value at planet's rising or setting.

It is therefore presumed that the *dṛṣṭiphala* varies as the hour angle.

The rule for the application of the *dṛṣṭiphala* is : When the planet's latitude is north, it should be subtracted from or added to the planet's longitude according as the planet is in the eastern or western half of the celestial sphere; and when the planet's latitude is south, it should be added to or subtracted from the planet's longitude according as the planet is in the eastern or western half of the celestial sphere.[2]

The *ayana-dṛkkarma* should be calculated and applied in the manner stated before. See *supra*, ch. VI, vs. 9.

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1. *Cf. SūSi*, vii. 8-9. It is probably the absence of this rule from the *Brāhma-sphuṭa-siddhānta* that Vaṭeśvara criticized him in vs. 43 of Chap. I, Sec. 10.
2. See *SūSi*, vii. 9.

REDUCTION OF MINUTES INTO CUBITS

10. At sunrise and sunset, seventy two minutes make a cubit; at midday, ninety six (minutes make a cubit). In between the two, proportion is to be used.

This rule is based on the assumption that at sunrise and sunset

$$1 \text{ } aṅgula = 3 \text{ minutes}$$

and at midday

$$1 \text{ } aṅgula = 4 \text{ minutes.}$$

For the proportion to be used to find the value of an *aṅgula* or a cubit in terms of minutes at any other time, see *supra*, ch. IV, vs. 40.

The above rule is meant to convert the distance in minutes between two planets at the time of their conjunction, into cubits.

2. *SAMAPROTĪYA-YUTI*

When two planets lie on the same secondary to the prime vertical the conjunction is called *Samaprotīya-yuti*. This type of conjunction was first studied by Brahmagupta who called it true conjunction. Vaṭeśvara deals with this conjunction now.

INTRODUCTION

11. Whatever happens in the case of apparent conjunction of Citrā (Spica) and Svātī (Arcturus) at rising and whatever reverse happens at setting also happens, in the same way, in the case of the planets. I shall therefore describe the (true) conjunction (conjunction along a secondary to the prime vertical) of the planets, where computation and observation tally.[1]

Bhaṭṭotpala[2] says: "Whatever has been said by Āryabhaṭa I and others regarding the conjunction of two planets is not very accurate, because even those planets whose longitudes are not equal up to minutes are said to have conjunction. For example, take the case of Citrā and Svātī. Citrā is deflected towards the south of the ecliptic from its position at 3°

1. *Cf. BrSpSi*, ix. 12; *ŚiDVr*, x. 14; *SiŚe*, xi. 19-20.
2. In the opening lines of his commentary on *KK*, II, chap. vi.

of Libra; and Svātī is deflected towards the north of the ecliptic from its position at 19° of Libra. Even then they are seen in conjunction (along a secondary to the prime vertical) every day. During the course of their day, the rising of Svātī occurs first and that of Citrā later; whereas the setting of Citrā occurs first and that of Svātī later. Similarly, in the case of conjunction of the planets too, conjunction of two planets with unequal longitudes does occur, and their conjunction lasts for many days. So equality of longitudes is not a criterion of conjunction."

So also says Mallikārjuna Sūri[1] :

"When the degrees intervening between two planets with equal longitudes, one lying to the south of the ecliptic and the other to the north of the ecliptic, are too many, then their conjunction in the eastern and western halves of the celestial sphere is of a different character. If one asks how it is so, the answer is that at the time of their rising (when they are situated on the horizon running north to south) there does not exist equality of their longitudes. At other time when there is equality of their longitudes up to minutes, they are not situated north-to-south (i. e., they do not lie on a great circle joining the north and south cardinal points). This is so in the case of conjunction of two planets. In the case of conjunction of two stars, which are separated by a large distance, too, even when there is equality of their longitudes, their north-to-south situation does not agree with observation (i. e., they do not appear north-to-south). If one asks how it is that conjunction of two stars (with unequal longitudes) is seen to occur, then the answer is : The north-south distance between Citrā and Svātī is 39 degrees. To the south of the Vindhyas, Svātī rises later than Citrā. So their conjunction does not occur there. To the north of the Vindhyas, Citrā rises later than Svātī. So there the conjunction of Citrā and Svātī occurs in the forenoon of their day. In the afternoon, Citrā, leaving Svātī behind, sets earlier; Svātī sets later. In the case of Abhijit and other stars, which are situated to the north of the ecliptic and have a large distance between them, too, the rising occurs differently. For this reason, the method for knowing the time of their conjunction of that kind (i. e., along a secondary to the prime vertical) in which computation and observation tally, is being stated."

What is meant is that conjunction in longitude (*kadambaprotīya-yuti*) is not the appropriate type of conjunction; firstly, because the two planets

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1. In his commentary on *ŚiDVr*, x. 14.

which are in conjunction in longitude are not observed north-to-south (i. e., along the same secondary to the prime vertical); secondly, because Citrā and Svātī are observed in conjunction along the same secondary to the prime vertical, even though they do not have conjunction in longitude. Hence, the proper type of conjunction is the *samaprotīya-yuti* (i. e., conjunction along a secondary to the prime vertical) rather than the *kadambaprotīya-yuti* (i. e., conjunction in longitude).

GHAṬĪS ELAPSED SINCE OR TO ELAPSE BEFORE *SAMAPROTĪYA YUTI*

12-14. Multiply the duration of day for the planet with smaller day-length by the time (in *ghaṭīs*) elapsed since the rising of the planet with greater day-length, and divide by the duration of day for the planet with greater day-length. When the resulting time is greater than the time elapsed since the rising of the planet with smaller day-length, (it should be understood that) the conjunction of the two planets is to occur; in the contrary case, (it should be understood that) the conjunction has already occurred.[1]

The difference of the two times (in terms of *ghaṭīs*) is the "first." A similar difference derived from the "*ghaṭīs* arbitrarily chosen" (for *ghaṭīs* elapsed since or to elapse before conjunction) is the "second." When the "first" and the "second" both correspond either to conjunction past or conjunction to occur, divide the product of the "first" and the "arbitrarily chosen *ghaṭīs*" by the *ghaṭīs* of the difference between the "first" and the "second"; in the contrary case (i. e., when out of the "first" and the "second", one corresponds to conjunction past and the other to conjunction to occur) divide that product by (the *ghaṭīs* of) the sum of the "first" and the "second". The resulting *ghaṭīs* are the *ghaṭīs* elapsed since or to elapse before the conjunction of the two planets, depending on whether the "first" relates to conjunction past or conjunction to occur.[2] (The process should be iterated if necessary.)

The conjunction referred to in the above rule is that along the same secondary to the prime vertical (*samaprotīya-yuti*). The calculation is supposed to be made when the two planets are in conjunction in celestial longitude.

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1. When the two are equal, it should be understood that the two planets are in conjunction. See *BrSpSi*, ix. 25; *KK*, II, vi. 4; *ŚiDVr*, x. 20; *SiŚe*, xi. 32. This criterion is based on the fact that the planet with smaller day-length is swifter than the other.
2. *Cf.* *BrSpSi*, ix. 22-24; also x. 51-56; *KK*, II, vi. 1-3; *ŚiDVr*, x. 15-19; *SiŚe*, xi. 29-31.

The time elapsed since the rising of a planet is equal to the oblique ascension of the portion of the ecliptic that lies between the rising ecliptic point at that time and the rising ecliptic point at the time of the planet's rising. In other words, it is equal to the sum of (1) the oblique ascension of the traversed part of the sign occupied by the rising ecliptic point at that time, (2) the oblique ascension of the untraversed part of the sign occupied by the rising ecliptic point at the time of the planet's rising and (3) the oblique ascensions of the intervening signs.

The length of day of a planet is equal to the oblique ascension of that part of the ecliptic that lies between the *udayalagna* and the *astalagna* of the planet. It is equal to the sum of (1) the oblique ascension of the untraversed part of the sign occupied by the *udayalagna*, (2) the oblique ascension of the traversed part of the sign occupied by the *astalagna*, and (3) the oblique ascensions of the intervening signs. (See *supra*, vs. 11)

Mallikārjuna Sūri[1] describes the whole process in detail, which (with certain modifications to suit our text) runs thus :

"First, determine the times of rising of the two planets which are in conjunction in longitude, then find out their *udayalagna* and *astalagna*. If one asks how this is to be done, then the answer is : First find out the true longitudes of the Sun and of those planets for the time of sunrise, then take the sum of (1) the oblique ascension of the untraversed part of the sign occupied by the Sun, (2) the traversed part of the planet's own sign, and (3) the oblique ascensions of the intervening signs. The *ghaṭīs* etc., obtained in this way, give the time, reckoned since sunrise, when that planet will rise on that day. By those *ghaṭīs* (severally) multiply the daily motions of the Sun and those two planets and divide (each product) by 60 ; and add the resulting minutes etc. to the respective longitudes of the Sun and the planets. Iterate this process until the *ghaṭīs* lying between the Sun and the planets (taken severally) are fixed. The longitudes of those planets made instantaneous for the time given by those *ghaṭīs* become the longitudes for the times of their own risings. These longitudes corrected for the visibility corrections for rising, in the manner stated before, become the *udayalagnas* of the respective planets. When 6 signs are added to the *udayalagnas* and the visibility corrections for setting are applied,

1. In his commentary on *ŚiDVr*, x. 15-20.

they became the *astalagnas* of the respective planets.[1]

In the case of conjunction of stars, multiply the degrees of the latitude for the star's *dhruva* ("polar longitude") by the equinoctial midday shadow and divide by 12 : the result will give the so called *akṣadṛkkarma* in terms of degrees etc. These degrees etc. should be subtracted from the star's polar longitude, provided the latitude is north. If the latitude is south, the same should be added to the star's polar longitude. Then will be obtained the star's *udayalagna*. Again, add the same degrees to the star's polar longitude if the latitude is north; if the latitude is south, subtract the same from the star's polar longitude. The star's polar longitude, which has been thus increased or diminished by the degrees etc. of the *akṣadṛkkarma*, should be increased by 6 signs : then will be obtained the star's *astalagna*. Then having applied the precession of the equinoxes to the *udayalagna* and *astalagna*, take the sum of (1) the oblique ascension of the untraversed part of the sign occupied by the *udayalagna*, (2) the oblique ascension of the traversed part of the sign occupied by the *astalagna*, and (3) the oblique ascensions of the intervening signs. This sum, in terms of *ghaṭīs* etc., gives the length of day in the case of the planets Mars etc., or the length of day in the case of the stars Aśvinī etc. Half of that is half the length of day. The difference between half the day and the day elapsed gives the hour angle (*nata*).

When the time is different from rising and setting, then one should multiply the degrees etc. of the *akṣadṛkkarma* (as obtained above) by the (planet's or star's) own hour angle and divide (the resulting product) by the semi-duration of the (planet's or star's) own day : the result will be the true *akṣadṛkkarma* for that time, in terms of degrees etc. This should be applied to the longitude in the case of the planets Mars etc. or to the polar longitude in the case of the stars, depending on whether the latitude of the planet or star is north or south, in the manner stated before. In this way one should know the time of conjunction of the planets and stars (and therefrom one should calculate the *udayalagna* for that time for each planet or star). The sum of (1) the oblique ascension of the traversed part of the sign occupied by the *udayalagna* for that time, (2) the oblique ascension of the untraversed part of the sign occupied by the *udayalagna* at the

1. This is a rough method for finding the planet's *astalagna*. To be more accurate find the planet's longitude for the time of its setting, apply the visibility corrections to it, and then increase it by six signs.

In general : To find the planet's *udayalagna*, apply to the planet's longitude the visibility corrections for rising; and to find the planet's *astalagna*, apply to the planet's longitude the visibility correction for setting and then add six signs.

time of rising of that planet or star, and (3) the oblique ascensions of the intervening signs, will give the elapsed amount of the day of that planet or star. This rule will hold whether it is conjunction of two planets, or of a planet and a star, or of two stars."

"When the distance between the two planets is small then the day-length for the planet lying to the south is small and the day-length for the planet lying to the north is large. Also, the northern planet rises first (above the horizon) and the southern planet later. So the "day elapsed" for the planet with smaller day-length is smaller (than that for the planet with greater day-length). Multiply the greater "day elapsed" (i.e., the day elapsed for the planet with greater day-length) by the smaller day-length (i.e., the day-length for the planet with smaller day-length) and divide by the greater day-length (i.e., the day-length of the planet with greater day-length). If the quotient is greater than the "day elapsed" for the planet with smaller day-length, (it must be understood that) the (true) conjunction is yet to occur; if the quotient is smaller, (it must be understood that) the (true) conjunction has already occurred. Having thus known that the (true) conjunction has already occurred or is yet to occur, find the difference between the above quotient and the "day elapsed" for the planet with smaller day-length. This difference is the "first quantity".

In case the (true) conjunction has already occurred, choose ten-ten or or five-five *ghaṭīs* and subtract them from the "day elapsed" for each of the two planets at the time of their longitudinal equality; in case the (true) conjunction is to occur, add the chosen ten-ten or five-five *ghaṭīs* to the "day elapsed" for each of those two planets (at the time of their longitudinal equality). (Treat this difference or sum as the new "day elapsed" for the two planets.) Then multiply the "day elapsed" for the planet with greater day-length by the smaller day-length and divide by the greater day-length. In case the quotient is greater than the day elapsed for the planet with smaller day-length, the (true) conjunction is yet to occur; in case the quotient is smaller, the (true) conjunction has already occurred. The difference between this quotient and the "day elapsed" for the planet with smaller day-length is the "second quantity';.

If the first and the second quantities both correspond either to conjunction past or to conjunction to occur, then the difference of the first and second quantities is the "divisor". If, out of those two quantities, one corresponds to conjunction past and the other to conjunction to occur, then the sum of the first and second quantities is the "divisor".

Then multiply the "first quantity" by the assumed *ghaṭīs* and divide by the "divisor" : the result is in terms of *ghaṭīs* etc. These *ghaṭīs* give the *ghaṭīs* elapsed since (true) conjunction or to elapse before (true) conjunction, according as the "first quantity" corresponds to conjunction past or to occur. In case the *ghaṭīs* are *ghaṭīs* elapsed, subtract them from the "day elapsed" for each of the two planets, as obtained by calculation for the time of their longitudinal equality; in case the *ghaṭīs* are *ghaṭīs* to elapse, add them to the calculated "day elapsed" for each of the two planets. Then multiply the "day elapsed" corresponding to greater day-length by the smaller day-length and divide by the greater day-length : the result is in terms of *ghaṭīs* etc. This process should be iterated until these *ghaṭīs* etc. become equal to those of the "day elapsed" for the planet with smaller day-length.

Thus, before or after the time of longitudinal equality, whenever the "day elapsed" for the two planets become equal, the two planets are seen in conjunction, situated north to south. (It should be noted that) while finding the "second quantity" from the *ghaṭīs* obtained from the "divisor" at the various stages, the initial "first quantity" itself is to be taken as the "first quantity".

The rule stated in the text was first devised by Brahmagupta and later adopted by Lalla, Vaṭeśvara and Śrīpati.

Bhāskara II has given rules for conjunction in celestial longitude as well as conjunction in polar longitude. But as there is no star at the pole of the ecliptic, conjunction in celestial longitude, says he, does not create confidence in the observer, while there being one at the pole of the equator conjunction in polar longitude is better suited for observation. However, conjunction of two planets, in his opinion, really occurs when the two planets are nearest to each other and this happens when the two planets are in conjunction in celestial longitude only.[1] He has given no credit to conjunction along a secondary to the prime vertical. He has not even mentioned this conjunction.

Munīśvara has criticised conjunction along a secondary to the prime vertical on the ground that the time of such a conjunction differs from place to place and so it creates confusion in making astrological predictions.[2]

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1. See *SiŚi*, I, x. 4(c-d)-5, gloss.
2. See *SiSā*, *Bhagrahayuti*, vs. 15, p. 543.

Section 2

Conjunction of Star and Planet

POLAR POSITIONS OF JUNCTION-STARS OF *NAKṢATRAS*

1-3(a-b). (The junction-stars of) the *nakṣatras* Aśvinī etc. come in conjunction with the planets in Aries at 8 and 20 degrees, in Taurus at the same distances diminished by half a degree, in Gemini at 3 and 7 degrees, in Cancer at 3, 16 and 18 degrees, in Leo at 8 and 27 degrees, in the sixth sign at 4 and 20 degrees, in Libra at 3 and 18 degrees, in Scorpio at 2, 14 and 19 degrees, in Sagittarius at 1, 14, 20 and 27 degrees, in Capricorn at 8 and 20 degrees, in Aquarius at 20 and 27 degrees, and in Pisces at 7 and 30 degrees.[1]

Table 30. Polar longitudes of the Junction-stars of the *Nakṣatras* according to Hindu astronomers

Junction-star of	Polar longitude according to				
	Brahmagupta[2], Śrīpati[3] Bhāskara II[4]	Lalla[5]	*Sūrya-Siddhānta*[6]	Vaṭeśvara	Āryabhaṭa II[7]
Aśvinī	8°	8°	8°	8°	12°
Bharaṇī	20°	20°	20°	20°	24°23′
Kṛttikā	37°28′	36°	37°30′	37°30′	38°33′
Rohiṇī	49°28′	49°	49°30′	49°30′	47°33′
Mṛgaśirā	63°	62°	63°	63°	61° 3′
Ārdrā	67°	70°	67°20′	67°	68°23′
Punarvasu	93°	92°	93°	93°	92°53′
Puṣya	106°	105°	106°	106°	106°

1. Similar constants have been given in *BrSpSi*, x. 1-3; *KK*, I, ix. 4-6; xii. 1-2; *SiŚe*, xii. 1-2; *SiŚi*, I, xi. 1-3.
2. *BrSpSi*, x. 1-3, 35, 40; *KK*, I, ix. 4-6.
3. *SiŚe*, xii. 1-2, 10, 20.
4. *SiŚi*, I, xi. 1-3, 7.
5. *ŚiDVr*, xi. 1-3, 7(c-d), 8.
6. *SūSi*, viii. 2-6(a), 10.
7. *MSi*, xii. 1-4; ix. 8(c-d).

Junction-star of	Polar longitude according to				
	Brahmagupta, Śrīpati, Bhāskara II	Lalla	*Sūrya-siddhānta*	Vaṭeśvara	Āryabhaṭa II
Āśleṣā	108°	114°	109°	108°	111°
Maghā	129°	128°	129°	128°	126°
Pūrvā Phālgunī	147°	139°20′	144°	147°	140°23′
Uttarā Phālgunī	155°	154°	155°	154°	150°23′
Hasta	170°	173°	170°	170°	174°3′
Citrā	183°	184°20′	180°	183°	182°53′
Svātī	199°	197°	199°	198°	194°
Viśākhā	212°5′	212°	213°	212°	211°33′
Anurādhā	224°5′	222°	224°	224°	224°53′
Jyeṣṭhā	229°5′	228°	229°	229°	230°3′
Mūla	241°	241°	241°	241°	242°44′
Purvāṣaḍhā	254°	254°	254°	254°	252°33′
Uttarāṣaḍhā	260°	267°20′	260°	260°	260°23′
Abhijit	265°	267°	266°40′	267°	263°
Śravaṇa	278°	283°10′	280°	278°	280°3′
Dhaniṣṭhā	290°	296°20′	290°	290°	296°33′
Śatabhiṣak	320°	313°20′	320°	320°	319°53′
Pūrva-Bhādrapadā	326°	327°	326°	327°	334°53′
UttaraBhādrapadā	337°	335°20′	337°	337°	347°
Revatī	0	359°	359°50′	0	0
Canopus	87°	87°	90°	87°	85°
Sirius	86°	86°	80°	80°	

POLAR POSITIONS OF CANOPUS AND SIRIUS

3(c-d). The polar position of Canopus is at 27 degrees in Gemini,[1] and that of Sirius at 20 degrees in the same sign.[2]

CONJUNCTION PAST OR TO OCCUR

4(a-c). When the longitude of a planet (corrected for *ayanadṛk-karma*)[3] is less or greater than the polar longitude (of the junction-star of a *nakṣatra*), their conjunction is to occur or has already taken place, (respectively): the rest is similar to that stated in the case of conjunction of two planets.[4]

This rule is applicable when the planet has direct motion. If the planet is retrograde, the reverse will be the result.

POLAR LATITUDES OF JUNCTION-STARS OF *NAKṢATRAS*

4(d)-6. The polar latitudes (lit. deviations from the declination-end) of (the junction-stars of) the *nakṣatras* Aśvinī etc. are: 10, 12, 5, 5, 10, 11, 6, 0, 7, 0, 12, 13, 11, 2, 37, 2, 3, 4, 9, 6, 6, 64, 30, 36, 0, 24, 26 and 0 degrees respectively.

7. 30 minutes should be diminished in the case of (the junction-stars of) Rohiṇī, Jyeṣṭhā, Mūla, Anurādhā and Kṛittikā; 40 minutes in the case of (the junction-stars of) Uttarāṣāḍhā, Pūrvāṣāḍhā and Viśākhā; and 20 minutes in the case of (the junction-star of) Citrā.[5]

8. Three *nakṣatras* beginning with Rohiṇī, Āśleṣā, two *nakṣatras* beginning with Hasta and six *nakṣatras* beginning with Viśākhā : these have their junction-stars deviated towards the south of the ecliptic. (The junction-stars of) the remaining *nakṣatras* are deviated towards the north (of the ecliptic).[6]

POLAR LATITUDES OF SIRIUS AND CANOPUS

9(a-d). Sirius is deviated towards the south (of the ecliptic) by 40 degrees and Canopus by 77 degrees.[7]

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1. Same is given by Brahmagupta; see *BrSpSi*, x. 35(c-d).
2. Same is given in *SūSi*, viii. 10.
3. See *SiŚi*, I, xi. 9.
4. *Cf. BrSpSi*, x. 4; *KK*, I, ix. 7; *ŚiDVṛ*, xi. 4; *SūSi*, viii. 14-15; *SiŚe*, xi. 3. Also see *MSi*, xii. 9.
5. *Cf. BrSpSi*, x. 5-9; *KK*, I, ix. 8-10, 11-12; *SiŚe*, xii. 4-6.
6. *Cf. KK*, I, ix. 13; *ŚiDVṛ*, xi. 9-10(a-b); *SiŚe*, xi. 7.
7. *Cf. BrSpSi*, x. 35, 40; *MSi*, ix. 8(c-d); *SiŚi*, I, xi. 7.

Table 31. Polar latitudes of the Junction-stars of the *Nakṣatras* according to Hindu astronomers

Junction-star of	Polar latitude according to					
	Brahmagupta[1], Śrīpati[2]	Lalla[3]	*SūSi*[4]	Vaṭeśvara	Āryabhaṭa II[5]	Bhāskara II[6]
Aśvinī	10° N	10° N	10° N	10° N	10° N	10° N
Bharaṇī	12° N	12° N	12° N	12° N	12° N	12°N
Kṛttikā	4°31′ N	5° N	5° N	4°30′ N	5° N	4° 30′ N
Rohiṇī	4°33′ S	5° S	5° S	4°30′ S	5° S	4°30′ S
Mṛgaśirā	10° S	10° S	10° S	10° S	10° S	10° S
Ārdrā	11° S	11° S	9° S	11° S	11° S	11° S
Punarvasu	6° N	6° N	6° N	6° N	6° N	6° N
Puṣya	0	0	0	0	0	0
Āśleṣā	7° S	7° S	7° S	7° S	7° S	7° S
Maghā	0	0	0	0	0	0
Pūrvā Phālgunī	12° N	12° N	12° N	12° N	12° N	12° N
Uttarā Phālgunī	13° N	13° N	13° N	13° N	13° N	13° N
Hasta	11° S	8° S	11° S	11° S	10° S	11° S
Citrā	1°45′ S	2° S	2° S	1°40′S	2° S	1°45′ S
Svātī	37° N	37° N	37° N	37° N	37° N	37° N
Viśākhā	1°23′ S	1°30′ S	1°30′ S	1°20′ S	1°30′ S	1°20′ S
Anurādhā	1°44′ S	3° S	3° S	2°30′ S	3° S	1°45′ S
Jyeṣṭhā	3°30′ S	4° S	4° S	3°30′ S	4° S	3°30′ S
Mūla	8°30′ S	8°30′ S	9° S	8°30′ S	9° S	8°30′ S

1. *BrSpSi*, x. 5-9, 35, 40; *KK*, I, ix. 8-13.
2. *SiŚe*, xii. 4-7, 10, 20.
3. *ŚiDVr*, xi. 4 (c)-8, 9-10.
4. viii. 6-11 (a-b).
5. *MSi*, xii. 6-8; ix. 8 (c-d).
6. *SiŚi*, I, xi. 4-7.

Pūrvāṣāḍhā	5°20′ S	5°20′ S	5°30′ S	5°20′ S	5° S	5°20′ S
Uttarāṣāḍhā	5° S	5° S	5° S	5°20′ S	5° S	5° S
Abhijit	62° N	64° N	60° N	64° N	63° N	62° N
Śravaṇa	30° N	30° N	30° N	30° N	30° N	30° N
Dhaniṣṭhā	36° N	36° N	36° N	36° N	36° N	36° N
Śatabhiṣak	0°18′ S	0°20′S	0°30′ S	0°	0°20′S	0°20′ S
P.-Bhādrapadā	24° N	24° N	24° N	24° N	24° N	24° N
U.-Bhādrapadā	26° N	26° N	26° N	26° N	26° N	26° N
Revatī	0	0	0	0	0	0
Canopus	77° S	80° S	80° S	77° S	77° S	77° S
Sirius	40° S	40° S	40° S	40° S		40° S

For the identification of the junction-stars of the *nakṣatras*, see *Bhāratīya-Jyotiṣaśāstra* by S. B. Dikshit.

BHEDA OR OCCULTATION

9(c-d). When the difference between the latitudes of like directions is less than half the sum of the diameters (of the two bodies concerned), there is occultation.

It should be noted that the diameter of a star is supposed to be negligible. Hence Brahmagupta says :

"A planet lying to the same side of the ecliptic (as a junction-star) occults the junction-star when the true latitude of the planet exceeds the latitude of the junction-star minus the semi-diameter of the planet but falls short of the latitude of the junction-star plus the semi-diameter of the planet."[1]

OCCULTATION OF ROHIṆĪ AND ITS JUNCTION-STAR

10. The planet, whose latitude at 17 degrees of Taurus amounts to $1\frac{1}{2}$ degrees south, occults the cart of Rohiṇī.[2]

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1. *BrSpSi*, x. 10; *KK*, I, ix. 14.
2. In place of $1\frac{1}{2}$ degrees south, Brahmagupta prescribes "greater than 2° south". See *BrSpSi*, x. 11; *KK*, I, ix. 15. Āryabhaṭa II, Śrīpati and the author of the *Sūrya-siddhānta* are in agreement with Brahmagupta in this respect. See *MSi*, xii. 13 (a-b); *SiŚe*, xii. 8; *SūSi*, viii, 13. According to Lalla, the Moon occults the cart of Rohiṇi when its longitude is 16° 40′ and latitude 2° 40′ south. See *ŚiDVr*, xi, 11 (a-b).

11. The Moon with its greatest latitude south (i. e., 43°0′S) covers the junction-star of Rohiṇī.[1]

The cart of Rohiṇī is the V-shaped Hyades cluster. The junction-star of Rohiṇī is the star Aldebaran.

NUMBER OF STARS IN THE *NAKṢATRAS*

12-13. 3, 3, 6, 5, 3, 1, 4, 3, 6, 5, 2, 2, 5, 1, 1, 4, 4, 3, 11, 2, 3, 3, 3, 4, 100, 2, 2, and 32 : these are the number of stars in the *nakṣatras* Aśvinī etc , including Abhijit (Lyra).

The biggest (i. e., the brightest) of all the stars (in a *nakṣatra*) is to be taken as the junction-star (of that *nakṣatra*).[2]

The following table gives the number of stars in the various *nakṣatras* according to Vaṭeśvara and other Hindu authorities :

Table 32. Number of Stars in the *Nakṣatras*

Nakṣatra	Number of stars according to				
	Varāhamihira[3]	Brahma-gupta[4]	Lalla[5]	Vaṭe-śvara	Śrīpati[6]
1 Aśvinī	3	2	3	3	3
2 Bharaṇī	3	3	3	3	3
3 Kṛttikā	6	6	6	6	6
4 Rohiṇī	5	5	5	5	5
5 Mṛgaśirā	3	3	3	3	3
6 Ārdrā	1	1	1	1	1
7 Punarvasu	5	2	4	4	4
8 Puṣya	3	1	3	3	3

1. The same statement is made by Lalla in *ŚiDVr*, xi. 11(a-b). Brahmagupta calls the star occulted as the third star of Rohiṇī. See *BrSpSi*, x. 12(a-b); *KK*, I, ix. 16(a-b).
2. *Cf. KK*, I, ix. 3.
3. See *BṛSaṁ*, xcvii. 1-2.
4. See *KK*, I, ix. 1-2.
5. See *Ratna-koṣa*.
6. See *Jyotiṣa-ratna-mālā*, vi. 87.

9 Āśleṣā	6	6	5	6	5
10 Maghā	5	6	5	5	5
11 Pūrvā Phālgunī	8	2	2	2	2
12 Uttarā Phālgunī	2	2	2	2	2
13 Hasta	5	5	5	5	5
14 Citrā	1	1	1	1	1
15 Svātī	1	1	1	1	1
16 Viśākhā	5	2	4	4	4
17 Anurādhā	4	4	. 4	4	4
18 Jyeṣṭhā	3	3	3	3	3
19 Mūla	11	2	11	11	11
20 Pūrvāṣāḍhā	2	4	2	2	4
21 Uttarāṣāḍhā	3	4	2	3	3
22 Abhijit		3		3	
23 Śravaṇa	3	3	3	3	3
24 Dhaniṣṭhā	5	5	4	4	4
25 Śatabhiṣak	100	1	100	100	100
26 Pūrva-Bhādrapadā	2	2	2	2	2
27 Uttara-Bhādrapadā	8	2	2	2	2
28 Revatī	32	1	32	32	32

SHAPES OF *NAKṢATRAS*

14-15. **The (twenty eight) *nakṣatras* Aśvinī etc., (in their respective order), resemble (1) a horse's head, (2) the female generating organ, (3) a rajor, (4) a cart, (5) a deer's head, (6) a precious stone or jewel, (7) a house or temple, (8) an arrow, (9) a wheel, (10) a wall or rampart, (11) a bed, (12) a dais, (13) a hand, (14) a pearl, (15) a coral, (16) an arched doorway, (17) *bali* or heaps of offering of cooked rice, (18) an ear-ring, (19) a lion's tail, (20) an elephant's tusk, (21) a bed, (22) an elephant's ear, (23) three feet, (24) a drum (*paṭaha*), (25) a circle, (26) a dais, (27) a cot and (28) a drum (*mṛdaṅga*), (respectively).**[1]

1. *Cf. MuCi*, ii. 58-59; *JC*, i. 138.

Table 33. Shapes and Regents of the *Nakṣatras*

Nakṣatra	Shape	Regent	*Nakṣatra*	Shape	Regent
1 Aśvinī	Horse's head	Aśvinī twins	15 Svātī	Coral	Wind
2 Bharaṇī	Uterus	Yama	16 Viśākhā	Arched doorway	Indra and Agni
3 Kṛttikā	Rajor	Agni	17 Anurādhā	Heaps of offering	Mitra
4 Rohiṇī	Cart	Brahmā	18 Jyeṣṭhā	Ear-ring	Indra
5 Mṛgaśirā	Deer's head	Moon	19 Mūla	Lion's tail	Nirṛti (demons)
6 Ārdrā	Jewel	Rudra	20 Pūrvāṣāḍhā	Bed	Āpa
7 Punarvasu	House	Aditi	21 Uttarāṣāḍhā	Elephant's tusk	Viśvedevāḥ
8 Puṣya	Arrow	Jupiter	22 Abhijit	Elephant's ear	Brahmā
9 Āśleṣā	Wheel	Serpents	23 Śravaṇa	Three feet	Viṣṇu
10 Maghā	Wall	Manes	24 Dhaniṣṭhā	Drum	Vasus
11 Pūrvā Phālgunī	Bed	Bhaga	25 Śatabhiṣak	Circle	Varuṇa
12 Uttarā Phālgunī	Dais	Aryamā	26 Pūrva-Bhādrapadā	Dais	Aja ekapāt
13 Hasta	Hand	Sun	27 Uttara-Bhādrapadā	Bed	Ahirbudhnya
14 Citrā	Pearl	Tvaṣṭrā	28 Revatī	Drum	Pūṣā

THE POLE STAR

16. The Pole Star is a faint star lying in the constellation (of the Ursa Minor) which resembles a fish.

In the *Dhruvabhramaṇākhyaṭikā*, the Polar Fish is described as follows :

"Around that one sees a constellation of stars consisting of twelve stars and looking like a fish. It is named as "the Polar Fish". From a distance one sees a pair of bright stars, one at its mouth and the other at its tail. Of these, the one that lies at the mouth is 3 degrees off the Pole Star and the other that lies at the tail is 9 degrees off. The two stars are separated by 6 degrees from each other."[1]

The Persian scholar Al-Bīrūnī says :

"The Hindus tell rather ludicrous tales when speaking of the figure in which they represent this group of stars, viz. the figure of a four-footed aquatic animal which they call *Śakvara* and also *Śiśumāra.* I suppose that the latter animal is the great lizard, for in Persia it is called *Susmār*, which sounds much like the Indian *Śiśumāra.* Of this kind of animals there is also an aquatic species, similar to the crocodile and the skink."[2]

The diurnal revolution of the Polar Fish has been generally adduced as an argument to refute the Jaina conception of two Suns and two Moons rising one after the other alternately. Thus Brahmagupta says :

"According to what Jina has said, there are fifty four *nakṣatras,* two Suns and two Moons: this is false because the Polar Fish makes one complete revolution in a day."[3]

So also says Lalla :

"If there are two Suns and two Moons which rise alternately, how can the Polar Fish complete its revolution in a day?"[4]

Bhāskara II also has made a similar statement.[5]

STAR'S *UDAYALAGNA* AND *ASTALAGNA*

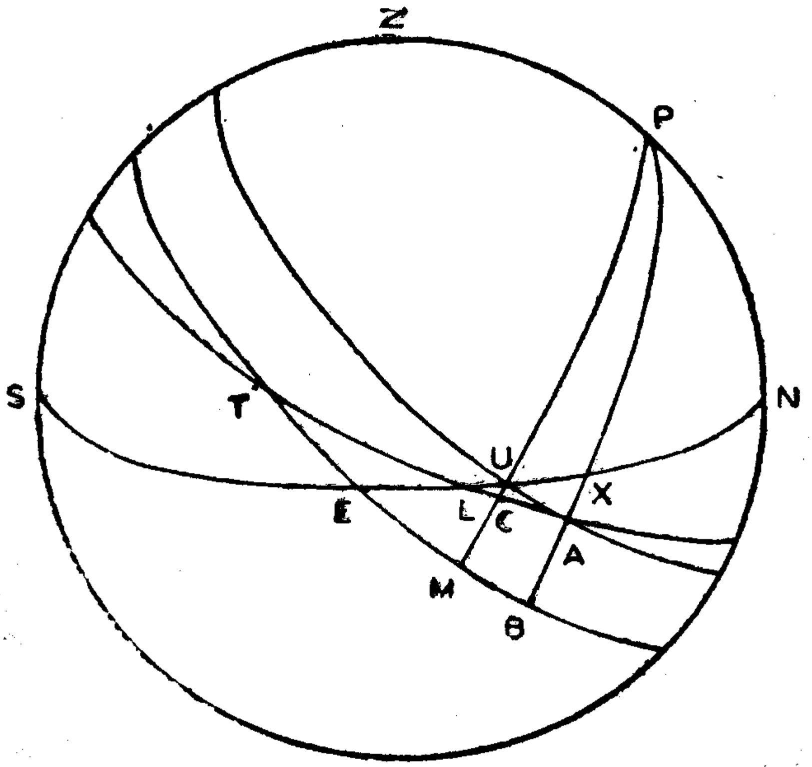
17-19(a-b). The declination of a star, calculated from the polar longitude of the star, when diminished or increased by the polar latitude of the star, as in the case of the Moon,[6] gives the true declination of the star (*i.e.*, the declination of the actual position of the star).[7] From the mean declination[8] and the true declination of the star, (severally

1. Quoted by Durga Prasad Dvivedi in his *Upapattīnduśekhara,* p. 510.
2. Alberuni's India, I, p. 241.
3. *BrSpSi*, xi. 3.
4. *ŚiDVr*, vii. 44.
5. See *SiŚi*, II, iii. 10.
6. *Vide supra,* ch. VI, vs. 21.
7. *Cf. BrSpSi*, x. 15.
8. The mean declination of a star is the declination calculated from the polar longitude of the star.

obtain the *asus* of the ascensional difference ; and take their sum or difference according as they are of unlike or like directions. Then are obtained the *asus* of true ascensional difference. From them subtract the *asus* of rising of (as many) signs (and parts thereof) (as possible) in the reverse order when the star's latitude is north or in the serial order when the star's latitude is south (beginning with the sign occupied by the star). The result (i.e., the signs etc. the *asus* of whose risings have been thus subtracted) should be subtracted from the polar longitude of the star when the latitude of the star is north or added to that when the latitude of the star is south, provided it is in the eastern half of the celestial sphere; in the western half of the celestial sphere, one should apply that (result) as a positive or negative correction to the polar longitude of the star as increased by six signs. Then are obtained the longitudes of the (*udaya* and *asta*) *vilagnas* (of the star).[1]

The *udaya-vilagna* or *udaya-lagna* of a star is that point of the ecliptic which rises on the eastern horizon simultaneously with the star, and the *asta-vilagna* or *asta-lagna* of a star is that point of the ecliptic which rises on the eastern horizon when the star sets on the western horizon.

The figure below represents the celestial sphere for a place in latitude ϕ. SEN is the horizon, S, E, N being the south, east and north cardinal points; Z is the zenith. X is the position of a star when it rises on the



1. *Cf. BrSpSi*, x. 18-20; *ŚiDVr*, xi. 12-15; *SiŚe*, xi. 13-15.

eastern horizon. TEB is the equator and P its north pole; TLA is the ecliptic and L is the point of the ecliptic which rises with the star X, i.e., the star's *udayavilagna*. The point T where the ecliptic intersects the equator is the first point of Aries. PXAB is the hour circle of the star X and A the point where it intersects the ecliptic. U is the point where the diurnal circle of A intersects the horizon, and PUM is the hour circle of U.

Now arc EB is the ascensional difference due to the true declination, (arc XB), of the star, and arc EM the ascensional difference due to the mean declination, (arc AB), of the star. The difference of these ascensional differences, which is designated as the true ascensional difference, gives the *asus* of the arc MB. The arc of the ecliptic that rises during the *asus* of the arc MB is the arc CA of the ecliptic. Vaṭeśvara has taken this arc as the approximate value of the arc LA, the star's *akṣadṛkkarma*. Thus, according to Vaṭeśvara,

longitude of the star's *udayavilagna* L, i.e., arc TL

=arc TA–arc LA

=arc TA–arc CA, approx.

=star's polar longitude—arc of the ecliptic that rises during the star's true ascensional difference.

This explains the rule for the *udayavilagna* when the star is to the north of the ecliptic. Other cases may be explained similarly.

HELIACAL RISING AND SETTING OF STARS

19(c-d)-20. When the longitude of the Sun is equal to the longitude of a star's *udayalagna* as increased by the arc of the ecliptic that rises at the place during the 14 time-degrees for the star's own visibility, the star rises heliacally; and when the longitude of the Sun is equal to the longitude of a star's *astalagna* as diminished by the arc of the ecliptic corresponding to the time-degrees of the star's visibility and (also) by half a circle, the star sets heliacally.[1]

When a star's *udayalagna* or *astalagna* is at a lesser distance from the Sun, the star is invisible; in the contrary case, it is visible.

That is to say, a star rises heliacally when

1. *Cf. KK*, II, v. 8-10; *ŚiDVr*, xi. 16-17; *SiŚe*, xii. 16-17. Also see *SiŚi*, I, xi. 12-14 (a-b).

Sun's longitude = long. of star's *udayalagna* + arc of ecliptic that rises at the place during 14 time-degrees,

and sets heliacally when

Sun's longitude = long. of star's *astalagna* − arc of ecliptic that rises at the place in 14 time-degrees − 6 signs.

Also, a star is invisible if

Sun's longitude—long. of star's *udayalagna*

< arc of ecliptic that rises at the place during 14 time-degrees;

or if

long. of star's *astalagna* − Sun's longitude

< arc of ecliptic that rises at the place during 14 time-degrees + 6 signs;

otherwise it is visible.

Vaṭeśvara, following other Hindu astronomers,[1] takes 14 degrees as the time-degrees for the visibility of a star.

The time-degrees of visibility for Canopus and Sirius, according to Brahmagupta and Bhāskara II, are 12 and 13 degrees respectively.[2]

Mallikārjuna Sūri[3] explains the above rule as follows :

"The time-degrees for the visibility of a star are 14. Multiply them by 1800 and divide by the *asus* of oblique ascension of the sign occupied by the star's *udayalagna* : the result is in degrees etc. Add it to the longitude of the star's *udayalagna*. When the Sun's longitude is equal to that, then that star rises in the east at sunrise. Again, multiply those 14 time-degrees by 1800 and divide by the oblique ascension of the sign occupied by the star's *astalagna* : the result is in degrees etc. Subtract them from the longitude of the star's *astalagna* and from what is obtained further subtract 6 signs. When the Sun is equal to that, then that star sets in the west at sunset."

ASTĀRKA AND *UDAYĀRKA*

We have already defined the terms *udayalagna* and *astalagna* in relation to a star. The *udayalagna* of a star is that point of the ecliptic which rises

1. See, e.g., *BrSpSi*, x. 38(a-b); *KK*, II, v. 7(a-b); *ŚiDVr*, xi. 16(c); *SiŚe*, xi. 16(c); *SiŚi*, I, xi. 8(c-d).
2. See *BrSpSi*, x. 36; *SiŚi*, I, xi. 8.
3. In his commentary on *ŚiDVr*, xi. 16-17

when the star rises ; and the *astalagna* is that point of the ecliptic which rises when the star sets. Bhāskara II says : "On account of the motion of the celestial sphere, a heavenly body rises every day when its *udayalagna* rises and sets when its *astalagna* rises."[1] He further says : "When a planet sets in the west, the *lagna* which rises in the east is called the *astalagna*."[2]

We shall now define the terms *Udayārka* (or *Udaya-sūrya*) and *Astārka* (or *Asta-sūrya*). These terms are defined in relation to the rising or setting of a star. The *Udayārka* is the position of the Sun when a star rises heliacally; and the *Astārka* is the position of the Sun when a star sets heliacally.

Computation of Udayārka.

Method. Calculate the *lagna* ("rising point of the ecliptic") by taking the star's *udayalagna* for the Sun's longitude and assuming that the time elapsed since sunrise is equal to the star's *kālāṁśa-ghaṭīs* ("time-degrees in terms of *ghaṭīs*"). This *lagna* itself is the *Udayārka*. Obviously, *Udayārka* > star's *udayalagna*.

Computation of Astārka.

Method. Calculate the *lagna* by taking the star's *astalagna* for the Sun's longitude and the star's *kālāṁśa-ghaṭīs* for the time to elapse before sunrise, and add six signs to that *lagna*. Whatever is thus obtained is the *Astārka*.[3]

Now we shall prove two theorems relating to *Udayārka* and *Astārka*.

Theorem 1. If for a star *Astārka* > *Udayārka*, the star will never set.[4]

Proof. When Sun = *Udayārka*, the star rises heliacally. Thereafter as the Sun moves, the distance of the Sun from the star's *udayalagna* increases and the star remains visible. Since *Astārka* > *udayārka*, the same happens when Sun = *Astārka*. The star therefore does not set even when Sun = *Astārka*. The setting of the star is thus impossible in this case.

This case happens when the star's latitude is north and considerable, such that

star's *akṣadṛkkarma* > star's *kālāṁśa* (reduced to ecliptic arc).[5]

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1. *SiŚi*, I, vii. 9.
2. *SiŚi*, I, vii. 6-8 com.
3. For the above methods of calculating *Udayārka* and *Astārka*, see Bhaṭṭotpala's commentary on *KK*, II, vii. 8-10 or Bhāskara II's commentary on *SiŚi*, I, xi. 12-13.
4. See *infra*, vs. 21(c-d).
5. See Mallikārjuna Sūri's commentary on *SiDVṛ*, xi. 20.

For, in this case,

Udayārka = Star's polar longitude − *akṣadṛkkarma* + *kālāṁśa* (reduced to ecliptic arc)

< Star's polar longitude

and

Astārka = Star's polar longitude + *akṣadṛkkarma* − *kālāṁśa* (reduced to ecliptic arc)

> Star's polar longitude,

so that

Astārka > Star's polar longitude > *Udayārka*.

Theorem 2. If for a star *Astārka* < *Udayārka*, then the star will rise and also set. [The star will remain set when *Astārka* < Sun < *Udayārka* and will remain visible when Sun < *Astārkā* but > *Udayārka*.]

Proof. When Sun = *Astārka*, the star sets heliacally. As the Sun's longitude increases, the distance between the Sun and the star's *astalagna* diminishes and the star remains heliacally set. This happens until Sun = *Udayārka*, when the star rises heliacally.

Hence in this case the star remains set until the Sun lies between *Astārka* and *Udayārka*, i. e., until

Astārka < Sun < *Udayārka*,

and when the Sun goes beyond this limit it is heliacally visible.

This case happens when the star's latitude is north and star's *akṣadṛkkarma* < star's *kālaṁśa* (reduced to ecliptic arc). For, in this case,

Udayārka = Star's polar longitude − *akṣadṛkkarma* + *kālāṁśa* (reduced to ecliptic arc)

> Star's polar longitude

and

Astārka = Star's polar longitude + *akṣadṛkkarma* − *kālāṁśa* (reduced to ecliptic arc)

< Star's polar longitude,

so that

Astārka < Star's polar longitude < *Udayārka*.

This case happens also when the star's latitude is south. For then

Udayārka = Star's polar longitude + *akṣadṛkkarma* + *kālāṁśa* (reduced to ecliptic arc)

> Star's polar longitude

and

Astārka = Star's polar longitude − *akṣadṛkkarma* − *kālāṁśa* (reduced to ecliptic arc)

< Star's polar longitude,

so that

Astārka < Star's polar longitude < *Udayārka.*

DAYS OF STAR'S INVISIBILITY

21(a-b). Subtract the *Astārka* (i. e., the longitude of the Sun at the time of the star's heliacal setting) from the *Udayārka* (i. e., the longitude of the Sun at the time of the star's heliacal rising) and reduce the difference to minutes. Divide those minutes by the minutes of the Sun's daily motion. Then are obtained the number of days during which the star remains set heliacally.[1]

As shown above, a star remains heliacally set until

Astārka < Sun < *Udayārka.*

Hence the above rule.

HELIACALLY EVER-VISIBLE STARS

21(c-d). If (for a star) the *Astārka* is greater than the *Udayārka*, then the star does not set heliacally (and is always visible).[2]

This has been already proved. See Theorem 1 above.

According to the *Sūrya-siddhānta*,[3] *Abhijit* (lat. 64° N), Brahmahṛdaya (lat. 30° N), Svātī (lat. 37° N), Śravaṇa (lat. 30° N), Dhaniṣṭhā (lat. 36° N) and Uttara-Bhādrapadā (lat. 26° N) are the stars which remain permanently visible heliacally, because of their large celestial latitudes.

1. *Cf. BrSpSi*, x. 39; *KK*, II, v. 12; *ŚiDVṛ*, xi. 19; *SiŚe*, xii. 19.
2. *Cf. BrSpSi*, x. 38(c-d); *KK*, II, v. 11(c-d); *ŚiDVṛ*, xi. 20(a-b); *SiŚe*, xii. 21(a-b); *SiŚi*, I, xi. 15.
3. ix. 11.

CIRCUMPOLAR OR ALWAYS VISIBLE OR INVISIBLE STARS

Lalla and Bhāskara II give the condition for stars being circumpolar. Lalla says :

"A star (of southern declination) whose declination corrected for its celestial latitude and the latitude of the place exceeds 90° is not visible there."[1]

Bhāskara II is more explicit. He says :

"The stars for which the true declination of the northern direction exceeds the colatitude (of the local place) remain permanently visible (at that place). And the stars such as Sirius and Canopus etc. for which the true declination of the southern direction exceeds the colatitude (of the place) remain permanently invisible (at that place)."[2]

Thus if ϕ be the latitude of a place and δ the true north declination of a star, then that star will be permanently visible at that place if

$$\delta > 90° - \phi.$$

And if ϕ be the latitude of a place and δ the true south declination of a star, then that star will be invisible at that place if

$$\delta > 90° - \phi.$$

Brahmagupta has made a similar statement for the Sun. He says :

"When the Sun is in the three signs beginning with Aries, it ceases to set wherever the Rsine of the local latitude equals the radius of the Sun's diurnal circle, or, (what is the same thing), the colatitude equals the Sun's declination, and it remains continuously visible until the same situation arises when the Sun is in the three signs beginning with Cancer. Similarly, when the Sun is in the three signs beginning with Libra, it ceases to rise wherever the Rsine of the local latitude equals the radius of the Sun's diurnal circle, or the colatitude equals the Sun's (southern) declination, and it remains continuously invisible until the same situation arises when the Sun is in the three signs beginning with Capricorn. The difference between the Sun's longitude when it ceases to set or rise and the Sun's longitude when

1. *ŚiDVr*, xi. 20(c-d). 2. *SiŚi*, I, xi. 16.

it again begins to set or rise, when divided by the Sun's daily motion, gives the days of the Sun's continuous visibility or invisibility."[1]

MEAN DECLINATION AND TRUE DECLINATION

22(a-b). When the sum of the ascensional differences (corresponding to mean declination and true declination) is taken, the direction (of mean declination differs) from that of true declination; when their difference is taken, the same direction is to be understood (for mean declination and true declination).[2]

HELIACAL RISING AND SETTING OF CANOPUS

One view

22(c-d). According to some astronomers, Canopus rises heliacally when the Sun is in the sign Leo at a distance equal to the latitude of the place.

That is, Canopus rises heliacally when

$$\text{Sun's longitude} = 120^\circ + \phi,$$

and likewise sets heliacally when

$$\text{Sun's longitude} = 180^\circ - (120^\circ + \phi) = 60^\circ - \phi,$$

where ϕ is the latitude of the place.

This view was held by Varāhamihira and Sumati, and probably also by Āryabhaṭa I.

Varāhamihira says :[3]

"Canopus sets heliacally when the Sun's longitude is equal to 2 signs diminished by the latitude of the place, and rises heliacally when the Sun's longitude is equal to 6 signs minus that."

1. *BrSpSi*, xv. 55-56. Also see *SiŚe*, iv. 118.
2. The reader is referred to the rule stated above in vss. 21-23(a) of chap. VI which involves addition or subtraction of the ascensional differences corresponding to the mean and true declinations.
3. See *BṛSaṁ*.

According to Sumati,[1] Canopus rises when

$$\text{Sun's longitude} = \frac{235\,\phi}{43}\ \text{degrees},$$

and sets when

$$\text{Sun's longitude} = \frac{11\,(90^\circ - \phi)}{21}\ \text{degrees},$$

where ϕ denotes the latitude of the place in terms of degrees.

This too is in agrement with the above, because according to Sumati $\phi = 27^\circ$, so that according to him Canopus rises when

$$\text{Sun's longitude} = \frac{192 \times 27}{43} + \phi$$

$$= 120 + \phi \text{ degrees, approx,}$$

and sets when

$$\text{Sun's longitude} = \frac{990 + 10 \times 27}{21} - \phi$$

$$= 60 - \phi \text{ degrees.}$$

The last verse of Chapter VI of the *Khaṇḍakhādyaka*, Part 1, (which Bhaṭṭotpala has excluded from the text on the ground that it does not yield accurate result), is:

राशिचतुष्केण यदा स्वाक्षांशयुतेन भवति तुल्योऽर्कः ।
उदयोऽगस्त्यस्य तदा चक्रार्धच्छिोधितेऽस्तमयः ॥

i. e., "Canopus rises when the Sun's longitude is equal to 4 signs plus the latitude of the place, and sets when the Sun's longitude amounts to half a circle minus that."

This also expresses the same view as held by the astronomers named above.

Since the above-mentioned verse is in agreement with Sumati's teachings, which were based on the old *Sūrya-siddhānta*, there is no justification

1. See *SMT*.

to exclude it from the text of the *Khaṇḍakhādyaka*. The *Khaṇḍakhādyaka* after all being a summary of the *Āryabhaṭa-siddhānta* belongs to the *Old Sūrya-siddhānta* school.

The above-mentioned view has been cited by Lalla also, who says:[1]

"Some other (astronomers) say that Canopus sets heliacally when the Sun's longitude is equal to two signs minus the latitude of the place; and that it rises heliacally when the Sun's longitude amounts to six signs minus that."

According to Mallikārjuna Sūri these other astronomers referred to by Lalla were "some of the pupils of Āryabhaṭa I".

Another view

23-24. Other astronomers hold the view that Canopus becomes invisible or visible according as the Sun's longitude is equal to 76° or 98°, respectively diminished and increased by the result obtained on multiplying the equinoctial midday shadow by 42 and dividing by 5.

That is, Canopus rises heliacally when

$$\text{Sun's longitude} = 98^\circ + \frac{42\,P}{5} \text{ degrees}$$

and sets heliacally when

$$\text{Sun's longitude} = 76^\circ - \frac{42\,P}{5} \text{ degrees,}$$

where P stands for the equinoctial midday shadow in terms of *aṅgulas*.

This view was held by Mañjula, Bhāskara II and Gaṇeśa Daivajña.

According to Mañjula,[2] Canopus rises heliacally when

$$\text{Sun's longitude} = 97^\circ + 8\,P \text{ degrees}$$

and sets heliacally when

$$\text{Sun's longitude} = 77^\circ - 8\,P \text{ degrees;}$$

1. See *ŚiDVr*, xi. 21. Also see *KPr*, vii. 8.

2. See *LMā*, iv. 4.

and according to Bhāskara II[1] and Gaṇeśa Daivajña,[2] Canopus rises heliacally when

$$\text{Sun's longitude} = 98^\circ + 8\,P \text{ degrees}$$

and sets heliacally when

$$\text{Sun's longitude} = 78^\circ - 8\,P \text{ degrees.}$$

The above rules have been derived by substitution from the following formulae:[3]

Udayārka = Star's polar longitude + *akṣadṛkkarma* + *kālāṁśa*

Astārka = Star's polar longitude − *akṣadṛkkarma* − *kālāṁśa*, approx.

SHADOW ETC. OF PLANETS AND STARS

25. One should find the shadow etc. of the planets and (the junction-stars of) the *nakṣatras* as in the case of the Moon according to the methods taught in the chapter on "Three Problems".

The stars which are not mentioned here should be determined by making all possible efforts.

PLANETS' *SAṄKRĀNTI*

26. Some astronomers have dealt with the time of a planet's *saṅkrānti* (i.e., the time when a planet goes from one sign to the next). In cases where calculation agrees with observation, the time obtained by calculation is correct; in cases where it is not so, there is error.

REVOLUTIONS OF THE SEVEN SAGES

27. (According to some astronomers) the Seven Sages, starting at the beginning of Kaliyuga, make a complete round of the *nakṣatras* in every 2700 years; according to others, the *nakṣatras* traversed by the Seven Sages are obtained by diminishing the elapsed years of Kaliyuga by 14 and dividing the remainder by 100.

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1. See *KKu*, vi. 15.
2. See *GL*, ix. 22.
3. See above under vss. 19(c-d)-20.

28. First Marīci, then Vasiṣṭha, then Aṅgirā, then Atri, then Pulastya, then Pulaha and then Kratu — in this order of succession the Sages move through the (twenty eight) *nakṣatras* one after the other.[1]

The constellation of the Seven Sages (now known as the Ursa Major or the Great Bear) is composed of seven stars which, stated in the east-to-west order, are Marīci, Vasiṣṭha, Aṅgirā, Atri, Pulastya, Pulaha and Kratu.[2]

The statement that the Seven Sages make a complete round of the *nakṣatras* in 2700 years amounts to saying that the Seven Sages perform 1600 revolutions in a *yuga*. This is contradictory to Vaṭeśvara's earlier statement[3] that the Seven Sages make 1692 revolutions in a *yuga*. It must therefore be understood that Vaṭeśvara is simply giving here the views of certain other astronomers.

It is, however, true that according to Vaṭeśvara the Seven Sages were in the beginning of the asterism Aśvinī at the commencement of Kaliyuga. For a rule given by him in his *Karaṇasāra* for finding the position of the Seven Sages is based on this assumption [4]

According to the author of the *Śākalya-saṁhitā*, the westernmost star Kratu of the constellation of the Seven Sages was at the first point of the asterism Aśvinī in the beginning of the current *yuga*. He says :

"In the beginning of the *yuga*, the (westernmost) star Kratu of the constellation of Viṣṇu (i.e., the constellation of the Seven Sages) occupied the initial point of the cycle of the *nakṣatras*."

Kamalākara has adopted this view by including the above passage of the *Śākalya-saṁhitā* in his own work, the *Siddhānta-tattva-viveka*.[5]

The second view is indeed due to Lalla, who says : "Subtract 14 from the years elapsed since the beginning of Kaliyuga and divide the remainder

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1. *Cf. BṛSaṁ*, xiii. 5-6(a-b).
2. See *BṛSaṁ*, xiii. 5-6(a-b). Also see *SiTVi*, xi. 26-27; *SuSi*, I, viii. 13.
3. *Vide supra*, chap. I, sec. 1, vs. 15.
4. See *supra*, chap. I, sec. 1, vs. 15 footnote.
5. See *SiTVi*, xi. 26.

by 100. The quotient, the learned say, gives the *nakṣatras* Rohiṇī etc. traversed by the Seven Sages, Marīci etc., who are the ornaments of the sky."[1]

According to this view, the Seven Sages had entered the fourth *nakṣatra* Rohiṇī 14 years after the beginning of Kaliyuga (i.e., in 3088 B.C.); and likewise they were in the tenth *nakṣatra* Maghā from 2488 B. C. to 2388 B.C. So during the reign of King Yudhiṣṭhira which, according to Kalhaṇa[2] (the author of the *Rājataraṅgiṇī*), started in 2449 B.C., the Seven Sages were in the *nakṣatra* Maghā. Kalhaṇa says :

"During the reign of Yudhiṣṭhira the Sages were in (the *nakṣatra*) Maghā. In the beginning of the Śaka Era, 2526 years had passed since he assumed kingship."[3]

There are still others who hold the view that the Seven Sages had entered the *nakṣatra* Kṛttikā, the first *nakṣatra* of the Vedic *nakṣatra* cycle, on the first *tithi* of the light half of Caitra when 25 years of Kaliyuga had passed. This view is the basis of the Saptarṣi Era (also called the *Laukika-kāla*) which was started in Kashmir 25 years after the beginning of Kali-yuga. There was a popular saying in Kashmir which ran : "When 25 years of Kaliyuga had passed, the (Seven) Sages had entered Kṛttikā, the *nakṣatra* presided over by Agni. The wise (astronomers) have adopted it as the beginning of the Saptarṣi Era in their *Saṁvatsara-patrikās* issued for popular use."[4] But according to this view the Seven Sages were in Āśleṣā (and not in Maghā) in the time of Yudhiṣṭhira.[5]

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1. *ŚiDVṛ*, I, xi, 22.

2. See *RājT*, i. 51 which reads: "The Kauravas and the Pāṇḍavas flourished 653 years after the beginning of Kaliyuga."

3. *RājT*, i. 56. Also see *BṛSaṁ*, xiii. 3.

4. कलेर्गतैः सायकनेत्र (25) वर्षैराग्नेयमृक्षं मुनयः प्रयाताः ।
लोके हि सम्वत्सरपत्रिकायां सप्तर्षिमानं प्रवदन्ति सन्तः ॥
This verse occurs in a manuscript (Acc. No. 1663) belonging to the Akhila Bharatiya Sanskrit Parishad, Lucknow.

5. If, however, we are to understand that 25 years after the beginning of Kaliyuga the Seven Sages had reached the last point of the *nakṣatra* Kṛttikā, then indeed they were in the *nakṣatra* Maghā in the time of Yudhiṣṭhira.

The time of Yudhiṣṭhira, however, is controversial, for, according to the *Mahābhārata*,[1] the Mahābhārata War took place towards the end of Dvāpara and the beginning of Kaliyuga and according to the *Bhāgavata-Purāṇa*,[2] Yudhiṣṭhira proceeded on the last journey when he came to know that Kaliyuga had commenced.

ARUNDHATĪ OR MIZAR

29. (To the north of Vasiṣṭha there is) a dim star known after the name of Arundhatī, the devoted wife of Sage Vasiṣṭha, the mother of the world;[3] those who catch sight of it become free from sins and go to the *Grahaloka* ("the planetary world").

CONCLUSION

30-31. Mean and true motion of the planets, three problems, eclipses, rising of the planets, rising of the Moon, conjunction of two planets, and conjunction of star and planet — all these topics have been treated in this book giving the various alternatives. One should read them with devotion, for one who is proficient in (*Graha*) *Gaṇita* (mathematical astronomy) and Gola (spherics) acquires great prosperity and glory.

1. *Ādiparva*, ch. 2, vs. 13.
2. *Skandha* 1, ch. 15, vs. 37.
3. *Cf. BṛSaṁ*, xiii, 6(c-d).

VAṬEŚVARA'S GOLA

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TRANSLATION AND NOTES

Chapter I

Appreciation of *Gola* or Spherics

INTRODUCTION

1. As one cannot have proper knowledge of the various celestial motions of the planets, such as the mean motion and so on, without spherics, so I proceed to compose methodically a treatise on spherics aiming at the exposition of the desired subject (of spherics).

APPRECIATION OF SPHERICS

2. No astronomical text-book (is complete) without a section on spherics, just as the chest of a woman without breasts (is devoid of charm), the night without the Moon is not (lovely), and a meal without milk, sugar and clarified butter (is not enjoyable).[1]

3. Fie upon the disputant who is ignorant of grammar, upon the physician who is incompetent in his profession, upon the priest who has not learnt how to recite the Vedas loudly, and upon the astronomer who has not learnt spherics for fear of the labour involved in it.[2]

4. One who has studied mathematics knows fully well the science of spherics and one who has studied the science of spherics knows the motion of the heavenly bodies (too). But one who is ignorant of mathematics as well as spherics does by no means know the motion of the planets.[3]

5. One who demonstrates the motions, the mean and so on, (of the planets) as if submitted to the eye possesses true knowledge of the science of spherics and is regarded as Ācārya amongst the learned (astronomers).[4]

SPHERICS AND WHAT IT TEACHES

6. "From this (science of spherics) one learns and understands the celestial sphere" —this is how the learned (scholars) explain the meaning

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1. *Cf. LG* (=Lalla's *Gola*), i. 2; *SiŚe*, xv. 2; *SiŚi*, II, i. 3.
2. *Cf. LG*, i. 3; *SiŚe*, xv. 3; *SiŚi*, II, i. 4.
3. *Cf. BrSpSi*, xxii. 3; *LG*, i. 4; *SiŚe*, xv. 4; *SiŚi*, II, i. 5-6.
4. *Cf. BrSpSi*, xxii. 1; *LG*, i. 5-6; *SiŚe*, xv. 5.

of the term *Gola* ("spherics"). If one asks: What is it that one learns from this (science)? the answer is: One learns about the realities (of astronomy) such as the positions of the planets, Earth and the stars and so on with the help of unreal things. This may be explained as follows: Just as the physicians learn (surgery) by dissecting the nerves of the lotus-stalk etc., the priests learn the sacrificial rites etc. by means of fire altars constructed with dry bricks etc., the grammarians learn correct words by means of *rūpa*, *sarga*, *āgama*, *pratyaya* and *aṅga* etc., in the same way the astronomers learn the positions and the distances (lit. hypotenuses) of the planets and stars by means of arc, Rsine, Rversed-sine, base, upright, hypotenuse and perpendicular, quadrilateral, triangular and rectangular figures and thread etc.

AIM OF THE PRESENT WORK

7. This treatise on spherics is being attempted because one who desires to deal with the entire subject of astronomy cannot accomplish that without the treatment of spherics and one should not fail to write fully on the practical aspect of the subject.

Chapter II

Graphical Demonstration of Planetary Motion through Eccentrics and Epicycles

1. Draw the planet's own orbit or *kakṣyāvṛtta* with radius equal to the semi-diameter of the planet's orbit and graduate it with the divisions of signs, degrees and minutes. At the centre thereof imagine the Earth, the Earth which is capable of supporting all, men and others.[1]

2. From the centre of the Earth towards the planet's own *ucca* stretch a thread of length equal to the Rsine of the planet's maximum correction, and at the extremity thereof draw (a circle equal to) the planet's *kakṣyāvṛtta*.[2]

3-4(a-b). This circle is known as *pratimaṇḍala*, *kendravṛtta* or *nirakṣavṛtta* (eccentric). (In the case of *manda-prativṛtta*) the true planet, starting from the *mandocca*, traverses it anticlockwise with its true (geocentric angular) velocity. (In the case of *śīghra-pratimaṇḍala*) the planet, starting from its *śīghrocca*, traverses it clockwise with the same motion as it has in its *kakṣyāvṛtta*.[3]

4(c-d). Next, draw the planet's *nīcoccavṛtta* (epicycle) with its centre at the planet in the *kakṣyāvṛtta*. (In the case of *manda-nīcocca-vṛtta*, the planet traverses it clockwise starting from the planet's *mando-cca*; and in the case of *śīghra-nīcoccavṛtta*, the planet traverses it anti-clockwise starting from the planet's *śīghrocca*.)

5. In case the planet traverses the *prativṛtta* clockwise, its true velocity is greatest when it is at the *ucca*; and in case it traverses the *prativṛtta* anticlockwise, its true velocity is least at the *ucca*. Its motion in the two circles, (*kakṣyāvṛtta* and *prativṛtta*), is always the same, in terms of *yojanas*.[4]

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1. *Cf. LG*, i. 7; *SiŚe*, xvi. 1; *SiŚi*, II, v. 10.
2. *Cf. LG*, i. 8; *SiŚe*, xvi. 2; *SiŚi*, II, v. 12; *SuSi*, II, iii. 7.
3. *Cf. LG*, i. 12(a-b), 13(a-b); *BrSpSi*, xxi. 24(c-d); *SiŚe*, xvi. 5(c-d); *SiŚi*, II, v. 13 (d), 30.
4. *Cf. BrSpSi*, xxi. 25(c-d).

6. When the mean planet is equal to its apogee (*tuṅga or ucca*) the planet is at the apogee of its orbit. Similarly, when the mean planet is six signs greater than that the planet is at the perigee (*nīca*).

Since a planet is sometimes distant and sometimes near from the Earth, depending on the length of the (planet's) hypotenuse, therefore the planet is sometimes said to be small and sometimes large.[1]

7. When a planet lies at the intersection of the *kakṣyāvṛtta* and *mandaprativṛtta*, its mean motion itself is its true motion. When a planet is at its *mandocca* or *mandanīca*, its mean position is the same as its true position.[2]

8. Since (in other positions of the planet) one sees the true planet behind the mean planet or in advance of it, therefore the minutes of arc intervening between the two, which constitute the *mandaphala* (i.e., equation of the centre), are respectively applied as a negative or positive correction to the mean planet (to get the true planet). The *śīghra-phala* is applied contrarily.[3]

9. In the half-orbit beginning with the anomalistic sign Cancer, the *koṭijīvā* (=Rcosine of anomaly) lies below the *paramaphalajyā* (=Rsine of the maximum correction); and in the half-orbit beginning with the anomalistic sign Capricorn, (the *koṭijīvā* lies) above (the *paramaphalajyā*). Their difference or sum is therefore the upright (*agrakā* or *koṭi*) (in the two cases, respectively). The square-root of the sum of the squares of that (upright) and the *bhujajyā* (=Rsine of anomaly) is the hypotenuse.[4]

10. In the half-orbit beginning with the anomalistic sign Capricorn, the *koṭiphala* lies above the radius; and in the half-orbit beginning with the anomalistic sign Cancer, (the *koṭiphala* lies) below (the radius). It is for this reason that the *koṭiphala* of a planet is added to or subtracted from the radius (in the two cases, respectively) (to obtain the upright).

1. *Cf. LG*, i. 9(c-d), 10, 11(a-b); *SiŚe*, xvi. 3-4; *SiŚi*, II, v. 21-22; *SuSi*, II, ii. 14.
2. *Cf. LG*, i. 13(c-d)-14(a-b); *SiŚe*, xvi 6(c-d). The first half of the verse has been criticized by Bhāskara II; see *SiŚi*, II, v. 39 and Bhāskara II's com. on it.
3. *Cf. LG*, i. 15; *BrSpSi*, xxi. 26; *SiŚe*, xvi. 8.
4. *Cf. LG*, i. 16; *MSi*, iii. 24; *SiŚe*, xvi. 18; *SiŚi*, II, v. 15-16(a-b); *SuSi*, II, iii. 13.

The hypotenuse is then obtained from tbe *bhujaphala* and the upright in the manner stated before.[1]

11. If one obtains the *bhujaphala* in the *kakṣyāvṛtta* from the radius, what then should one get from the hypotenuse ? Since this crooked proportion is inverse, that is why one gets smaller *śīghraphala* when the hypotenuse is larger (than the radius) and larger *śīghraphala* when the hypotenuse in smaller (than the radius).[2]

12. By proceeding according to the (prescribed) method of the *mandakarma* (in which the *mandakarṇa* is obtained by iteration) one arrives at accuracy in the resulting longitude of a planet, and by using that longitude of the planet one arrives at accuracy in the planet's motion, and further there is agreement between computation and observation when use is made of the iterated *mandakarṇa*. This is why non-iteration is not prescribed (under the Indian eccentric or epicyclic theory) for finding the hypotenuse in the case of *mandakarma*.

13. The use of (proportion with) the hypotenuse in finding the *mandaphala* is not made (under the Indian eccentric or epicyclic theory) because (when proportion with the hypotenuse is not made) the velocity of a planet begins to increase from the planet's *mandocca* (as it should be), the true position and velocity of a planet come out to be accurate as before, and there is certainly agreement between computed and observed positions.

The determination of the *mandakarṇa* by iteration and the omission of proportion with the hypotenuse in the computation of the *mandaphala* ("the equation of the centre") are related problems.

The Indian astronomers believe that the *manda* epicycles, stated in the Indian works on astronomy, are their mean values corresponding to the mean distances of the planets. To obtain their true values corresponding to the true distances of the planets it is necessary to obtain the true distances of the planets which can be obtained by iteration only.

Further, since in finding the *mandaphala* the Indian astronomers use the mean values of the *manda* epicycles and not the true values, proportion with the hypotenuse is not made.

1. See *VSi*, ch. II, sec. 2, vs. 3. *Cf. LG*, i. 17; *BrSpSi*, xxi. 27; *MSi*, iii. 25; *SiŚe*, xvi. 19; *SiŚi*, II, v. 28(c-d)-29(a-b); *SuSi*, II, iii. 17.
2. For a similar rule see *SuSi*, II, iii. 18(a-b).

For details on this topic, the reader is referred to my paper entitled "Use of hypotenuse in the computation of the equation of the centre under the epicyclic theory in the school of Āryabhaṭa," published in the *Indian Journal of History of Science*, vol. 8, nos. 1 and 2, 1973, pp. 43-57.

In the opinion of Vaṭeśvara, iteration of the hypotenuse in the case of *mandakarma* and computation of the *mandaphala* without applying proportion with the hypotenuse lead to accurate results.

14. Since in the *mandakarma* (under the Indian eccentric or epicyclic theory) the hypotenuse of a planet, i e., the distance between the Earth and the planet, is obtained by the process of iteration, that is why (in order to find the mean longitude of a planet from its true longitude) one reversely applies, again and again, the correction due to the *ucca* to the longitude of the (true) planet.

15. The distance between the Earth and the planet is called here (in Indian astronomy) by the term "hypotenuse". So whatever is the (angular) distance between the hypotenuse and the mean planet is the correction. When the true planet is ahead of the mean planet, this correction is added to the longitude of the mean planet; and when the true planet is behind the mean planet, this correction is subtracted from the longitude of the mean planet.

Chapter III

Construction of the Armillary Sphere

1. *KHAGOLA* OR SPHERE OF THE SKY

1. The vertical circle passing through the west and east cardinal points is the first circle : this is called the *samamaṇḍala* or the prime vertical. Another similar (vertical) circle (called the *yāmyottara-vṛtta* or the meridian) passes through the north and south cardinal points. Two (vertical) circles (called the *vidig-vṛtta*) similarly pass through the intermediate cardinal points (viz. north-east and south-west, north-west and south-east points).[1]

2. The great circle which goes round them, dividing each of them into two equal parts, is called "*harija*" or "*kṣitija*" (horizon). This is the circle on which the rising and setting of the stars and planets take place towards the east and west, respectively.[2]

3. Passing through the two points of intersection of the prime vertical and the horizon, lying below the south cardinal point by the degrees of the local latitude, fastened to the horizon, and lying above the north cardinal point, passing through the north celestial pole, is the *unmaṇḍala* ("the six o'clock circle"), the cause of decrease and increase of the day and night.[3]

4. The vertical circle which goes through the planet is the *dṛṅmaṇḍala* ("the planet's vertical circle"). The vertical circle that passes through the central ecliptic point which lies three signs behind the *vilagna* or the rising point of the ecliptic is the *dṛkkṣepavṛtta*.

These (above-mentioned) eight circles which are graduated by the divisions of signs and degrees lie on the *Khagola* or "Sphere of the sky".[4]

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1. *Cf. LG*, ii. 1; *Ā*, iv. 18(a-b); *BrSpSi*, xxi. 49; *SiŚe*, xvi. 29; *SiŚi*, II, vi. 3 (a-b).
2. *Cf. LG*, ii. 2; *Ā*, iv. 18(c-d); *SiŚe*, xvi. 29(d); *SiŚi*, II, vi. 3(c-d), vii. 2(c-d).
3. *Cf. LG*, ii. 3; *Ā*, iv. 19; *BrSpSi*, xxi. 50; *SiŚe*, xvi. 30; *SiŚi*, II, vi. 4; *SuSi*, II, iv. 4.
4. *Cf. Ā*, iv. 21; *SiŚe*, xvi. 37; *SiŚi*, II, vi. 6-7; *SuSi*, II, iv. 14.

It may be mentioned that the centre of the sphere of the sky lies at the observer.

Lalla[1] has mentioned only the following six great circles as lying on the sphere of the sky: (1) the prime vertical, (2) the meridian, (3 and 4) two vertical circles through the intermediate cardinal points, (5) the horizon, and (6) the six o'clock circle.

2. *BHAGOLA* OR SPHERE OF THE ASTERISMS

5. The sphere of the asterisms lies within the sphere of the sky. The great circle (of the sphere of the asterisms) which lies towards the south of the zenith by an amount equal to the degrees of the (local) latitude and towards the north of the nadir by the same amount and which is graduated with the divisions of *nāḍīs* is called the *viṣuvad-vṛtta* or the equator.[2]

6. Surrounding it on all sides like the horizon is another great circle of this sphere called the meridian. Fastened to the north and south poles of that (equator) is the polar axis which is fixed in position. At the centre of the sphere of the asterisms lies the Earth.[3]

7. Fastened to the so called *nāḍīvṛtta* or the equator at the first points of Aries and Libra and lying 24 degrees to the south (of the equator) at the first point of Capricorn and 24 degrees to the north (of the equator) at the first point of Cancer, is the great circle called the *apakrama-vṛtta* or the ecliptic.[4]

8. The Sun moves incessantly on this circle; so does the Earth's shadow at a distance of half a circle from the Sun; and so do also the nodes of the planets, in the opposite sense. The Moon etc. move on their own orbits (called *vimaṇḍala*).[5]

9. One half of the planet's orbit (*vimaṇḍala*) beginning with the so called *pāta* or ascending node lies inclined to the north (of the ecliptic)

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1. See *ŚiDVṛ*, II, ii. 1-4.
2. *Cf. LG*, ii. 5; *SiŚe*, xvi. 31; *SiŚi*, II, vi. 10(c-d); *SuSi*, II, iv. 3.
3. *Cf. LG*, ii. 6; *SiŚi*, II, vi. 10(a-b).
4. *Cf. LG*, ii. 7; *Ā*, iv. 1; *BrSpSi*, xxi. 52; *SiŚe*, xvi. 32; *SiŚi*, II, vi. 12; *SūSi*, II, iv. 6 (a-b).
5. *Cf. LG*, ii. 8; *Ā*, iv. 2; *BrSpSi*, xxi. 53; *SiŚe*, xvi. 33, 35; *SiŚi*, II, vi. 11.

by the degrees of its greatest celestial latitude; and the second half of the planet's orbit beginning with its descending node (lit. ascending node plus six signs) lies inclined to the south (of the ecliptic) by the degrees of its greatest celestial latitude.[1]

10. Displaced (northwards) from the equator by the degrees of their declinations there are three diurnal circles corresponding to the end-points of the (first three) signs, Aries etc.; the same in the reverse order are those for the first points of (the next three signs), Cancer etc.; Similarly, (displaced southwards from the equator there are three diurnal circles) for the end points of the six signs, Libra etc. The diurnal circle for the given declination should be constructed at the distance of the given degrees of declination.[2]

3. *GRAHAGOLA* OR SPHERES OF THE PLANETS

11. In the plane of the equator (of the sphere of the asterisms) fix a circle equal to the planet's orbit (*kakṣyāvṛtta*): this is the equator in the sphere of the planet. Similarly, fix the meridian and also another circle in the plane of the horizon (each equal to the planet's orbit). Similarly, fix the ecliptic; and in this (ecliptic) at the *kendra* (defined by the position of the mean planet), fix the (*manda*) epicycle in the manner stated before.[3]

12. (In this way) there are seven circles representing the (*manda*) eccentrics of the (seven) planets and (seven) circles representing the (*manda*) epicycles (of the seven planets). There are also ten circles due to the *śīghroccas* (viz. five *śīghra* eccentrics and five *śīghra* epicycles for the five star-planets, Mars etc.). This is how the spheres of the planets are constructed.[4]

13. The (small) circle which is fastened to the eastern and western halves of the six o'clock circle at the distance of the planet's declination from the equator and to the meridian at the distance of the degrees of the planet's meridian zenith distance from the zenith is called the circle of the planet's diurnal motion.[5]

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1. *Cf. LG*, ii. 9; *Ā*, iv. 3; *BrSpSi*, xxi. 54; *SiŚe*, xvi. 34; *SiŚi*, II, vi. 14.
2. *Cf. LG*, ii. 10-11; *BrSpSi*, xxi. 57-58; *SiŚe*, xvi. 36; *SuSi*, II, iv. 13.
3. *Cf. LG*, ii. 12-13; *SiŚi*, II, vi. 25(c-d).
4. *Cf. LG*, ii. 14.
5. *Cf. LG*, ii. 15.

The shadow (due to a planet) falls in the direction which is diametrically opposite to that of the planet.

4. *SĀMĀNYA GOLA* OR THE GENERAL CELESTIAL SPHERE

Definitions

14. The Rsine of the arc of the horizon lying between the prime vertical and the diurnal circle of the planet is the Rsine of the *agrā* of the rising point (of the planet); and the Rsine of the degrees of the diurnal circle lying between the six o'clock circle and the horizon is the *bhūjyā* or the earthsine.[1]

15. (The Rsine of) the arcual distance between these (viz. the six o'clock circle and the horizon), measured along the R-circle (*trijyāvṛtta*) or the great circle of the celestial sphere supposed to be of radius 3438′, is the *carādhajīvā* or the Rsine of the ascensional difference.[2] The thread tied to the extremities of the *agrā* on the eastern and western halves of the horizon (and stretched tightly between them) is called the *udayāsta-sūtra* or the rising-setting line for the planet.[3]

16. The point of intersection of the horizon and the ecliptic in the eastern half of the celestial sphere is called the *prāglagna* or the rising point of the ecliptic; the same in the western half is called the *astalagna* or the setting point of the ecliptic.[4] Counted from the rising sign the seventh one is the setting sign. The time of setting of that (seventh sign) is equivalent to the time of rising of this (rising sign); (and *vice versa*).[5]

17. Whether the heavenly body be on the prime vertical, on the intermediate vertical, on the *dṛkkṣepa-vṛtta*, on the meridian, or on any vertical circle, the distance (in degrees) between the heavenly body in the sky and the horizon gives the degrees of the altitude; and 90 minus those degrees give the degrees of the zenith distance. In all positions of the heavenly body, the Rsine of the altitude is called *nara* (*nṛ* or *śaṅku*) and the Rsine of the zenith distance is called *dṛgjyā*.[6]

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1. *Cf. LG*, ii. 18-19(a-b); *SiŚi*, II, vii. 39(a-b), 1.
2. *Cf. LG*, ii. 19(c-d); *SiŚi*, II, vii. 1(c-d).
3. *Cf. SiŚi*, II, vii. 39(c-d).
4. *Cf. LG*, ii. 20; *SiŚe*, xvi. 45; *SiŚi*, II, vii. 26.
5. *Cf. SiŚi*, II, 59(c-d); II, vii. 24.
6. *Cf. LG*. ii. 21-23.

18. At noon on the equinoctial day, the *nara* (i.e., the Rsine of the Sun's altitude) and the *dṛgjyā* (i. e., the Rsine of the Sun's zenith distance) are equal to the Rsines of colatitude and latitude (respectively).[1] The distance on the ground between the foot of the *nara* and the rising-setting line is the *narāgra* (*śaṅkvagra* or *śaṅkutala*).

19(a-b). Between the top of the *nara* and the rising-setting line lies the *dhṛti* qualified by the words "*sva*" or "*tat*."

Right-angled Triangles

19(c-d). The triangle in which the *nara* and *dṛgjyā* are the upright and base (respectively) and the radius is the hypotenuse is a right-angled triangle.

20. The triangle in which the base and upright are equal to the *śaṅkutala* and *śaṅku* (respectively) and the hypotenuse is equal to the *dhṛti* is another right-angled triangle. The triangle which has the *sama-śaṅku* for the upright, the *agrā* at rising for the base, and the *svadhṛti* for the hypotenuse is another right-angled triangle.

21. The triangle in which the base and upright are equal to the earthsine and the Rsine of declination (respectively) and the hypotenuse is equal to the *agrā* at the planet's rising is another right-angled triangle. The (triangular) figure which has the Rsine of declination and the Rsine of codeclination for the base and upright (respectively) and the radius for the hypotenuse is also said to be so.[2]

22. Another right-angled triangle is the one in which the earthsine is the base, one-half of the rising-setting line is the upright, and the Rsine of codeclination is the hypotenuse. Still another is the one in which the *agrā* is the base, one-half of the rising-setting line is the upright, and the radius is the hypotenuse.

23. Still another is the one which has the shadow (of the gnomon) for the base, the gnomon for the upright, and the hypotenuse of shadow for the hypotenuse. Hundreds of such right-angled triangles may be contemplated by those whose intellect has been purified by the knowledge of spherics.

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1. *Cf. LG*, ii. 24; *SiŚe*, xvi. 44; *SiŚi*, II, iii. 12.
2. The two right-angled triangles stated here have been mentioned by Śrīpati also. See *SiŚe*, xvi. 48, 49(c-d).

Denoting the altitude, zenith distance and declination of the Sun by *a*, *z* and δ respectively, the right-angled triangles mentioned above may be briefly described as follows:

	Base	Upright	Hypotenuse
(1)	Rsin *a*	Rsin *z*	R
(2)	*śaṅkutala*	*śaṅku*	*dhṛti*
(3)	*agrā*	*samaśaṅku*	*svadhṛti*
(4)	earthsine	Rsin δ	*agrā*
(5)	Rsin δ	Rcos δ	R
(6)	earthsine	½(rising-setting line)	Rcos δ
(7)	*agrā*	½(rising-setting line)	R
(8)	shadow	gnomon	hypotenuse of shadow

Of these right-angled triangles, (2), (3), (4) and (8) are known as *akṣakṣetras* ("latitude-triangles") and (5) is known as *krānti-kṣetra* ("declination-triangle").

Āryabhaṭa II and Bhāskara II have also given lists of latitude-triangles.[1]

The *Lambana* Triangle

24. (In one triangle) the *dṛgjyā* and the *madhyajyā* are the lateral sides, and the sum of the smaller earth-segment and the larger earth-segment is the base. (In two triangles) the segments of the base (viz. the smaller earth-segment and the larger earth-segment) are the bases, and the *dṛkkṣepa* is the altitude.

(In the *lambana* triangle) the *dṛkkṣepa* becomes the *avanati* (i.e., parallax in latitude) and the larger earth-segment becomes the east-west *lambana* (i.e., parallax in longitude).

On the horizon, this *dṛkkṣepa* (of the *lambana* triangle) amounts to half the diameter of the Earth; and when the planet is on the horizon (and the ecliptic is vertical), the larger earth-segment, measured from the zenith, amounts to half the diameter of the Earth (in the *lambana* triangle). (What is meant is this: When the ecliptic coincides with the horizon, the *avanati* or parallax in latitude amounts to half the diameter

1. See *MSi*, iv. 4(c-d)-7; *SiŚi*, I, iii. 13-17.

of the Earth; and when the ecliptic is vertical and the planet is on the horizon, the *lambana* or parallax in longitude amounts to half the diameter of the Earth.)

Of the three triangles described in the first half of the above passage, two are based on the following relations :

$$(d\d{r}gjy\bar{a})^2 = (d\d{r}kk\d{s}epajy\bar{a})^2 + (\text{larger earth-segment})^2$$

$$(madhyajy\bar{a})^2 = (d\d{r}kk\d{s}epajy\bar{a})^2 + (\text{smaller earth-segment})^2.$$

The third one is formed by putting the other two in juxtaposition.

The *lambana* triangle is the right-angled triangle whose sides are : *lambana* ("parallax in longitude"), *avanati* ("parallax in latitude") and *dṛglambana* ("parallax in zenith distance"). This is supposed to be similar to the triangle whose corresponding sides are : larger earth-segment (or larger *dṛggati*), *dṛkkṣepajyā* and *dṛgjyā* ("Rsine of zenith distance").

At Laṅkā and Poles

25. For the people residing at Laṅkā, the (Sun's) *nara* at midday is equal to the day-radius and the Sun's *dṛgjyā* at midday is equal to the the Rsine of the degrees of the (Sun's) declination. For the gods and demons, the (Sun's) *dṛgjyā* is said to be equal to the day-radius and the (Sun's) *śaṅku*, equal to the Rsine of the degrees of the (Sun's) declination.[1]

This is evident because when $\phi = 0$, then at midday

$$a = 90 - \delta \text{ and } z = \delta$$

and when $\phi = 90°$, then

$$a = \delta \text{ and } z = 90 - \delta,$$

a, z and δ being the altitude, zenith distance and declination of the Sun, and ϕ the latitude of the place.

Fixed or Immovable Circles

26. The meridian, the prime vertical, the horizon, the six o'clock circle, and the equator — these five circles are fixed in the case of (the

1. *Cf. LG*, ii. 28.

spheres of) the asterisms and the planets; their dimensions are equal to their own orbits.[1]

Movable Circles

27-28. The seven *manda* epicycles (for the seven planets, Sun etc.); the five *śīghra* epicycles (for the five star-planets, Mars etc.); the same number of *manda* and *śīghra* eccentrics (for those planets); the *dṛkkṣepa*, vertical and declination circles, one each for the (seven) planets, Sun etc; and six *vimaṇḍalas* (for the six planets, Moon etc.)—these fifty-one circles in all are stated to be the movable circles (in the spheres) of the planets.[2]

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1. *Cf. LG*, ii. 30; *BrSpSi*, xxi. 67; *SiŚe*, xvi. 39(b-d); *SuSi*, II, iv. 16(a-b).
2. *Cf. LG*, ii. 31-32; *BrSpSi*, xxi. 68-69; *SiŚe*, xvi. 38-39(a).

Chapter IV

Spherical Rationale

MEAN MOTION

1. Since a civil day exceeds a sidereal day by as many *asus* as there are minutes in the Sun's daily motion, therefore the number of risings of a star (in a *yuga*) plus the number of revolutions of the Sun (in a *yuga*) is equal to the number of civil days (in a *yuga*).[1]

(1) Length of a civil day = length of a sidereal day $+ 59\frac{8}{60}$ *asus*

(2) Civil days in a *yuga*

= risings of a star in a *yuga* + revolutions of the Sun in a *yuga*

= sidereal days in a *yuga* + Sun's revolutions in a *yuga*.

Bhāskara II[2] has shown that formula (1) above gives the length of a mean civil day and has given the following formula for the length of a true civil day :

(3) length of a true civil day = length of a sidereal day

$$+\frac{(\text{Sun's daily motion}) \times (\text{oblique ascension of Sun's sign})}{1800}$$

$$= 60\ \textit{nāḍīs} + \frac{(59\tfrac{8}{60})' \times (\text{obl. asc. of Sun's sign})}{1800}\ \textit{asus}.$$

2. As one sees the conjunction of the Sun and Moon after an interval of one civil month diminished by 28 *nāḍikās* and 10 *palas*, so that interval is (called) a lunar month.[3]

1 lunar month = time-interval from one new moon to the next

= 1 civil month − 28 *nāḍīs* 10 *palas*

= 29 days 31 *nāḍīs* 50 *palas*.

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1. *Cf. LG*, iii. 1; *SiŚe*, xv. 63; *SiŚi*, II, iv. 5-8.
2. *SiŚi*, II, iv. 5-8.
3. *Cf. LG*, iii. 2; *SiŚe*, xv. 64; *SiŚi*, II, iv. 9.

3. Since the length of a civil month increased by 26 *nāḍīs* and 18 *palas* gives the length of a solar month and since this exceeds the length of a lunar month, that is why an intercalary month happens to fall after every 976 (civil) days.[1]

Since

1 solar month = 30 days 26 *nāḍīs* 18 *palas*

1 lunar month = 29 days 31 *nāḍīs* 50 *palas*

and their difference = 54 *nāḍīs* 28 *palas*,

therefore one intercalary month will fall after every

$$\frac{(29 \text{ days } 31 \textit{ nāḍīs } 50 \textit{ palas}) \times 30 \text{ days}}{54 \textit{ nāḍīs } 28 \textit{ palas}}$$

$$= 975 \frac{750}{817} \text{ or } 976 \text{ days, approx.}$$

4. And this is why the number of solar months (in a *yuga*) increased by the number of intercalary months (in a *yuga*) gives the number of lunar months (in a *yuga*). The number of lunar days (in a *yuga*) too, when diminished by the number of civil days (in a *yuga*), gives the number of omitted lunar days (in a *yuga*). Other things (pertaining to mean motion) may be explained similarly.[2]

PLANETARY CORRECTIONS

5. Since to the east of the prime meridian the Sun rises earlier and to the west of the prime meridian the Sun rises later, that is why the correction for the longitude (of the local place) is subtracted (if the place is to the east of the prime meridian) and added (if the place is to the west of the prime meridian). And it is for the same reason that the planetary positions for the epoch and the additive parameters are stated for the time corresponding to the former (i. e., for sunrise at Laṅkā).[3]

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1. *Cf. LG*, iii. 3-4.
2. *Cf. LG*, iii. 5; *SiŚe*, xv. 65-66; *SiŚi*, II, iv. 10-12.
3. *Cf. LG*, iii. 6; *MSi*, xvii. 61-62; *SiŚe*, xv. 67.

6. When the Sun's equation of the centre is subtractive, the true Sun rises earlier than the mean Sun by as many *asus* as correspond to the Sun's equation of the centre; and when the Sun's equation of the centre is additive, the true Sun rises later (than the mean Sun by as many *asus* as correspond to the Sun's equation of the centre). It is for this reason that the (*bhujāntara*) correction (for a planet) which is obtained by proportion (from the Sun's equation of the centre) is respectively subtracted from or added to the longitude of a planet.[1]

7. Since the times of rising and setting of the Sun depend on the Sun's declination and the six o'clock circle and since the six o'clock circle due to its passing through the polar axis lies above or below the horizon, hence the necessity of the correction for the Sun's ascensional difference.[2]

8-9. In the northern hemisphere, the horizon lies below the six o'clock circle; and in the southern hemisphere, the horizon lies above that (six o'clock circle). So, in the northern hemisphere, the Sun comes to sight earlier than it does at the equator by an amount of time equal to the Sun's ascensional difference and goes to set later (by the same amount of time); and in the southern hemisphere, it rises later (and sets earlier than it does at the equator by the same amount of time).[3] **It is for this reason that the (*cara*) correction, which is obtained by proportion from the *asus* of the Sun's ascensional difference and the planet's own daily motion, is subtracted from or added to the longitude of the planet (according as the Sun is in the northern or southern hemisphere). It is also for the same reason that the day and night are respectively of longer and shorter duration and of shorter and longer duration (in the northern and southern hemispheres).**[4]

10(a-c). At Laṅkā the six o'clock circle itself is the horizon, therefore the *cara*-correction does not exist there and likewise there is (always) equality of day and night.[5]

RIGHT ASCENSIONS OF THE SIGNS

10(d)-11. Although there is absence of the degrees of latitude at Laṅka, (the times of rising of the zodiacal signs are not the same).

1. *Cf. LG*, iii. 7; *SiŚe*, xvi. 23; *SiŚi*, II, v. 43.
2. *Cf. LG*, iii. 8.
3. *Cf. LG*, iii. 9-10(a-b); *SiŚe*, xvi. 25.
4. *Cf. LG*, iii. 10(c-d)-11; *SiŚe*, xvi. 28, 26.
5. *Cf. LG*, iii. 12; *SiŚe*, xvi. 27.

The signs Aries and Taurus, being inclined to the equator by their own declinations, rise in a shorter time, while the sign Gemini, lying towards the end of the quadrant, though of lesser declination, rises in a longer time, because of being (almost) parallel to the equator.[1]

The declination of a sign is obtained by taking the difference of the declination of the first and last points of that sign.

OBLIQUE ASCENSIONS OF THE SIGNS

12. In a place where the Rsine of latitude exists, the first and last quadrants of the circle of asterisms rise in one-fourth of a day diminished by the (corresponding) ascensional difference and the second and third (quadrants) rise in one-fourth of a day increased by the (corresponding) ascensional difference, directly and reversely.[2]

13. Since the six signs beginning with Capricorn are inclined northwards and the six signs beginning with Cancer are inclined southwards, therefore on account of the diurnal motion of the circle of asterisms the signs (beginning with Cancer and those beginning with Capricorn) take longer and shorter times to rise (at the local place than those they take at Laṅkā.)[3]

It is noteworthy that in Indian astronomy all places are supposed to be towards the north of the equator.

RISING AND SETTING OF THE SIGNS

14. The times of rising of the signs inclined northwards are the same as the times of rising of the signs inclined southwards; and the times of rising of the signs inclined southwards are the same as the times of setting of the signs inclined northwards.[4]

That is to say :

time of rising of Capricorn = time of setting of Cancer
time of rising of Aquarius = time of setting of Leo
time of rising of Pisces = time of setting of Virgo

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1. *Cf. LG*, iii. 13-14.
2. *Cf. LG*, iii. 15; *SiŚe*, xvi. 45; *SiŚi*, II, vii. 26.
3. *Cf. LG*, iii. 16; *SiŚe*, xvi. 53.
4. *Cf. LG*, iii. 17; *SiŚe*, xvi. 54.

time of rising of Aries	=	time of setting of Libra
time of rising of Taurus	=	time of setting of Scorpio
time of rising of Gemini	=	time of setting of Sagittarius
and		
time of rising of Cancer	=	time of setting of Capricorn
time of rising of Leo	=	time of setting of Aquarius
time of rising of Virgo	=	time of setting of Pisces
time of rising of Libra	=	time of setting of Aries
time of rising of Scorpio	=	time of setting of Taurus
time of rising of Sagittarius	=	time of setting of Gemini.

15. The signs which rise on the eastern horizon in the time-intervals obtained on diminishing or increasing their right ascensions by their own ascensional differences, set on the western horizon in the time-intervals respectively obtained on increasing or diminishing their right ascensions by their own ascensional differences.[1]

Let $\alpha_1, \alpha_2, \alpha_3$ be the right ascensions and c_1, c_2, c_3 the ascensional differences of Aries, Taurus and Gemini, respectively. Then the times of rising and setting of the signs are as exhibited below :

Sign	time of rising	time of setting	Sign
Aries	$\alpha_1 - c_1$	$\alpha_1 + c_1$	Pisces
Taurus	$\alpha_2 - c_2$	$\alpha_2 + c_2$	Aquarius
Gemini	$\alpha_3 - c_3$	$\alpha_3 + c_3$	Capricorn
Cancer	$\alpha_3 + c_3$	$\alpha_3 - c_3$	Sagittarius
Leo	$\alpha_2 + c_2$	$\alpha_2 - c_2$	Scorpio
Virgo	$\alpha_1 + c_1$	$\alpha_1 - c_1$	Libra

VISIBILITY AND INVISIBILITY OF THE SIGNS

16. The sign whose right ascension is equal to its ascensional difference is always visible at that place, and that sign remains (permanent-

1, *Cf LG*, iii. 18; *SiŚe*, xvi. 55.

ly) invisible at that place which is at the same declination (southwards) as the sign which is always visible there.

This rule was first given by Lalla,[1] and then by Vaṭeśvara and Śrīpati.[2]

The logic behind this rule seems to be as follows : Since the time of rising of a sign at a place

= right ascension of the sign — ascensional difference of the sign, (1)

therefore when

right ascension of the sign = ascensional difference of the sign

the right hand side of (1) reduces to zero, meaning thereby that the sign does not take any time in rising at that place. Vaṭeśvara infers from this that the sign remains permanently visible at that place.[3]

Bhāskara II has shown this to be incorrect. He writes :

"Lalla has declared that a sign would always be visible at a place where the right ascension of the sign is equal to its ascensional difference, but this assertion is without reason. If it were so, then at a place in latitude 66°, the right ascensions of the signs being the same as their ascensional differences, all the signs would always be seen simultaneously there. But this is not the fact. His assertion is therefore false."[4]

In fact, a sign having δ ($<24°$) for the declination of its initial point will be permanently visible at a place in north latitude $\phi = 90° - \delta$. Thus the sign Gemini (and also Cancer) will be permanently visible at a place in latitude 90° — 20°40′ or 69°20′N approx., the signs Taurus and Gemini (and also Cancer and Leo) will be permanently visible at a place in latitude 90° — 11°45′ or 78°15′N approx., and all the six signs beginning with Aries will be permanently visible at the north pole. Likewise the signs Sagittarius and Capricorn will be permanently invisible at a place in latitude 69°20′N, the signs Scorpio, Sagittarius, Capricorn and Aquarius will be permanently invisible at a place in latitude 78°15′N, and the six signs beginning with

1. *Cf. LG*, iii. 19, 21; *SiŚe*, xvi. 56, 59.
2. See *SiŚe*, xvi. 56, 59.
3. The correct inference is that the sign rises simultaneously at that place.
4. *SiŚi*, II, vii. 31. Also see com. on it.

Libra will be permanently invisible at the north pole. (But this will be the case when obliquity of the ecliptic is taken to be 24° and precession of the equinoxes is disregarded.)

17. Where the latitude amounts to 66 degrees, there the signs Capricorn and Sagittarius are not visible; and where the latitude amounts to 75 degrees, there the signs Aquarius, Scorpio, Sagittarius and Capricorn are always invisible.

This statement was also made for the first time by Lalla,[1] and then by Vaṭeśvara and Śrīpati[2].

The figures 66 degrees and 75 degrees mentioned in the statement are both incorrect and Bhāskara II has criticised Lalla for giving them. He says :

"Lalla has idly declared in his *Gola* (Spherics) that in latitude 66½° (? 66°), Sagittarius and Capricorn, and in latitude 75°, Scorpio and Aquarius too, would always remain invisible. Prompted by what consideration, say O proficient in Spherics, has he stated the figures lessened by three degrees."[3]

Varāhamihira, however, rightly says that Sagittarius and Capricorn never rise in latitude 69°24′;[4] and Scorpio, Sagittarius, Capricorn and Aquarius never rise in latitude 78°14′.[5]

So also says Bhāskara II :

"In those places where the latitude amounts to 69°20′, the signs Capricorn and Sagittarius are never visible, but the signs Cancer and Gemini are always visible. And in those places where the latitude amounts to 78°15′, the four signs Scorpio, Sagittarius, Capricorn and Aquarius are never seen but the four signs Taurus, Gemini, Cancer and Leo are always seen risen above the horizon."[6]

Vṛddha Vasiṣṭha too says the same.[7]

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1. See *LG*, iii. 20.
2. See *SiŚe*, xvi. 57.
3. *SiŚi*, II, vii. 32.
4. See *PSi*, xiii. 23.
5. See *PSi*, xiii. 24.
6. *SiŚi*, II, vii. 28-29.
7. See *VVSi*, xii. 109-111.

In view of the fact that the correct statements in regard to the permanent visibility or invisibility of the signs were made by Varāhamihira much earlier than the time of Lalla, it is difficult to account for the above erroneous statement of Lalla. Probably he did not have the opportunity of seeing the *Pañca-siddhāntikā* of Varāhamihira. As far as Śrīpati and Vaṭeśvara are concerned, they have behaved here as blind followers of Lalla.

18. Multiply the Earth's circumference (in *yojanas*) by the degrees of declination (of the heavenly body) and divide the product by 360 : the result gives the distance in *yojanas* at which that heavenly body rises (on the equatorial horizon) towards the north or south (of the east cardinal point). In case the declination pertains to the end-point of a sign, then the above result gives the distance in *yojanas* at which the end-point of that sign rises on the (equatorial) horizon towards the north or south (of the east cardinal point).

Lalla says: "The Sun, when at the end of the sign Aries, rises on the (equatorial) horizon 107 *yojanas* north (of the east cardinal point); when at the end of the sign Taurus, 189 *yojanas* north; and when at the end of the sign Gemini, 220 *yojanas* north."[1]

A similar statement has been made by Śrīpati.[2]

SOLAR ECLIPSE

19. The observer stationed at the centre of the Earth sees the Sun eclipsed by the Moon at the time of geocentric conjunction of the Sun and Moon; the observer situated on the surface of the Earth does not see him so (at that time).[3]

20. (The observer on the Earth's surface) sees with his eye the Sun eclipsed by the Moon in the eastern hemisphere prior to the time of geocentric conjunction of the Sun and Moon and in the western hemisphere after the time of geocentric conjunction of the Sun and Moon, because he is elevated above the Earth's centre.[4]

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1. *LG*, iii. 22. This result easily follows from Vaṭeśvara's rule by taking 3300 *yojanas* for the Earth's circumference and 703′, 1238′ and 1440′ for the declinations of the end-points of Aries, Taurus and Gemini, respectively.
2. See *SiŚe*, xvi. 60.
3. *Cf. LG*, iii. 23; *SiŚe*, xviii. 1(a-b).
4. *Cf. LG*, iii. 24-25(a-b); *SiŚe*, xviii. 1(c-d)-2(a-b).

21. It is for this reason that the *lambana* amounting to the Earth's semi-diameter, in terms of minutes of arc, is subtracted in the eastern hemisphere and added in the western hemisphere (when the solar eclipse happens to occur at the horizon).[1] At midday (when the Sun is at the zenith) there is no *lambana*, because then the lines of sight of the observers at the centre and surface of the Earth coincide.[2]

22. Whatever rationale has been given above for the *lambana*, measured east-west (i.e., along the ecliptic), when the *avanati* is supposed to be absent, a similar rationale holds also for the *avanati*, measured north-south (i.e., perpendicular to the ecliptic) when the *lambana* is absent.[3]

PHASES OF THE MOON

23. The Sun's rays reflected by the Moon destroy the thick darkness of the night just as the Sun's rays reflected by a clean mirror destroy the darkness inside a house.[4]

24. On the new moon day the Moon is dark; in the middle of the bright fortnight it is seen moving in the sky half-bright; on the full moon day it is seen completely bright as if parodying the face of a beautiful woman.[5]

25. In the dark and bright fortnights the dark and bright portions of the Moon (gradually) increase as the Moon respectively approaches and recedes from the Sun. The Sun (on the other hand) always looks bright (due to its own light).[6]

26. The asterisms, the Earth, and the planets including the stars, being illuminated by the rays of the Sun, shine (towards the Sun) like the Moon. On the other sides of their globes (which are not reached by the Sun's rays), they are indeed dark due to their own shadows.[7]

This is the general conception of the ancient Hindu astronomers. Āryabhaṭa I writes :

1. *Cf. LG*, iii. 25(c-d); *SiŚe*, xviii. 2(c-d).
2. *Cf. LG*, iii. 26; *SiŚe*, xviii. 3.
3. *Cf. LG*, iii. 27; *SiŚe*, xviii. 5.
4. *Cf. LG*, iii. 39; *PSi*, xiii. 36; *SiŚe*, x. 3.
5. *Cf. LG*, iii. 38(a-b).
6. *Cf. LG*, iii, 38(c-d).
7. *Cf. LG*. iii. 40; *Ā*, iv. 5; *SiŚe*, xviii. 14.

"Halves of the globes of the Earth, planets and stars are dark due to their own shadows, and the other halves, which face the Sun, are bright in proportion to their size."[1]

Lalla is more specific when he says that the Sun is the only source of light in the universe. He writes :

"The Sun alone gives light to all who live in this hollow of the universe—to all the regions described above, whether around the Earth, below the earth or above the earth ; to all, whether gods, demons, *Rākṣasas*, *Bhūtas*, *Piśācas*, *Kinnaras*, *Vidyādharas*, *Nāyakas*, *Pitṛs*, *Siddhas*, sages or men ; and to all the heavenly bodies."[2]

27. One half of the stars and planets, which lie above the Sun (i.e., which are at a greater distance than the Sun), is illumined by the Sun and is seen bright by the people. Mercury and Venus too, even though they are below the Sun (i. e., at a lesser distance than the Sun), do not look dark like the Moon: this is so because they are near the Sun (and wholly covered by the dazzling light of the Sun).[3]

ELEVATION OF LUNAR HORNS

28. The (sum or) difference (as the case may be) of the *bhujas* of the *lagna* ("horizon ecliptic point") and the Moon, in the eastern or western hemisphere, is the base (of the *śṛṅgonnati* triangle) and the Rsine of the Moon's altitude is the upright (of the same triangle). This is why the square-root of the sum of their squares is called the hypotenuse.[4]

This is applicable when the elevation of the Moon's horns is obtained for the time of sunrise or sunset.

29. When the Moon is in the western hemisphere, the upright is towards the east; and when the Moon is in the eastern hemisphere, it is towards the west. This is the reason that the Moon's horn lying towards the horizon ecliptic point is elevated (and the other one depressed).[5]

1. *Ā*, iv. 5.
2. *LG*, vi. 45.
3. *Cf. LG*, iii. 41-42; *Ā*, iv. 5; *SiŚe*, xviii. 15.
4. *Cf. LG*, iii. 43. Also see *ŚiDVṛ*, ix. 10-11.
5. See *ŚiDVṛ*, ix. 15(c-d).

APPEARANCE OF THE SUN

30. At midday the observer is brought nearer to the Sun by half the Earth's diameter, even then he does not comfortably see the Sun. This is so because the central globe (of the Sun) is lost within the brilliant rays of the Sun.[1]

31. When the Sun is on the horizon it is at a greater distance, but, its rays being obstructed by (the atmosphere of) the Earth, it is comfortably seen (by the observer). The same Sun which looks reddish and large (on the horizon) becomes brilliant and tiny when in the middle of the sky, because then it is surrounded by the multitude of its rays.[2]

VISIBILITY CORRECTION : PLANETARY CONJUNCTION

32. The visibility correction is similar to the *cara* correction. It should be obtained for the eastern or western horizon from the planet's celestial latitude and applied to the planet concerned according to the prescribed rules.[3]

When the celestial latitudes of two planets, (which are in conjunction in longitude), are equal in magnitude and direction, they move along the same diurnal circle (lit. path).[4]

CONCLUDING STANZA

33. This rationale has been briefly told by me. A different one may be added by seeing the armillary sphere. Although the teachings of a scientific work are brief, one enlarges and elaborates them with one's own intellect, just as (a drop of) oil spreads (extensively) on water.[5]

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1. *Cf. LG*, iii. 44; *SiŚe*, xviii. 11.
2. *Cf. LG*, iii. 46(c-d)-47(a-b); *SiŚe*, xviii. 13.
3. *Cf. LG*, iii. 48(c-d); *SiŚe*, xviii. 16(c-d).
4. *Cf. LG*, iii. 49(a-b).
5. *Cf. LG*, iii. 50; *SiŚe*, xviii. 17; *SiŚi*, II, x. 6.

Chapter V

The Terrestrial Globe

CAUSE OF EARTH'S CREATION

1. The stationary glode of the Earth, which is made of sky, air, fire, water and earth and is well surrounded by the stars and planets, has been created in the sky on account of the good and bad deeds of the human beings.[1]

It was believed that the Earth was *karmabhūmi* where people performed good or bad deeds and suffered the consequences of those deeds.

EARTH SPHERICAL AND SUPPORTLESS IN SPACE, NOT FALLING DOWN

2. Just as an iron ball surrounded by pieces of magnet does not fall though standing (supportless) in the sky, in the same way this (globe of the) Earth though supportless does not fall as it is prevented by (the attraction of) the stars and planets.[2]

3. If you are inclined to believe that it falls down, say what is up and down for an object standing in space. The globe of the Earth does not come in contact with the planets and the stars, in what direction should it then fall ?[3]

4. The quarters where the setting and rising of the Sun take place are those of Varuṇa and Indra (i. e., west and east), (respectively); those of Yama and Soma (i. e., south and north) are dependant on the Pole Star (Dhruva); vertically below that (Pole Star) are situated the demons; and above the Earth there are asterisms and the expanse of the sky.[4]

The intention of saying this is that the directions east, west, north, south, above and below are defined with respect to the observer on the Earth's surface. Actually, there is no direction which may be called

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1. *Cf. Ā*, iv. 6, but Āryabhaṭa I does not say that the Earth is stationary; *BrSpSi*, xxi. 2; *LG*, iv. 1; *SiŚe*, xv. 23; *SiŚi*, II, iii. 2(a-b); *Golasāra*, ii. 1.
2. *Cf. LG*, iv. 2; *PSi*, xiii. 1; *SiŚe*, xv. 22; *SiŚi*, II, iii. 2(a-c).
3. *Cf. LG*, vii. 38. Also *cf. MSi*, xvi. 8(a-b).
4. This seems to have been substituted in place of *LG*, iv. 5.

above or below (up or down) in relation to the Earth. So the question of the Earth falling down does not arise.

5. If the Earth is supported by *Śeṣa*, tortoise, mountains, and elephants etc., how do they stand supportless (in space)? If they are believed to be endowed with some power, why is not the same power assigned to the Earth ?[1]

6. Why should the Earth be supported by others, when it itself supports the entire human beings ? That it does not move also on account of the boon of the Lotus-born has been stated by the other learned people.[2]

7. As here in our locality a flame of fire goes aloft in the sky and a heavy mass falls towards the Earth, so is the case in every locality around the Earth. As there does not exist a lower surface (for the Earth to fall upon), where should it fall ?[3]

Since every heavy thing was seen falling towards the Earth and the Earth itself was heavy, the followers of the Buddha thought that, like all heavy things, the Earth was also falling down (although this was not felt by the people on the Earth). This conception of the followers of the Buddha has been contradicted in vss. 2, 3 and 7 above.

8. Just as a house lizard runs about on the surface of a pitcher lying in open space, so do the human beings move about comfortably all around the Earth.[4]

9. Just as warmth is the natural property of the Sun and fire, motion that of water, coolness that of the Moon, and hardness that of stone, in the same way to remain suspended in space is undoubtedly the natural attribute of the Earth.[5]

CITIES ON THE EQUATOR AND ABODES OF GODS AND DEMONS

10. Diametrically below Laṅkā lies Siddhapura, towards the east (of Laṅkā) lies Yamakoṭi, towards the west lies Romaka, towards the

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1. *Cf. LG*, vii. 41; *SiŚi*, II, iii. 4.
2. *Cf. LG*, iv. 10.
3. *Cf. LG*, iv. 8; *SiŚi*, II, iii. 6.
4. *Cf. LG*, iv. 7; *MSi*, xvi. 7(c-d); *SiŚe*, xv. 28.
5. *Cf. LG*, iv. 9; *SiŚe*, xv. 21; *SiŚi*, II, iii. 5.

south lies the habitat of the demons, and towards the north lies Meru, the abode of the gods.[1]

11. The abode of the demons is amidst water and the Meru is in the midst of land. Men who reside on the (common) boundary of land and water, at the distance of one-fourth of the Earth's circumference from each other, mutually consider themselves as standing at right angles.[2]

The ancient people believed that half the Earth lying north of the equator was land and the other half lying south of the equator was water. The cities known as Laṅkā, Yamakoṭī, Siddhapura and Romaka were supposed to be on the equator successively eastwards at the distance of one-fourth of the Earth's circumference. The abodes of the gods and the demons were supposed to be the north and south poles, respectively.

EARTH LIKE THE BULB OF THE KADAMBA FLOWER

12. Just as (the bulb of) the Kadamba flower is covered all around by filaments, so is the Earth surrounded on all sides by gods, men, reptiles, and other creatures.[3]

1. *Cf. LG*, iv. 3; *MSi*, xvi. 6(c-d); i. 3(a-b), 57; *SiŚe*, xv. 30.
2. *Cf. LG*, iv. 4; *Ā*, iv. 12(a-b); *MSi*, xvi. 7(a-b).
3. *Cf. LG*, iv. 6; *Ā*, iv. 7; *SiŚe*, xv. 8(c-d); *SiŚi*, II, iii. 2(b)-3; *SuSi*, II, i. 22.